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FOUNDATIONS OF ENGINEERING SCIENCE

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PREFACE

This book gives an account of the fundamental principles of Engineering Science.

It is based largely upon lectures I have given to National Certificate students at the South-West Essex Technical College, but also contains certain other material not normally included in such courses, being coloured to an extent by lectures I gave to engineering trainees during part of the late war, and by my own experience as a practising engineer.

The work should prove suitable for those taking National Certificate or similar courses, preparing for the examinations of the professional engineering institutions, or studying privately. The requirements of the Junior Technical School have also been kept in mind.

In particular, its scope is such as to cover the syllabuses of most National Certificate courses (including any preliminary work required) up to at least the end of the second year. It includes all the Engineering Science normally taken during the complete Ordinary National Certificate course in Electrical Engineering. Owing to variations in the syllabuses of different colleges, &c., it may be found that it also covers the bulk of the syllabus of many third-year courses in Mechanical Engineering. However, it is not intended specifically to include all the work in Applied Mechanics required for the Section A examinations of the professional engineering institutions. The Section A examination syllabus is often used as a basis for many third-year Ordinary National Certificate courses in Mechanical Engineering, and may be used during certain Higher National Certificate courses. This book should, however, cover the syllabus in Elementary Mechanics (and the appropriate parts of Elementary Physics) of the Common Preliminary Examination of the Engineering Joint Examination Board. This Common Preliminary Examination may, in general, be taken as an alternative to the National Certificate examination in

order to qualify for studentship and other grades of the various professional engineering institutions.

Owing to limitations of space, elementary electricity has not been covered in this volume, although it is included in the initial stages of some National Certificate Engineering Science courses before specialization begins.

A large number of examples, both worked and unworked, have been included. Many of these have been taken from first- and second-year National Certificate sessional examination papers set by such representative bodies as the Union of Lancashire and Cheshire Institutes, the Union of Educational Institutions, Birmingham, the South-West Essex Technical College, Walthamstow, &c. Others have been taken from the papers in Elementary Mechanics and Elementary Physics of the Common Preliminary Examination of the Engineering Joint Examination Board.

In some cases, where the subject-matter of a chapter may be applicable to more than one year of a course, it will be seen that the exercises have been divided into two sections.

The source of question has been acknowledged by the use of appropriate initials after each, and thanks are tendered to the bodies concerned for the kind permission given me to use their questions.

It is hoped that the book will prove of use and interest to all requiring a knowledge of the Foundations of Engineering Science.

L. PILBOROUGH.

WOODFORD GREEN,
1952.

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CHAPTER I

Introductory

1. Engineering

To the man in the street, the word "engineering" conjures up a variety of impressions. Pictures come readily to his mind of a speeding steam locomotive, or of a motor car. Mixed with this is perhaps a recollection of a busy engineering works, with its humming wheels, the smell of oil, and a certain measure of dirt, both of the necessary and of the avoidable kind. Maybe he thinks of a bridge spanning a river, or of a power station, or of an aeroplane in flight. Very few would immediately think of the small components in a radio set, the manufacture of the parts of a clock, or an electric lamp. All of the above, and many more things besides, come within the scope of engineering.

Also to the man in the street the term "engineer" usually implies a figure clad in overalls, rather dirty and oily. Much confusion has arisen around this point, because, although a true engineer may wear overalls, and may be dirty (in fact at some time during his career this will most certainly have been the case), it does not necessarily follow that someone in this condition is an engineer, just because he is associated with engineering. Quite often he is a skilled mechanic, an artisan, or even just a machine-minder. An engineer may also be a mechanic, but a mechanic is not necessarily an engineer.

The true engineer is able to direct and control the forces of nature by designing, constructing, and maintaining various machines and devices. He is able to appreciate the laws that govern their operation. Perhaps the best idea of his function is that implied by the Charter of the Institution of Civil Engineers—one conversant with "the art of directing the great sources of power in Nature for the use and convenience of man".

2. Main branches of engineering

We may divide the work of the engineer into three main branches—civil, mechanical and electrical engineering.

The term “civil engineering” was originally intended to embrace all forms of engineering other than the purely military side (in fact it still does). To-day, however, the term is commonly used in a more restrictive sense to imply work in connection with roads, docks, water supply, and structures generally. As his work is often closely connected with earth, and the movement of earth, the civil engineer is sometimes impolitely but expressively referred to as a “muck-shifter”.

The “mechanical engineer” is concerned with the design, construction, and operation of power plant, machinery, and transport of all kinds, and especially with the methods employed in all branches of manufacture.

The “electrical engineer” is concerned with the design, construction, and operation of all forms of plant and equipment making use of electrical principles. Two main subdivisions are apparent here:

1. Work in connection with the generation, transmission, distribution, and utilization of electrical energy—sometimes termed “heavy-current electrical engineering”.

2. Work in connection with electrical communications, involving telephone and telegraph systems, radio, and radar.

The term “light-current electrical engineering” is used to designate work in connection with electrical appliances, meters, instruments, signalling and lighting equipment, but it is often meant to include electrical communication work also.

3. The three main groups of engineering personnel

The technical personnel of the engineering industry can be regarded as falling into three main groups:*

- (1) Craftsmen, whose skill is essentially manual.

- (2) Technicians, whose skill lies in the practical application of a knowledge of engineering technique which is limited to a particular field. This group embraces draughtsmen, erection engineers, technical assistants, and testers.

- (3) Professional engineers, with ability to direct progress,

* See the report by The Institution of Electrical Engineers on “Education and Training for Engineers”.

devise methods for the solution of the problems which arise in connection with every new development, and control the application of these methods in accordance with scientific principles.

It is to this last, and smallest, group of fully qualified professional engineers that the true engineer, as defined by the Charter of The Institution of Civil Engineers, most rightly belongs.

4. Engineering institutions

A fully qualified member of the engineering profession is usually a corporate member* of at least one of the three major engineering institutions—The Institution of Civil Engineers, The Institution of Mechanical Engineers, and The Institution of Electrical Engineers.

The object of these institutions is to promote the general advancement of engineering science and its applications, and to facilitate the exchange of information and ideas. To this end, meetings are held in various centres, at which developments are discussed, and reports and other publications are issued.

The premier engineering institution is that of the civil engineers. It was founded in 1818, and incorporated by Royal Charter in 1828. The other two institutions are also incorporated by Royal Charter, hence a corporate member of these chartered institutions is often referred to as a “chartered civil, mechanical or electrical engineer”.

The qualifications required for corporate membership are, in general, threefold: involving recognized periods of technical education, practical training, and responsible experience in the profession. Junior grades of membership, such as “students” and “graduates”, are open to those engaged in the profession, a “graduate” being a person who has obtained the necessary technical education, but who has not yet obtained the required training or experience.

Everyone taking up engineering seriously can aim at becoming a corporate member of the appropriate institution. The educational requirements can be met by taking a university course in engineering, or a Higher National Certificate or similar course in engineering, and/or passing part or the whole

*Corporate member means (*inter alia*) an “associate member” or a “member” (e.g. A.M.I.C.E., M.I.E.E.)

of the particular institution's examinations. As the regulations are often altered, the appropriate institution should be consulted for the latest particulars.

The required practical training is usually obtained by means of some form of pupilage or apprenticeship in an engineering works.

After the acquisition of the necessary training and responsible experience, a graduate can apply for corporate membership, as an "associate member" of his particular institution.

The more specialized branches of engineering have their own institutions. A short, but not exhaustive, list would include those dealing with the following branches: structural, marine, aeronautical, radio, inspection, production and chemical engineering.

5. Science

Natural science is that branch of human knowledge which deals with the properties of the material world. It can be divided into three main sections: **physics** and **chemistry**, which deal with the properties of inert matter, and **biology**, which deals with the properties of living matter.

The engineer is not normally concerned with the study of biology (although occasions do arise when such knowledge is required), but a good knowledge of physics, and perhaps a more general knowledge of chemistry, is essential.

Physics deals with properties of inert matter such as its state and condition, and with the effects of forces on it; in particular, it deals with the relationship between matter and energy.

Chemistry deals with the composition of inert matter, and with changes in its composition which result in the formation of substances possessing properties different from the original. The line of demarcation between the two is not very distinct, there being for instance a branch known as *physical chemistry*.

Physical science has been called an organized knowledge of facts and numerical relationships regarding natural phenomena. Qualitative and quantitative relationships must therefore be known. It must be possible to observe the same phenomena when the same conditions are repeated. Consequently, methods of measuring the various physical quantities to a high standard of accuracy must be available. In fact, as once stated by Lord Kelvin, "science is measurement". Thus any branch of

knowledge which does not permit of measurement is not, strictly speaking, a science.

Engineering science

Engineering science deals with those parts of scientific knowledge which are essential to the understanding of the engineer's activities. It embraces physics (and sometimes a little chemistry) and its applications to engineering.

The branches of physics usually included in the study of engineering science, especially in those courses leading up to the award of a National Certificate or Diploma in Mechanical or Electrical Engineering, or to the examinations of the professional engineering institutions, are:

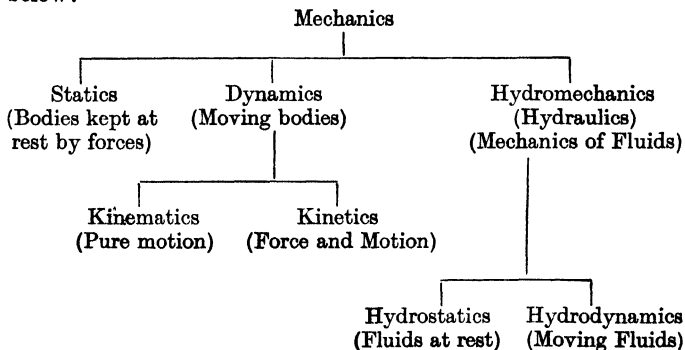
1. General Properties of Matter.
2. Mechanics (including Hydromechanics).
3. Heat (including Heat Engines).
4. Magnetism and Electricity.

A more general course may also include study of those branches dealing with Light and Sound.

Mechanics

Mechanics was originally defined as the "science of machines". Although this description is still apt, it is usual to-day to describe "mechanics" as that branch of physics dealing with the action of forces on bodies, and with the motion which these may produce.

An outline of the various branches of mechanics is given below:



Mechanics is one of the oldest branches of science, and many of its principles have been used since the birth of civilization. The lever, inclined plane, and screw were known and used in Ancient Egypt and Greece (and perhaps in China). The Greek philosophers (c. 200 B.C.) left writings concerning them.

It is not until the last three centuries, however, that any further substantial progress can be recorded. Galileo (c. 1600) experimented with falling bodies from the famous leaning tower of Pisa. Sir Isaac Newton, who can be called the father of modern mechanics, analysed the results of experiments on moving bodies and propounded his famous Laws of Motion, which were first published in his *Principia* in 1687. These laws form the basis of modern mechanics.

Heat

Heat being a form of energy, its study forms a branch of physics. It can conveniently be considered under the following heads:

1. Thermometry.—Dealing with temperature, its measurement, and its effects upon matter.
2. Calorimetry.—Dealing with quantity of heat and its measurement.
3. Thermodynamics.—Dealing with heat in motion and its relationship with energy.
4. Fuels and Combustion.—Dealing with sources of heat and the mechanism of burning.
5. Heat Engines.—Dealing with details of the methods and machines used in converting heat to work.

In this volume, after some discussion concerning matter, we will deal with elementary mechanics (including hydraulics) and heat.

6. Relationship between scientists and engineers

In many respects the education of the scientist (at least the physical scientist) and the engineer follow parallel lines. The scientist, however, usually goes more deeply into theory, and the engineer more deeply into practical applications. It is usual to consider the scientist as being concerned only with research and development, and the engineer with the putting of these ideas into practical form. This is not entirely true, as their functions overlap considerably, e.g. an engineer may be

employed on research work, or a scientist on practical operational work. There is no hard dividing line; it depends so largely on the individual. It should be realized, however, that their functions are complementary, both being necessary to the successful completion of the work.

A scientist may produce an idea and make a rough sketch of it (some scientists *cannot* draw). The engineer tries to understand this idea, makes practical suggestions, and then makes up practical working drawings (an engineer *can* draw). The engineer's practical knowledge is used to ensure that the article can be made, and that it can be made economically.

It sometimes happens that a person is both a scientist and an engineer. Usually, however, this person has specialized more deeply in one profession than in the other.

CHAPTER II

Fundamental Quantities, Units, and Dimensions

1. Fundamental quantities

Most physical quantities can be expressed in terms of what are known as the fundamental quantities of **distance**, **mass**,* and **time**. In order to consider these quantities we need to know something of their magnitude, and this can be determined by measuring them against some **standard of reference**. Thus, given a foot rule, we can obtain a measure of the distance across a room. Therefore the **foot** may be considered as a fundamental **unit** of distance.

2. Fundamental units

Two systems of fundamental units have been in common use for many years. These are:

1. The British system (or F.P.S. system).
2. The metric system (or C.G.S. system).

The fundamental units adopted in each case are:

	Distance	Mass	Time
British System	Foot	Pound	Second
Metric System	Centimetre	Gram †	Second

Multiples and submultiples of these units are used. In the British system, different names are given to them (e.g. inch, yard, mile), and their relationship to the fundamental unit

* Mass denotes the quantity of matter in a body, and is proportional to its weight (see Chap. III).

† Although a French word correctly spelt "gramme", it has become common practice in English-speaking countries to spell it as "gram".

does not follow a simple plan. Consequently tables of British units have usually to be learnt by heart.

With the metric system, however, as it is a **decimal** system, it is sufficient to learn the various prefixes employed with each unit (see Table 1).

TABLE 1

THE METRIC SYSTEM

Prefixes used for Multiples and Submultiples of Units

Prefix	Abbreviation	Multiple or Submultiple
Mega-	M.	Million (10^6)
Kilo-	k.	Thousand (10^3)
Hekto-	H.	Hundred (10^2)
Deka-	D. or dk.	Ten (10)
Deci-	d.	Tenth (10^{-1})
Centi-	c.	Hundredth (10^{-2})
Milli-	m.	Thousandth (10^{-3})
Micro-	μ .	Millionth (10^{-6})
Millimicro-	m μ .	Thousand-millionth (10^{-9})
Micro-micro (or pico)	$\mu\mu$. or p.	Million-millionth (10^{-12})

The prefix *myria*, meaning 10,000, is sometimes used, e.g. myria-metre.

It will be noticed that the **second** is the unit of time in both systems. This unit, known as the **mean solar second**, is obtained by means of solar observation. Depending as it does on natural phenomena, it may be termed a **natural unit**. The duration of a mean solar second is 1: 86,400 of the *average* length of a **solar day** (1 day = $24 \times 60 \times 60$ seconds). A solar day is the period between successive transits of the sun across the meridian at any point on the earth's surface. It varies according to the time of year; so an average value is taken.

It is of interest to note that the metric system was originally designed to be a natural system of units. The metre was originally defined as one ten-millionth part of the length of the arc of the earth's meridian between the equator and the pole, passing through Paris; and the gram was defined as the mass

of a cubic centimetre of water at 4° C. (its temperature of maximum density).

The original calculations have now been found to be slightly inaccurate, so therefore, for legal purposes, the **international metre** is defined as the distance between two lines on a platinum-iridium bar, kept at Sèvres near Paris, when measured at 0° C. (the temperature of melting ice), this bar being the original standard metre. Similarly, the **gram** is now legally defined by reference to the original standard **kilogram**, made of platinum-iridium, and preserved at Sèvres.*

It will thus be observed that these units in the metric system are "arbitrary units", as they do not depend upon any natural data.

The corresponding units in the British system are also arbitrary, and are defined by reference to certain standards kept by the Board of Trade. Thus, the **imperial standard yard** is the distance between two marks in two gold plugs inserted in a certain bronze bar, when measured at 62° F., whilst the **imperial standard pound** is the legal standard of weight, and hence for the comparison of mass.

Legend has it that the yard was fixed as the length of the arm of Henry I!

3. Derived units

Combinations of the three fundamental units are used to form other dimensional units, which are known as "derived units". Thus a combination of the units of distance and time gives a unit of velocity (speed); a combination of distance, mass, and time gives a unit of force, and so on. It will be observed that the squares of the units of distance give the units of area, and their cubes that of volume. Various derived units and their relationship with the fundamental units are given in Table 2 (p. 13), together with the actual units employed in the different systems of units. The significance of these units and expressions will be better appreciated at a later stage, when the various definitions have been mastered.

4. Conversion factors

The relationship between F.P.S. and C.G.S. units is obviously of importance to science and commerce. The exact ratio

* It having been discovered that the mass of this original standard kilogram was not exactly that of the intended 1000 c.c. of water at 4° C.

between the units of distance in the two systems has been twice determined with high experimental precision, once in 1895 by co-operation between the Standards Department of the Board of Trade and the Bureau International des Poids et Mesures, Sèvres, and again in 1922-4 by co-operation between the National Physical Laboratory and the same two authorities.

The results obtained were:

In 1895 1 metre = 39.370113 inches.

In 1922-4 1 metre = 39.370147 inches.

America has legalized the conversion factor

1 metre = 39.37 inches.

In Great Britain the legalized conversion factor for use in trade is

1 inch = 25.4 millimetres.

These two conversion factors agree with each other very closely, and certainly are sufficiently accurate for all engineering purposes.

The legal relations of other British Imperial to International Metric Measures adopted by the Standards Office of the Board of Trade are:

1 pound = 453.59243 grams.

1 kilogram = 2.20462 pounds.

1 Imperial gallon = 4.5459631 litres.

Both the gallon and the litre are measures of cubic capacity. An Imperial gallon contains 10 lb. weight of distilled water at 62° F. (and at an air pressure of 30 in. of mercury). The litre is the volume occupied by 1 kg. of distilled water at 4° C. (and at an air pressure of 760 mm. of mercury).

1 litre (1000 millilitres) = 1000.028 c.c.

If a cubic centimetre of water weighed *exactly* one gram*, then a litre would occupy a volume of exactly 1000 c.c. In all practical calculations this is assumed to be the case, but for accurate conversions the difference should be remembered.

N.B. It should be noted that 1 c.c. of water (at 4° C.) for all

* See footnote on p. 10.

practical purposes is taken as weighing 1 gram, but 1 millilitre (ml.) of water (at 4° C.) weighs, by definition, exactly 1 gram.

5. The M.K.S. system of units

This is a system of fundamental units based on the Metre, Kilogram, and Second which was recommended by the International Electrotechnical Commission in 1935, and subsequently adopted in 1938. At first sight it appears little different from the C.G.S. system, except that the magnitudes of many of its derived units are better from the practical engineering viewpoint. When its electrical units come to be considered, however, it will be found that they are precisely the same as the practical units in common use (volts, amperes, joules, watts, etc.), there being thus a straightforward relationship between these and the fundamental units.

6. Units and dimensions

Table 2 shows derived units expressed in terms of fundamental units.

Thus the unit of area	= (unit of length) ² = L ² .
„ „ volume	= (unit of length) ³ = L ³ .
„ „ density	= $\frac{\text{unit of mass}}{\text{unit of volume}} = \frac{M}{L^3}$ or ML ⁻³ .
„ „ velocity (speed)	= $\frac{\text{unit of length}}{\text{unit of time}} = \frac{L}{T}$ or LT ⁻¹ .
„ „ acceleration	= $\frac{\text{unit of velocity}}{\text{unit of time}} = \frac{L}{T} \div T$ = $\frac{L}{T^2}$ or LT ⁻² ,

and so on (refer to the table).

The *powers* to which these fundamental units are raised in order to obtain the unit of any other quantity are called the **dimensions** of that quantity, and the resulting expression is called its **dimensional equation**, e.g. the dimensions of density are 1 in mass, and -3 in length.

The dimensional equation can readily be found if the defining equation is known. This defining equation follows from the definition of the quantity concerned.

TABLE 2.—FUNDAMENTAL AND DERIVED UNITS

Unit	Symbol	Fundamental relationship (or dimensional equation)	Defining equation	Units		
				F.P.S.	C.G.S.	M.K.S.
Length	l	Fundamental		foot	cm.	metre
Mass	m	"		pound	gram	kg.
Time	t	"		sec.	sec.	sec.
Area	A	L^2	$l \times l$	sq. ft.	sq. cm.	sq. m.
Volume	V	L^3	$l \times l \times l$	cu. ft.	c.c.	cu. m.
Density	D	ML^{-3}	$\frac{m}{V}$	lb./cu. ft.	gram/c.c.	kg./cu. m.
Velocity	v	LT^{-1}	$\frac{l}{t}$	ft./sec.	cm./sec.	m./sec.
Acceleration	a (or f)	LT^{-2}	$\frac{v}{t}$	ft./sec./sec.	cm./sec./sec.	m./sec./sec.
Force	F	MLT^{-2}	$m \times a$	poundal	dyne	newton
Work or energy	W	ML^2T^{-2}	$F \times l$	ft.-lb.	erg	joule
Power	P	ML^2T^{-3}	$\frac{W}{t}$	ft.-lb./sec	erg/sec.	watt
Momentum	—	MLT^{-1}	$m \times v$	lb.-ft./sec.	g.-cm./sec.	kg.-m./sec.
Angle	α	$LL^{-1} = 1$ (Dimensionless)	$\frac{\text{arc}}{\text{radius}}$	degree and radian		
Moment (torque)	M	ML^2T^{-2}	$\frac{W}{\alpha}$	lb.-ft.	dyne-cm.	newton-metre

Dimensional equations may be used:

1. To check the accuracy of any expression or formula.
2. To convert from one system of units to another.

The subject can, however, be considered further at a later stage.

The rules for the calculation of areas and volumes are given below for reference.

7. Calculation of area (fig. 2.1)

Square.—Multiply lengths of 2 sides together. As these lengths are equal, then

$$\text{area} = a \times a = a^2.$$

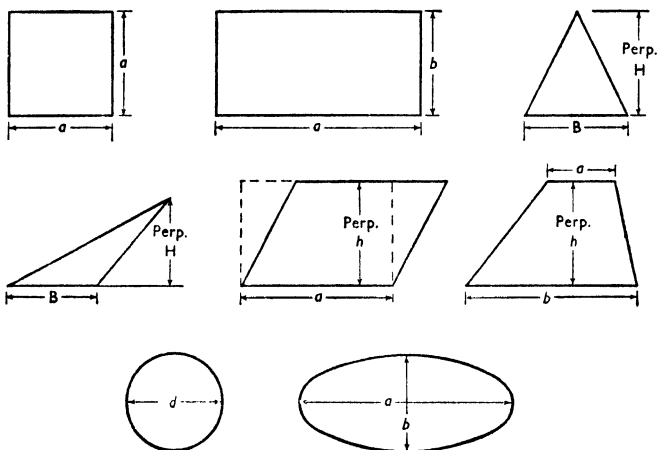


Fig. 2.1

Rectangle.—Multiply length by breadth.

$$\text{area} = a \times b.$$

Triangle.—Multiply $\frac{1}{2}$ (length of base) by perpendicular height.

$$\text{Area} = \frac{1}{2}B \times H.$$

Parallelogram.—Multiply length of one side by perpendicular distance between this side and opposite side.

$$\text{Area} = a \times h.$$

Trapezium.—Multiply $\frac{1}{2}$ (sum of parallel sides) by perpendicular distance between them.

$$\text{Area} = \frac{1}{2}(a + b) \times h.$$

Circle.—Multiply square of radius by π (pi) where $\pi = 3.1416$ (usually taken as 3.142 or $\frac{22}{7}$).

$$\text{Area} = \pi r^2, \text{ or } \frac{1}{4}\pi d^2, \text{ or } 0.7854d^2.$$

Ellipse.—Multiply product of major and minor axes by $\frac{1}{4}\pi$.

$$\text{Area} = \frac{1}{4}\pi(a \times b).$$

N.B. If dimensions are given in inches, then the areas will be in square inches.

Irregular Figure

Method 1.—Mean-ordinate rule (fig. 2.2).

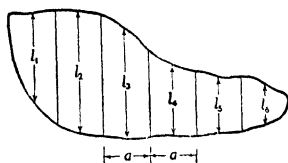


Fig. 2.2.—Area by mean-ordinate rule

Divide figure into any number of strips of equal width a . Measure the length of each strip along its mean ordinate (lengths l_1, l_2, l_3 , etc.), then

$$\text{Area} = a(l_1 + l_2 + l_3 + \text{etc.}).$$

Method 2.—Simpson's rule (fig. 2.3).

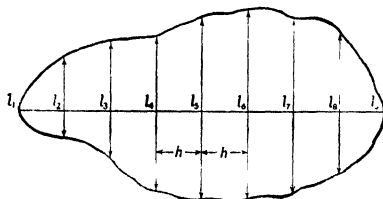


Fig. 2.3.—Area by Simpson's rule

Draw a line connecting the extreme points of the area.

Divide this into an even number of parts (e.g. 8 strips of width h in the figure). Erect ordinates at right angles to the line and measure their lengths (l_1, l_2, l_3 , etc.). Then

$$\text{Area} = \frac{1}{3}h \left[\begin{array}{l} \text{sum of first} \\ \text{and last} \\ \text{ordinates} \end{array} + 2 \left(\begin{array}{l} \text{sum of odd} \\ \text{ordinates} \end{array} \right) + 4 \left(\begin{array}{l} \text{sum of even} \\ \text{ordinates} \end{array} \right) \right].$$

In fig. 2.3,

$$\text{Area} = \frac{1}{3}h[(l_1 + l_9) + 2(l_3 + l_5 + l_7) + 4(l_2 + l_4 + l_6 + l_8)].$$

N.B. One or both of the end ordinates may be of zero length, but they must still be considered.

Method 3.—Determination of area by a planimeter (see Ch. V, § 16.)

Method 4.—Determination of area by counting squares.

Draw the irregular area on squared paper, or place squared tracing paper over it.

Count the number of small squares included in the area.

(Estimate the number of complete small squares which could be made up from incomplete small squares.)

Knowing the area of a small square, the area of the irregular figure is then readily obtained.

8. Calculation of volume

Cube.—If length of side is a units, then volume = a^3 .

Prism (Parallelopiped).—Multiply area of one end by length.

$$\text{Volume} = a \times b \times l.$$

Pyramid.—Multiply base area by $\frac{1}{3}$ (perpendicular height).

$$\text{Volume} = \frac{1}{3}h(\text{base area}).$$

Cylinder.—Multiply base area by length.

$$\text{Volume} = \pi r^2 h.$$

Cone.—Multiply base area by $\frac{1}{3}$ (perpendicular height).

$$\text{Volume} = \frac{1}{3}\pi r^2 h.$$

Sphere.—

$$\text{Volume} = \frac{4}{3}\pi r^3.$$

N.B. If the dimensions are given in inches, then the volumes will be in cubic inches.

9. Angular measure

The most commonly used unit of angle is the **degree**, which is $\frac{1}{90}$ of the divergence between two lines at right angles to each other. Therefore a circle contains 360 degrees ($^{\circ}$). This degree is divided into 60 parts called **minutes**, and again the minute is divided into 60 parts called **seconds**.

Another unit of angle, used to relate distance with angle, is the **radian**. This may be defined as the angle between two radii of a circle when joined by an arc at the circumference equal in length to the radius (fig. 2.4). The value of any angle expressed in radians is given by (length of arc $\div$ radius).

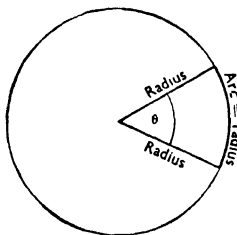


Fig. 2.4

As the circumference of a circle = $2\pi r$, and it encloses 360° , then $\frac{\text{length of arc}}{\text{radius}}$ for a complete circle = $\frac{2\pi r}{r} = 2\pi = \text{number of radians in } 360^{\circ}$. Then as

$$2\pi \text{ radians} = 360^{\circ},$$

$$1 \text{ radian} = \frac{360}{2\pi} = 57.29^{\circ} \text{ (approx.)}.$$

10. Trigonometrical ratios (see fig. 2.5)

Sine.—The ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ or $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{AB}$ is known as the sine of the angle theta (θ) (angle ABC), i.e.

$$\sin \theta = \frac{AC}{AB}.$$

Cosine.—The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ or $\frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AB}$ is known as the cosine of the angle θ , i.e.

$$\cos \theta = \frac{BC}{AB}.$$

Tangent.—The ratio $\frac{\text{opposite}}{\text{adjacent}}$ or $\frac{\text{perpendicular}}{\text{base}} = \frac{AC}{BC}$ is known as the tangent of the angle θ , i.e.

$$\tan \theta = \frac{AC}{BC}.$$

Cosecant.—The reciprocal of $\sin \theta$ is $\frac{1}{\sin \theta} = \frac{AB}{AC}$
 $= \text{cosecant } \theta \text{ (cosec } \theta \text{)}.$

Secant.—The reciprocal of $\cos \theta$ is $\frac{1}{\cos \theta} = \frac{AB}{BC}$
 $= \text{secant } \theta \text{ (sec } \theta \text{)}.$

Cotangent.—The reciprocal of $\tan \theta$ is $\frac{1}{\tan \theta} = \frac{BC}{AC}$
 $= \text{cotangent } \theta \text{ (cot } \theta \text{)}.$

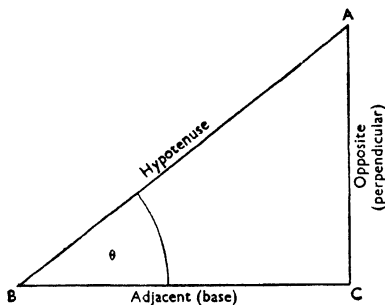


Fig. 2.5

Knowing the length of the hypotenuse AB, the lengths of the other two sides are:

$$AC = AB \sin \theta.$$

$$BC = AB \cos \theta.$$

Pythagoras' theorem states that "In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides",

i.e. $(AB)^2 = (BC)^2 + (AC)^2$ (see fig. 2.5).

Substituting from the previous paragraph,

$$(AB)^2 = (AB \cos \theta)^2 + (AB \sin \theta)^2$$

If the hypotenuse AB is considered to be of unit length, then

$$1^2 = (1 \cos \theta)^2 + (1 \sin \theta)^2,$$

or

$$\sin^2 \theta + \cos^2 \theta = 1.$$

EXERCISES ON CHAPTER II

1. Explain the difference between fundamental and derived units, giving examples of each.
2. Write a short account of the C.G.S. system of units. Explain its main distinction from the M.K.S. system.
3. Find the areas of: (a) A circle $4\frac{1}{4}$ in. diameter by the mid ordinate rule. (b) A triangle of $2\frac{1}{2}$ in. base $\times$ $3\frac{1}{4}$ in. height by counting squares. Check your results by calculation.
4. Convert (a) 1 ton into kilograms, (b) 3 miles into metres.
5. Convert (a) 60 m.p.h. into metres per second, (b) a pressure of 100 lb./sq. in. into kg./sq. cm.
6. A wheel turns through 56 revolutions. How many (a) degrees, (b) radians has it turned through?
7. A figure is divided into equal intervals of 10 cm. each and has ordinates in cm. as follows: 0, 100, 140, 120, 80, 60, 2
Find the total area in sq. cm. by Simpson's rule.

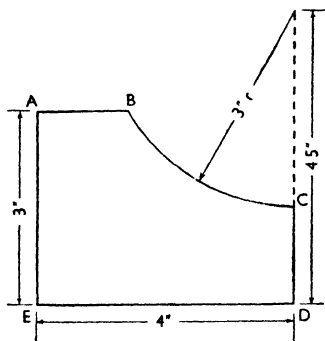


Fig. 2.6

8. Draw the figure ABCDE full size, and find its area by the mid-ordinate rule.
(S.W.E.T.C.)

9. Draw the given figure full size and determine its area.
(S.W.E.T.C.)

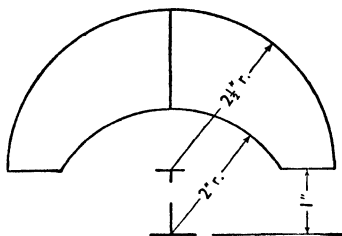


Fig. 2.7

10. A train travelling at uniform speed passes two points 480 ft. apart in 10 sec. Find the speed in m.p.h. Express this speed in metres per second.
(S.W.E.T.C.)

CHAPTER III

Matter, Energy and Force

1. Matter and energy

If we consider the universe around us, a little thought will tell us that its existence depends upon two fundamental entities—matter and energy.

Matter, or those material things which we can see, feel, smell, or taste, is often defined as “anything which exists and occupies space, and which can be made apparent to our senses”.

Energy is that agent which is capable of performing **work**, which it does in general by causing force of some kind to be exerted on matter, thereby initiating movement.

Matter on its own can be considered as something tangible, but lying dormant.* Energy, however, is intangible, and we are only aware of its presence whenever a change of **motion** or a transformation of matter takes place. We *cannot* see energy, although we *can* watch work being done on matter, as the result of the release of energy. The performance of work necessitates the presence of matter, and a change in its motion.

Matter and energy are therefore interrelated, the actual relationship being discussed further in Chap. IV.

Energy may be transferred from one body or system to another. It may manifest itself in many different forms, such as mechanical energy, heat energy, chemical energy, electrical energy, atomic energy. But these forms of energy are all interrelated and their relationships can be exactly determined. The subject is considered further in Chap. XXIII.

2. States of matter

Matter can exist in any one of three states—**solid**, **liquid** or **gaseous**, according to the conditions of temperature and pressure existing. For example, water is normally a liquid, but if

* But see Chap. IV.

heated sufficiently it becomes a gas (steam), and if cooled it becomes a solid (ice). Similarly, all substances can be changed in state by the application of suitable conditions of temperature and/or pressure, e.g. air can be liquefied and carbon dioxide (CO_2) can be solidified. Both these processes are carried out on a large scale commercially, and the resulting products, being intensely cold, are used as cooling agents in various industries. Solid CO_2 , known familiarly as "snow" or "Car-dice", is often used in tradesmen's cabinets and vehicles for keeping ice cream and foodstuffs cool. At the other extreme, we can, for instance, convert solid steel into a molten liquid by raising it to a sufficiently high temperature.

A **solid** has a definite shape and volume and resists any attempts to change these.

A **liquid** has a definite volume, but takes the shape of any containing vessel as far as its volume permits.

A **gas** has no definite volume or shape. It will fill any space in which it is placed, regardless of shape or volume, but of course the degree of concentration will decrease as the size of the space increases.

The liquid and gaseous states are sometimes referred to collectively as the **fluid** state. Some of the laws relating to fluids are dealt with in Chap. XXIX.

It is not always easy to classify a substance as either a solid, liquid or gas. For instance, tar and treacle are borderline cases between solids and liquids. If a substance can be stirred it is considered a fluid, but, of course, it is more difficult to stir some fluids than others.

The dividing line between liquids and gases is often indistinct. Consider a cloud up in the sky and a jet of steam, both composed essentially of water. The cloud consists of particles of the liquid suspended in the air, and is known as **vapour**. The steam may also be a vapour containing liquid water in suspension, but if its temperature is raised *well above* the **boiling-point** of water all the particles of the liquid will disappear and the steam becomes a **gas**.

3. Properties of matter

Amongst the general properties possessed by matter are the following:

Porosity.—There are minute spaces, or pores, between the particles of matter.

Compressibility.—As a result of porosity it follows that the particles in a portion of matter can be squeezed closer together. With some substances this property is very noticeable, with others, it is hardly measurable.

Inertia.—Matter is unable to move itself, or to change its state of motion.

From this last property it follows that force is necessary to overcome inertia; also that matter offers resistance to movement. This is evident if one collides with a solid object, or when swimming (liquid resistance), or riding in an open car without a windscreen (air resistance).

If we make a billiard ball run into another one, or direct a jet of water on to a ball, or blow at a paper object, another general property of matter is evident. This is the power of giving motion to other things by collision.

4. Mass of a body

A collection of matter bounded by known limits is termed a **body**.

The quantity of matter contained in a body is spoken of as its **mass**.

The mass of a body may be considered as a measure of the actual *quantity* of matter in it, and therefore it will not be affected by considerations of shape, volume, or state, or of its position in the universe, or of the effect of external forces. It is a *constant*.

It is usual in practice to measure mass by comparing with some standard. In Britain the mass of a body is compared with that of the **Imperial Standard Pound**. If exactly the same then the body is said to possess a mass of *one pound*. By experiment we also find that two bodies possess the same mass when they have the same weight. This *does not mean*, however, that mass and weight are the same.

5. Force

It is difficult to say exactly what force is, as its actual origin is something of a mystery. We are, however, very familiar with its effects. It may start or stop, increase or decrease, or change the direction of the motion of a body. It may stretch, compress, bend, or break an object.

It is usual therefore to define force as **that which, by acting**

upon a body, alters, or tends to alter, its state of rest or of uniform straight-line motion.

This definition is derived from Sir Isaac Newton's First Law of Motion, which is referred to in more detail in Chap. XXII.

Types of forces

One of the commonest manifestations of force in nature is that of **attraction**. Every body attracts every other body with a force directly proportional to the product of their masses and inversely proportional to the square of their distance apart, i.e.

$$\text{Force} \propto \frac{m_1 \times m_2}{d^2},$$

where m_1 and m_2 are the two masses, and d is their distance apart.

The sun, moon, earth and the planets all exert and are influenced by such forces. Due, for instance, to the pull of the moon and sun on the oceans of the earth we get the phenomenon of the tides.

The force tending to pull all bodies on or around the earth towards the earth's centre is well known. The equal and opposite force which tends to pull the earth towards the body is not so well known, because when movement occurs, it is always towards the more massive body. It is an illustration, however, of the fact that all forces occur in pairs—to every force there is an equal and opposite force or **reaction**.

Even the smallest particles in the universe obey the law of attraction, but it should be realized that movement does not always occur, as often the effects of attraction are swamped by the effects of other forces. For instance, a large body placed on a table will not draw a smaller body towards it because the force of attraction between these two bodies is swamped by the much greater pull of the earth, acting towards its centre, which holds the two bodies in place on the table.

Another type of force is that known as **repulsion**. Certain electrical and magnetic forces (charges) of like character tend to push each other apart. Other such charges of unlike character tend to cause attraction.

Other types of force, such as those due to pressure, tension, compression, friction, collision, explosion and electrical properties of matter, will be encountered.

Force due to Gravity

The earth exerts a force of attraction upon all bodies on or around it, which tends to cause all such bodies to "gravitate" towards the earth's centre.

The magnitude of this force on a body may be expressed as in the last paragraph, omitting the mass of the earth, as this will be a common factor. We then get:

$$\text{Force due to gravity} \propto \frac{\text{mass of body } (m)}{d^2}.$$

From the given definition of a force (see above), it follows that if gravity causes a body to fall, then this fall will occur, not at uniform speed, but with a changing or increasing speed. This increase of speed when falling is referred to as the **acceleration due to gravity**.

Force, therefore, can cause acceleration of a mass.

6. Weight

Gravity acting on the mass of a large body makes it difficult for us to lift this body. Consequently, we say that this body is heavy, or that it possesses the property of weight.

Therefore the weight of a body may be defined as a measure of the force due to gravity acting on it.

The greater the mass of the body the greater will be this force, and consequently, in like proportion, the greater will be its weight. Mass and weight are therefore proportional to each other, and if the units used are carefully chosen, may possess the same numerical value.

Distinction between mass and weight

Bearing in mind the formula $F \propto m/d^2$, and the definition of weight given above, we see at once the difference between mass and weight. One is *matter*, whereas the other is *force*. When speaking of a mass of one pound, we mean the amount of matter contained in a certain lump of some material; but, when speaking of a weight of one pound, we mean the force with which that lump of material is attracted by the earth.

Variation of weight

The mass of a body is a physical constant and does not vary. It will be the same in England as at the North Pole, or on the

sun. The weight of a body is apt to be regarded as constant, but this is not so, as an inspection of the formula previously given will reveal. It will be observed that the acceleration due to gravity, and hence weight, varies inversely as the square of the distance from the earth's centre (the mass of any body acts through its **centre of gravity**, which in the case of the earth is situated at its centre; see Chap. X). Consequently a body will weigh less when at the top of a high mountain, or high in the air, than when it is down a deep mine. Also, since the earth is not a perfect sphere, being flattened at the poles, gravity and weight will vary according to a body's position on the earth. It can actually be shown that a body weighing 191 ounces when near the equator will weigh 192 ounces if moved near to the poles.

To measure this difference in weight, it is necessary to use a spring balance, as this gives a true measure of the earth's pull on the body (see Chap. VI).

7. Standards of weight

Newton established that a practical measure of the mass of a body is its weight and that equal masses or weights would always counterpoise each other when placed in the two pans of a lever balance.

Most countries make use of some national "standard of weight" for commercial purposes. Such commercial standards are, strictly speaking, intended to be tests of mass. In commerce, then, an *Imperial Standard Pound*, for instance, is used as the British legal standard of comparison when measuring out a definite quantity of some commodity (i.e. it is not designed specifically to determine the quantity of matter which is attracted to the earth by a given force)—the actual comparison being carried out by means of a lever balance.

National standards of weight are kept for reference in the archives of the principal countries. They are made of a material such as platinum, to prevent corrosion, but are rarely referred to, the actual day-to-day work of comparison being carried out by means of certified substandard weights.

8. Units of force

In order to measure a force, we must have some **known** force with which we can compare it.

One method is to compare the *force* with the *weight of a body*. We can then speak of a *force* of so many *pounds weight*. When expressed in terms of weight, in this manner, we say we are using **gravitational units** of force. In common usage we usually abbreviate the expression "a force of so many pounds weight" to "a force of so many pounds".

It will be noted that a gravitational unit of force is not constant, but, so long as it is used to compare forces in the same locality, this does not matter. In practice, and where strict accuracy is not essential, gravitational units are much employed. For strictly scientific purposes, however, recourse must be made to **absolute units** of force, these being constant in magnitude. The various units of force in use, and their relationship with mass and weight, are dealt with in more detail in Chap. XXII.

EXERCISES ON CHAPTER III

1. What do you understand by (a) matter, (b) mass, and (c) a body?
2. Write a few words concerning the different states in which matter can exist.
3. What do you understand by force and energy? Give examples.
4. Distinguish between mass and weight. One of these quantities is variable. What causes this variation?
5. Explain the difference between the gravitational and absolute units of force, naming the common units used.

CHAPTER IV

Concerning Matter and its Structure

1. Construction of matter

By taking a portion of matter and continually dividing it into smaller and smaller portions (if it were possible to manipulate such small particles) a stage would eventually be reached when the resulting particles would no longer possess the properties of the original substance.

The smallest particle of matter which still retains the properties of the original substance is called a **molecule** of that substance. Thus there are as many different kinds of molecules as there are different substances, e.g. the smallest portion of chalk is thus a molecule of chalk.

The chemical name for chalk is calcium carbonate (in symbols CaCO_3). A portion of chalk could be divided by chemical means into its constituents: calcium (Ca), carbon (C) and oxygen (O). A molecule of chalk could be similarly divided into particles of calcium, carbon and oxygen (in this case three particles of oxygen). All of these particles are termed **atoms**.

An atom may be defined as the smallest portion of matter which is obtainable by chemical separation, or which can take part in a chemical reaction.

With few exceptions, most molecules normally consist of two or more atoms, e.g. two similar atoms of hydrogen form a molecule of hydrogen (H_2); two atoms of hydrogen and one of oxygen combine to form a molecule of water (H_2O); whilst the molecule of alum ($\text{K}_2\text{SO}_4\text{Al}_2(\text{SO}_4)_3\cdot 24\text{H}_2\text{O}$) consists of no less than 96 atoms of various kinds. Examples of molecules consisting only of one atom occur with the rare gases of the atmosphere, such as neon, argon and helium.

The term **element** is used to denote a substance composed entirely of atoms of the same type. Substances built up of different types of atom are termed chemical compounds. There are up to 92 different elements existing naturally on the earth, but the number of chemical compounds is indefinitely

large. Substances such as helium (He), carbon (C), iron (Fe), calcium (Ca) and gold (Au) are elements, whilst chalk (CaCO_3), sulphuric acid (H_2SO_4), and water (H_2O) are common compounds.

The atom was originally defined as the smallest particle of matter capable of separate existence. The researches of Sir J. J. Thomson (around 1900), however, proved that atoms, themselves minute, contained very much smaller particles, which carried a negative electric charge. These were eventually termed **electrons**. Later other small particles carrying a positive electric charge were discovered and termed **protons**. Consequently an atom is now defined as mentioned previously.

Electrons and protons may be regarded as elementary particles of matter, and as they carry equal charges of negative and positive electricity respectively, they may also be regarded as the elementary particles of electricity. The entire field of electrical engineering depends upon their behaviour.

The existence of a further elementary particle of matter, similar in size and mass to the proton, but without any electrical charge, was determined by Chadwick in 1932. This particle is termed a **neutron**.

2. Atomic construction

A very broad outline of modern ideas regarding the arrangement of the particles inside an atom will now be given, as it is of importance in the light of modern developments.

The complete atom may be regarded as a miniature solar system. In the centre, corresponding to the sun, is a comparatively large and heavy **nucleus**. Around this, corresponding to the planets, rotate from 1 to 92 electrons, depending upon the kind of atom being examined. These electrons describe definite elliptical orbits, and are themselves assumed to be spinning about their axes, in a similar way to the earth. The size of these orbits is so great in relation to the size of the atomic particles that the bulk of the atom can be regarded as largely empty space.

To give a rough idea, if an atom were magnified to the size of St. Paul's Cathedral in London, then the electrons could be represented by pinheads rotating around the dome, and the nucleus by the filament of a small flashlight bulb at the centre. In actual physical dimensions, the diameter of a typical atom is of the order of 10^{-8} cm. (one ten-millionth of a millimetre).

Atomic number

The nucleus, being located so far inside the atom, long defied the attempts of physicists to examine its construction. Now, however, it is regarded as being built up from a number of protons and neutrons. The number of protons is equal to the normal number of electrons circling around outside it. As an electron carries an equal, but opposite, electrical charge to a proton, it will be seen that in a normal atom these charges will cancel out, leaving the atom as a whole electrically neutral. The actual number of protons in a given atom (and hence the normal number of electrons) is termed the **atomic number** of that atom.

Atomic weight

The weight (mass) of an electron is less than 1/1800th part of that of a proton. Protons and neutrons are of almost equal weight; consequently the weight of an atom can be regarded as being concentrated largely in its nucleus. Scientists have long used a term called **atomic weight**, which is the relative weight of any atom when compared with that of the lightest atom, hydrogen. The atomic weight of hydrogen is therefore 1.* If a table of atomic weights is examined, it will be noticed that some of the values are approximately whole numbers, and may be taken as such in many instances (see Table 3, p. 33).

Now let us consider the construction of an atom and its relative weight. The simplest atom, and therefore the lightest, is hydrogen. It consists of one proton and one electron, and is shown diagrammatically in fig. 4.1.

As its weight is concentrated largely in the nucleus, and this consists of one proton only, then its atomic weight can be considered as unity, so corresponding approximately with the chemically determined value.

As the hydrogen atom contains only one proton its atomic number is therefore also unity.

The next lightest atom is the gas helium (see fig. 4.2). It consists of two electrons revolving around a nucleus containing two protons, thus securing electrical balance, and giving an atomic number of 2. But the weight of the helium atom has been found to be four times that of the hydrogen atom (i.e. its atomic

* In practice, for convenience, chemists use atomic weights based on oxygen as 16. Under this arrangement the atomic weight of hydrogen is 1.008.

weight is 4). Consequently its nucleus must weigh 4 times that of the hydrogen nucleus. To achieve this the helium nucleus is regarded as being built up of two protons *and* two neutrons. These neutrons, having no charge, do not affect the electrical balance, but having the same weight as a proton (approximately) they bring the weight of the atom up to four times that of hydrogen.

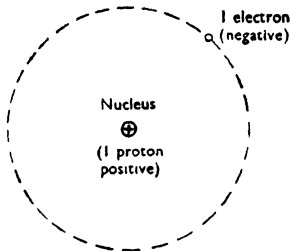


Fig 4.1.—Hydrogen atom

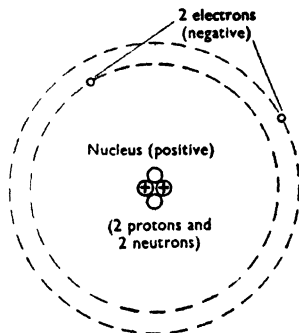


Fig. 4.2.—Helium atom

From the point of view of atomic physics then, atomic weight is the total number of protons and neutrons in the nucleus of an atom. This whole number is sometimes called the **mass number** of that atom.

Chemically determined atomic weights, however, usually differ from an exact whole number for reasons which will be given later.

Example.—Oxygen has an atomic weight of 16, and an atomic number of 8. Describe the make-up of its atom in terms of electrons, protons and neutrons.

Its atomic number being 8, its nucleus will contain 8 protons, and there will be 8 electrons revolving around it.

Its atomic weight being 16, its nucleus will contain 16 particles. As 8 of these are protons then

$$\begin{aligned}\text{number of neutrons} &= \text{atomic weight} - \text{atomic number} \\ &= 16 - 8 = 8.\end{aligned}$$

3. Periodic table of the elements

The heaviest naturally existing element is **uranium**. Its atom contains 92 protons and 92 electrons, and therefore its atomic number is 92.

All the elements can be listed numerically in terms of their atomic numbers. There is an element corresponding to each and every atomic number from 1 to 92 (i.e. there is a *complete* sequence of different elements, starting with hydrogen, whose atom contains 1 proton, and finishing with uranium, whose atom contains 92 protons).

A list of some of the elements arranged in this manner is given in Table 3. It includes all the commoner elements, and also certain valuable and rare elements of interest to engineers.

The systematic classification of the elements was originally due to Mendeléeff (around 1869). He found that, when all were listed in ascending order of their atomic weights, elements belonging to families possessing similar chemical properties were spaced at definite periodic intervals. Thus starting with lithium (3) and counting on 8, 8, 18, 18 (and then 32) down the *complete* list (not available to Mendeléeff) we get the alkali elements lithium, sodium, potassium, rubidium, caesium, etc. Similarly, starting from helium (2), the *same* numerical sequence gives the inert gases of the atmosphere—helium, neon, argon, krypton, xenon and radon.

Mendeléeff therefore by his periodic classification was able to predict the existence and properties of elements not then known, gaps being left in the correct places in his periodic table for these then unknown elements. Later investigations, carried out by Moseley (1914), enabled an adequate explanation of this table to be made, based on the structure of the atom.

4. Isotopes

It has been mentioned that chemically determined atomic weights are not all exactly whole numbers, as reference to the Periodic Table will show. One reason for this is that many atoms can exist in several slightly different forms, known as **isotopes**.

Thus the gas neon, of atomic number 10, can have two forms, one of which is much more rare than the other. Both forms will have an atomic nucleus containing 10 protons, surrounded

TABLE 3.—PART OF THE TABLE OF THE ELEMENTS
Showing the common elements and others of interest to the engineer
 (Atomic Weights based on Oxygen as 16)

Atomic number	Element	Symbol	Atomic weight	Period
* 1	Hydrogen	H	1.008	I
2	Helium	He	4.003	
3	Lithium	Li	6.94	II
4	Beryllium	Be	9.02	
5	Boron	B	10.82	
* 6	Carbon	C	12.01	
* 7	Nitrogen	N	14.008	
* 8	Oxygen	O	16	
9	Fluorine	F	19	
10	Neon	Ne	20.18	
* 11	Sodium	Na	22.997	III
* 12	Magnesium	Mg	24.32	
* 13	Aluminium	Al	26.97	
* 14	Silicon	Si	28.06	
* 15	Phosphorus	P	30.98	
* 16	Sulphur	S	32.06	
* 17	Chlorine	Cl	35.46	
18	Argon	A	39.94	
* 19	Potassium	K	39.10	IV
* 20	Calcium	Ca	40.08	
23	Vanadium	V	50.95	
24	Chromium	Cr	52.01	
25	Manganese	Mn	54.93	
* 26	Iron	Fe	55.85	
27	Cobalt	Co	58.94	
† 28	Nickel	Ni	58.69	
† 29	Copper	Cu	63.57	
† 30	Zinc	Zn	65.38	
36	Krypton	Kr	83.7	
42	Molybdenum	Mo	95.95	V
47	Silver	Ag	107.88	
48	Cadmium	Cd	112.4	
50	Tin	Sn	118.7	
74	Tungsten	W	183.92	VI
78	Platinum	Pt	195.23	
79	Gold	Au	197.2	
† 80	Mercury	Hg	200.6	
† 82	Lead	Pb	207.21	
88	Radium	Ra	226.05	VII
90	Thorium	Th	232.12	
92	Uranium	U	238.07	
93	Neptunium	Np	237 ‡	Artificially produced
94	Plutonium	Pu	239 ‡	Artificially produced

All the other elements, including those not listed, are rare or very rare.

* Common. † Comparatively expensive commercially. ‡ Principal isotope only.

by 10 revolving electrons; but one form (the commoner) contains 10 neutrons in its nucleus, and the other 12. Consequently the atomic weight of the predominating form will be $10 + 10 = 20$, and of the other $10 + 12 = 22$.

The chemically determined atomic weight is the atomic weight of a mixture of the two forms of neon, the naturally existing proportions of this mixture being such that its atomic weight comes to 20.18.

Atomic weights, it will be seen, thus range from roughly 1 for hydrogen to 238 for uranium. Actually both hydrogen and uranium exist in isotopic forms, but the proportion of these isotopes to the normal element is very small indeed; they are, however, of great importance in connection with atomic energy. The hydrogen isotope is present in natural hydrogen to the extent of 0.02 per cent. Its nucleus consists of 1 proton and 1 neutron, i.e. its atomic weight is 2. It is often called "heavy hydrogen" or deuterium, and its nucleus a deuteron. Natural uranium contains 3 isotopes, of atomic weight 234, 235 and 238 respectively. The 238 variety (U238) predominates to the extent of 99.3 per cent, most of the remainder being U235. This U235 had to be separated out from natural uranium by a long and costly process during the manufacture of the first atomic bombs. Its atom consists of 92 electrons surrounding a nucleus of 92 protons and $(235 - 92) = 143$ neutrons.

5. Radioactivity

Some elements, mainly those of high atomic number, and therefore of complex nuclear structure, are unstable. They are constantly emitting high-velocity particles, or radiations, and are therefore spontaneously changing or disintegrating and giving out energy. Such elements are termed **radioactive**. Three forms of such radiation are known, called alpha (α), beta (β) and gamma (γ) rays respectively. An alpha ray or particle is really the nucleus of a helium atom, and a beta ray or particle is really an electron. Consequently a radioactive element will, in the course of time, change its atomic weight and number, and be transformed into a different element. This transformation process will continue until the final product is a form of lead (an isotope of lead). The time taken to complete this process varies from billions of years for some elements, to fractions of a second for very unstable radioactive materials.

Uranium was the first element found to exhibit this property of radioactivity, its radiations affecting a photographic plate placed nearby. Later radium (of atomic number 88) was found to be even more radioactive.

Elements with atoms containing more than 92 electrons do not exist naturally on the earth (as far as is known). Some, however, can be produced artificially. For instance, an element of atomic number 93 has been produced, and has been called "neptunium". This element is a very unstable radioactive material; it quickly undergoes changes resulting in the emission of a beta particle and the formation of another new artificial element of atomic number 94. This new element is comparatively stable, and it has been given the name "plutonium".

Radioactive elements (both those naturally existing and those artificially produced) are of great importance in connection with atomic-energy production. They also have various uses in industry, medicine, and research. The first atomic bomb made use of uranium (actually U235); and the second one used derived its energy from the artificially produced element plutonium.

6. Conservation of matter and energy

The interrelationship between matter and energy, and in particular the fact that energy is given out by radioactive materials and results in a loss of mass, has already been noted.

Many years ago two fundamental principles were laid down:

1. The Principle of the Conservation of Matter.
2. The Principle of the Conservation of Energy.

At the time, these laws were considered as entirely separate and absolutely true. To-day, however, they should be regarded as complementary, owing to this interrelationship of matter and energy. This means that, however many physical or chemical processes take place, the total amount of matter and energy in a closed system remains the same. Consequently matter and energy, collectively, can neither be created nor destroyed, although they may be converted from one form to another or more forms.

For all ordinary purposes, the above two principles may be

considered separately and regarded as true. Thus the coal used in a power station is burned (combined with oxygen), and its matter (and that of the oxygen) converted into ash, dust and various gases. Using ordinary methods of measurement,* the total mass involved in the process is the same after burning as before. Similarly the chemical energy stored in the coal is converted by the plant into heat, mechanical and electrical energy. In both cases nothing has been lost, and a complete balance sheet of the original matter and of the energy can be made out.

For processes involving atomic energy (such as "atomic fission"), however, the mass remaining after the process is detectably less than before. The loss of mass has resulted in the liberation of a large amount of energy, i.e. matter has been transformed into energy.

As long ago as 1905 it was proved mathematically by Einstein that matter and energy were mutually convertible, and that the actual relationship between them was given by the formula

$$\text{energy (ergs)} = \text{matter (mass in grams)} \\ \times \text{velocity of light}^2 \text{ (centimetres per second),}$$

or

$$E = mc^2,$$

where $c = 30,000$ million cm./sec.

His calculations were beyond most people, and little attention was paid to his work outside scientific circles. However, after the use of the atomic bomb against Japan, it began to be more generally realized that, under certain conditions, it was possible to transform a small quantity of matter directly into a large amount of energy.

The possibility of using a similar process for the production of power (i.e. making use of the liberated energy slowly, instead of all at once, as in an explosion) was then pursued, and, as a result, atomic power plants are now possible.

We are only on the threshold of this field of scientific discovery, but it seems possible that the reverse process, i.e. transforming energy into matter, may be possible, and may have occurred at some period during the life of the universe.

* Modern ideas presume that an undetectably small loss of mass occurs during combustion.

7. Atomic energy *

An electrostatic force of attraction exists between the negatively charged electrons and the positively charged nucleus. This force tends to pull the electrons into the nucleus, because their mass is very small compared with that of the heavy nucleus. The energy of rotation of the electrons around the nucleus, however, counterbalances this electrostatic attraction, a state of equilibrium being reached when the outward centrifugal force on the electrons due to their rotation just balances this inward attraction.

Now consider the forces acting inside the nucleus. Here we have protons and neutrons in comparative proximity. The protons, being positively charged, will exert a strong force of repulsion on each other. There will also be a short-range force of attraction between all the particles, as they must obey the universal law of attraction ($\text{Force} \propto m_1 m_2 \div d^2$, see Chap. III, § 5). Careful measurement has shown that the total mass of the component parts of the nucleus slightly exceeds that of the built-up nucleus; therefore a slight amount of mass disappears in the building-up of a nucleus. But in accordance with Einstein's equation, it reappears as energy. This energy, known as "binding energy", holds the nucleus together, and its value varies with the particular element considered, some elements being more strongly bound than others.

It will be apparent, therefore, that the atom, besides containing the fundamental particles of matter, is also the seat of many forces performing, or capable of performing, work, i.e. energy manifests itself within the atom. The close relationship of matter and energy will therefore be seen. Without energy matter would collapse; without matter energy would have nothing to perform work on. In fact, the atom is delicately balanced matter and energy. The structure of matter is held together by energy. This is true for the infinitely small (the atom) and for the infinitely large (the universe).

Owing to the minuteness of the matter present, the ratio of the energy to the matter in an atom is enormous, judged by ordinary standards. Therefore, it will be seen that atoms are veritable storehouses of energy. The doors to these store-

* These remarks will be more fully understood after further study, and may be omitted at a first reading.

houses are very difficult to open; both the keys and the correct combination are required.

A door can sometimes be opened partially by electrical methods, and we can make use of some of the energy of the revolving electrons; e.g. in electric-discharge lighting the atoms of various gases or vapours are electrified, resulting in slight rearrangements of the orbits of the revolving electrons. The balance of matter and energy is upset, and some energy is released. This energy manifests itself as an electromagnetic radiation, part of which happens to be of such a frequency as to be visible (i.e. light energy is given out).

In order to open the door fully, and to get to the nucleus of the atom, very complex plant and methods are needed. When these are provided, and many other difficulties overcome, then **nuclear energy** becomes available for the use of man.

As an indication of the possibilities, consider the splitting-up of certain atomic nuclei by the process known as "fission".

If the nucleus of the uranium isotope U235 is bombarded by neutrons, fission may occur. The nucleus is then split into two fragments, but these fragments, instead of possessing the properties of uranium, may show the properties of barium and krypton. In addition to these fragments, a few neutrons are also emitted, which are able, if circumstances permit, to cause fission of further U235 nuclei, so maintaining the process, or "chain reaction" as it is called.

The total resulting mass of the fission products is considerably less than that of the original U235 nucleus. The mass that has been destroyed reappears as energy, in accordance with Einstein's equation $E = mc^2$, and it has been calculated from this that 1 gram of mass destroyed in this manner would produce energy sufficient to lift 25 battleships to the top of Mount Everest!

This energy can be released quickly for destructive purposes, as in the case of the atom bomb, or it can be released gradually, and utilized for purposes of power production. In the latter case, the heat energy released may be conveyed to, and used to operate, an orthodox heat engine.

EXERCISES ON CHAPTER IV

1. What do you understand by (a) an atom, (b) a molecule, (c) a chemical compound, (d) an element?
2. Give a brief account of the structure of a typical atom, explaining how the atom of one element differs from that of another.
3. By reference to the Periodic Table, determine the number of electrons, protons, and neutrons in an atom of (a) nitrogen, (b) sodium, (c) chromium.
4. State briefly what you understand by (a) atomic weight, (b) atomic number.
5. What is an isotope?
6. Give a short account of radioactivity, mentioning its connection with the structure of the atom.

CHAPTER V

Measuring Fundamental Quantities—Length

The methods adopted for the measurement of length depend upon: (a) the magnitude of the distance to be measured, (b) the degree of accuracy required.

A. COMPARATIVELY LARGE DISTANCES

1. Use of standard lengths

For the measurement of distance over the ground, the surveyor uses a chain, a steel band, or a tape.

Two types of chain are commonly used, each consisting of 100 links of steel or iron wire. Gunter's chain is 66 ft. long, and the engineer's chain is 100 ft. long. (With Gunter's chain, survey work involving square measure is facilitated, as 1 sq. chain = $22 \times 22 = 484$ sq. yd., and 10 sq. chains = 1 acre = 4840 sq. yd.)

For greater accuracy a "steel band" is used; and a steel tape measure, graduated in feet, inches, and eighths of an inch, is used for smaller distances.

The same general method of using a known length as the standard of comparison can be employed in measuring the depth of water, or "sounding" as it is called.

A length of line, known as a "lead line", is heaved over the side of a ship into the water. This lead line, which may be a hundred or more fathoms long as required (a fathom being 6 ft.), is suitably marked off at intervals.

The length of lead submerged (as indicated by the marks) will then indicate depth. It will be appreciated that, for an accurate result, the lead line must be vertically beneath the ship at the moment of measurement. This requires skill in "swinging the lead", owing to the tendency of the motion of the ship, and of any currents present, to cause the line to trail off at an angle.

2. Echo sounding

The modern, and more accurate, method of measuring depth is by means of some form of echo sounding gear. A sound (which need not necessarily be of an audible frequency) is transmitted from the hull of the ship, travels to the sea bed, and is reflected back again to the ship. This echo is picked up, and the time taken for the sound to travel to the sea bed and return is measured. Knowing the speed with which sound travels through sea water, the distance travelled, and hence the depth of water, can be determined, and is indicated automatically by the apparatus.

3. Distance and mileage recorders

Fig. 5.1 shows a simple little instrument called an opisometer. It can be used to measure the length of a curved line, and it is sometimes encountered in mapping work, when we require to know the distance between two points by a certain route. It consists of a wheel A which works along a finely divided screw thread B. Starting with the wheel at one end of the screw thread, it is rolled along the line to be measured. It is then transferred to a straight line and rolled back to the starting point of the thread. The distance travelled is marked off and then measured.

Alternatively, the number of revolutions made by the wheel can be counted (or is recorded on a dial). Knowing the circumference of the wheel ($2\pi \times \text{radius}$), then

$$\text{distance travelled} = \text{number of revolutions} \times 2\pi r.$$

The "cyclometer", or mileage recorder familiar to cyclists, operates in a similar manner. It is really a revolution counter worked off a road wheel. Each revolution of this wheel taps the counter round, and, knowing the size of the road wheel, the gearing of the cyclometer can be so designed that the instrument shows the mileage covered.

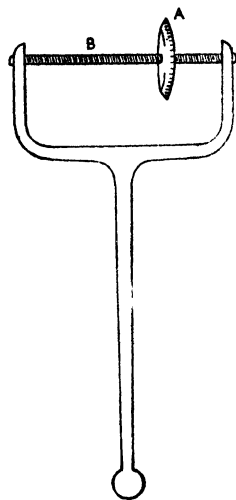


Fig. 5.1.—Opisometer

(a) The force due to the pressure exerted by the outside air, and caused by the passage of the aircraft through it. (This pressure is obtained from the Pitot tube, and is proportional to air speed.)

(b) The force due to the pressure exerted by an electrically driven fan.

If force *a* is the greater, the diaphragm will bulge downwards, and so close the electrical contacts, causing the fan motor to increase in speed. When force *b* is greater, it will cause the diaphragm to bulge upwards, and so open the contacts, causing the fan motor to decrease in speed.

When the diaphragm is in equilibrium, the fan motor will be rotating at a speed proportional to true air speed. A suitable counter, geared to this motor, can then be made to record the air mileage travelled.

It should be realized that both the patent log and the air mileage recorder indicate the distance actually travelled through the sea or air respectively. They do not take into account the effects of drift or wind. In order to obtain the true course of the ship or aircraft a "vector" of mileage (with compass bearing), and a vector of wind, drift, etc., should be plotted, and the resultant determined (see Chap. VIII).

B. WORKSHOP MEASUREMENTS

4. Importance of accuracy

Progress in engineering workshop practice has depended very largely on progress in the accuracy with which measurements can be made. A hundred years ago it would have been unlikely that a particular part from one engine or machine could have been fitted into a similar machine without a great deal of trouble. To-day, thanks to measurement, parts of similar machines can be made strictly interchangeable.

An old-time mechanic, who thought he was doing very well in working to the nearest eighth of an inch, would be surprised at a modern instrument-maker working to one hundred-thousandth of an inch.

It is recorded that, when James Watt first started building his steam engines, he had great trouble in getting the parts machined to the required dimensions. Pistons and cylinders, for instance, were not only of inaccurate dimensions, and con-

sequently ill-fitting, but they departed seriously from truly circular form.

His delight was great when, due to the invention of the "boring bar", it became possible to produce cylinders up to six feet in diameter, "not being worse from absolute truth in the worst part than a thick sixpence" (about $\frac{1}{16}$ in. at that time).

5. Units adopted in workshop measurements

Both the British and metric systems of units are employed, although in Britain and America the British system is by far the most common. For precision work, it is customary to express dimensions in decimal parts of an inch, whereas, for less accurate work, or for woodwork, structures, etc., fractions up to $\frac{1}{8}$ in. are employed.

Classes of measurement

There is no point in working to a higher standard of accuracy than necessary. The following degrees of accuracy are typical.

$\frac{1}{100}$ in.—Hand and ordinary machine work.

$\frac{1}{1000}$ in.—Good hand and machine work.

$\frac{1}{10,000}$ in.—High-class machine and grinding work.

$\frac{1}{100,000}$ in.—Very high-class instrument and gauge work.

6. The engineer's steel rule

Most people are acquainted with the engineer's rule. It differs from a wooden ruler in that its graduations extend right up to one end, so that it is possible to make measurements in awkward corners. It is divided up with a high degree of accuracy, the usual divisions being $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ in., with perhaps $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$ in., and also millimetres. With this rule it is possible to estimate dimensions to the nearest $\frac{1}{16}$ in. with a fair degree of accuracy.

By making use of **calipers**, of either the inside or outside patterns, the dimensions of any object can be transferred to the rule. Some skill is needed in obtaining the final adjustment of the calipers; this is usually done by tapping them lightly and then offering them to the object to be measured, until the sense of touch decides that contact is just made.

FINE MEASURING INSTRUMENTS

Where a greater degree of accuracy than is obtainable by the above method is required we may make use of either (i) a vernier caliper (or slide gauge), or (ii) a micrometer screw gauge.

7. The vernier

This method of accurate measurement was invented by the French mathematician Pierre Vernier early in the seventeenth century. The instrument used is fitted with two scales.

1. A main scale, graduated in normal dimensions.
2. A sliding, or vernier, scale capable of movement relative to the main scale, and graduated as described below.

In a vernier caliper one jaw is fixed to the main scale, and the other to the sliding scale (see fig. 5.3a).

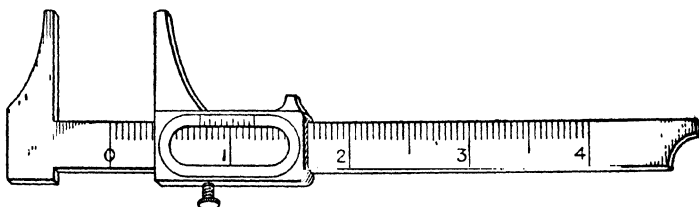


Fig. 5.3a—Vernier caliper

Fig. 5.3b shows a simple vernier which will read to an accuracy of $\frac{1}{100}$ in.

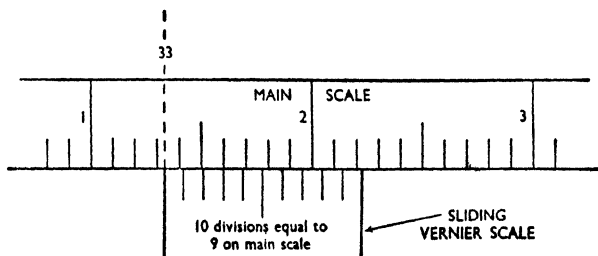


Fig. 5.3b

The main scale is divided into $\frac{1}{10}$ in., and the sliding scale consists of a length equal to 9 main scale divisions (0.9 in.), which has been subdivided into ten equal parts, each of .09 in.

The difference between the length of a main-scale division and of a sliding- or vernier-scale division is therefore

$$0.10 - 0.09 = 0.01 \text{ in. (or } \frac{1}{100} \text{ in.)}$$

This value is sometimes referred to as "the least count" of the instrument, and it represents the degree of accuracy of reading obtainable.

To use the instrument, the object to be measured is placed between the jaws, and the sliding portion brought into contact with it. (On better-class instruments the final adjustment is made by locking a portion of the slider and then turning an adjusting screw.)

The reading is obtained as follows (see fig. 5.3b):

1. Note the last main-scale division which has been passed (1.3).
2. Read the second decimal place by looking along the vernier scale, and noting the number of the division which coincides with a main-scale division (.03).
3. Add up these values (1.33 in.).

Example 1.—Obtain the reading of the vernier scale shown in fig. 5.4. Give an explanation to prove your result.

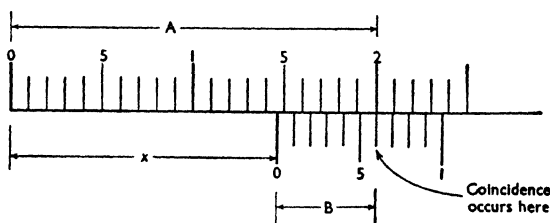


Fig. 5.4

Method. 1. Last main-scale division passed is 1.4.
 2. Vernier-scale division at which coincidence occurs is .6. Therefore reading is 1.46 in.

Proof. (See fig. 5.4).—Let the distance to be measured (corresponding to the opening between the jaws of an actual vernier caliper) be x in.

Let reading on main scale at point of coincidence be A in., and let actual length of vernier-scale reading up to point of coincidence be B in. Then

$$x = A - B.$$

As 1 vernier-scale division = $\frac{9}{10}$ of a main-scale division = 0.09 in., then distance $B = 6 \times 0.09 = 0.54$ in.

Since distance $A = 2$ in., $x = 2 - 0.54 = 1.46$ in.

Backward-reading vernier

It is possible to make a vernier scale by taking, say, 11 main-scale divisions, and dividing this distance into 10 equal parts on the vernier. Each vernier division will then be 1.1 times a main-scale division. Assuming a main-scale division to be 0.1 in., then a vernier-scale division will be 0.11 in., and the difference between a main-scale and a vernier-scale division will be $(0.11 - 0.10)$, or 0.01 in., as before.

The vernier scale on such an instrument will be numbered in the opposite direction to that of the normal type, i.e. from right to left. In consequence, this scale must be read backwards, hence the name.

8. Practical workshop vernier calipers

Two patterns of vernier are in common use, both permitting readings to be made to $\frac{1}{1000}$ in.

(a) *The English pattern*

In this the main scale is graduated into $\frac{1}{16}$ in. and then into $\frac{1}{40}$ in. (i.e. 40 small divisions per inch).

The vernier scale is made up by dividing the space occupied by 24 of these small main-scale divisions into 25 equal parts. Then

$$1 \text{ main-scale division} = \frac{1}{40} \text{ in.} = 0.025 \text{ in.}$$

$$1 \text{ vernier-scale division} = \frac{24}{40} \div 25 = 0.024 \text{ in.}$$

$$\therefore \text{least count of vernier} = \underline{\underline{0.001 \text{ in.}}}$$

(b) *The Rock (or Swiss pattern)*

The main scale is graduated into 0.2 in., and then into $\frac{1}{2} \times 0.2 = 0.040$ in. (i.e. 25 small divisions per inch).

The vernier scale is made up by dividing the space occupied by 39 of these small main-scale divisions into 40 equal parts.

Then

$$\begin{aligned}\text{one small main-scale division} &= \frac{1}{25} \text{ in.} = 0.040 \text{ in.} \\ \text{one vernier scale division} &= \frac{39}{25} \div 40 = 0.039 \text{ in.} \\ \therefore \text{least count of vernier} &= \underline{0.001 \text{ in.}}\end{aligned}$$

The Roch pattern is sometimes preferred because of its more open scale, which simplifies reading.

Verniers may be met with graduated in the metric system, or in fractional dimensions. A quiet study of these should enable the method of graduation and of reading to be mastered.

9. The micrometer screw gauge

This instrument is extensively used to-day for engineering workshop measurements, because of its convenience and accuracy. It is easier to read and manipulate than a vernier slide gauge or caliper. The principle of the micrometer has been known and used for many years. James Watt himself made an elementary screw gauge of this type. It was not until the latter half of the nineteenth century, however, that micrometers as we know them came to be manufactured on a commercial scale.

One of their earliest uses was for checking the gauge of sheet metal, as complaints regarding variations in this were common. To-day micrometers are fitted to many types of instrument, such as depth and height gauges; they may even be encountered in most unexpected places, such as in an instrument for the measuring of the wavelengths used in very high-frequency radar apparatus!

Principle of the micrometer

The micrometer is essentially a caliper whose jaws are adjusted by means of an accurately cut screw. One revolution of this screw will open or close the jaws by an amount equal to the **pitch** of this screw (pitch being the distance between corresponding and consecutive points on the same thread).

The construction of a typical micrometer (for measuring dimensions of up to one inch to an accuracy of $\frac{1}{1000}$ in.) is shown in fig. 5.5. The object to be measured is inserted between the measuring faces of the anvil and the spindle; then the thimble is turned until light contact is obtained. The leading screw has 40 threads per inch (i.e. its pitch is $\frac{1}{40}$ in. or 0.025 in.);

therefore one complete revolution will advance (or retract) the spindle by $\frac{1}{40}$ in. The bevelled edge of the thimble is divided

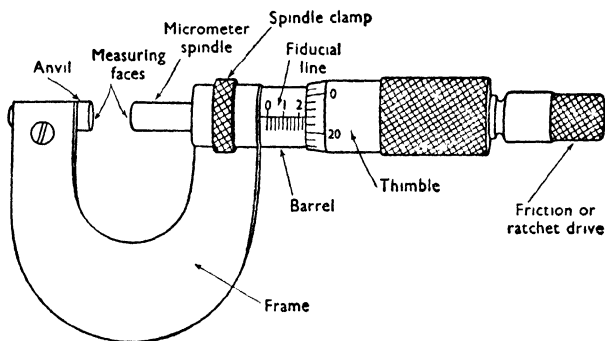


Fig. 5.5—Micrometer

into 25 equal divisions, numbered every fifth division, so that a movement of one division advances (or retracts) the spindle by

$$\frac{1}{25} \times \frac{1}{40} = \frac{1}{1000} \text{ in. (0.001 in.)}$$

A scale is engraved on the barrel to correspond with the pitch of the leading screw, being divided into $\frac{1}{40}$ in. (0.025 in.) and numbered every 0.1 in.

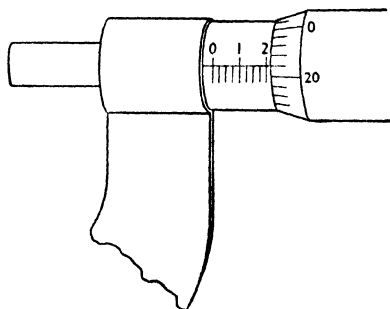


Fig. 5.6

Method of reading a micrometer

Example 2.—What is the reading of the micrometer shown in fig. 5.6?

Method.

1. Examine the barrel and note number of last main graduation uncovered (.2 in this case).	0.2
2. Note number of subdivisions above this (0 in figure) and add .025 in. for each	0.0
3. Examine thimble. Note the graduation closest to reading (or <i>fiducial</i>) line (21 in this case), and add this number of thousandths	0.021
3. Total	<u>0.221</u>

It will be observed that although this micrometer is graduated to read to thousandths of an inch, it is possible to estimate one-half and one-quarter thousandths on the thimble scale.

If a greater degree of accuracy is required, the micrometer may have a vernier engraved on the barrel, so enabling readings of ten-thousandths of an inch to be obtained. This vernier scale consists of ten divisions, which occupy the same space as nine divisions on the thimble. They are marked 0, 1, 2 to 9, 0.

To read such a micrometer, note the thousandths reading as before; then the number of the division on the vernier scale at which a line on the thimble is coincident. Then add this number of ten-thousandths.

A ten-thousandth micrometer should not be used for routine work not warranting such a high degree of accuracy, as wear will occur and affect the accuracy of the instrument.

Before taking a reading with any micrometer it is advisable to check the zero setting. First wipe the measuring faces by lightly gripping a piece of paper between them and pulling it through. Then close them, being careful not to use undue force. The micrometer should read exactly zero. If it does not, the error should be allowed for when making a measurement. If possible, the micrometer should be sent for readjustment. To ensure accurate work, all micrometers should be checked and adjusted periodically.

Two refinements often found fitted to a micrometer are:

1. *A ratchet drive.*—This enables the objects being measured to be subjected always to the same degree of pressure (about $1\frac{1}{2}$ lb.), so eliminating the human element and making for consistency of results. Should the operator attempt to apply excessive pressure the ratchet would slip.

2. *A spindle clamp.*—This, when turned, clamps the spindle, so enabling a particular setting to be preserved.

For measuring sizes greater than one inch, micrometers with larger frames are made (normally in sizes up to 12 in. and occasionally 24 in.). A set of standard lengths is usually provided, for insertion in place of the anvil, so enabling intermediate measurements to be made. It should be realized that the micrometer head itself only registers over a one-inch range.

The metric micrometer

In the metric micrometer the pitch of the leading screw is 0.5 mm., and the scale on the barrel is graduated accordingly into millimetres and $\frac{1}{2}$ mm.

The bevelled edge of the thimble is divided into 50 equal divisions. A movement through one of these divisions advances the spindle through $\frac{1}{50} \times 0.5 = 0.01$ mm. Therefore this type of micrometer can be read to an accuracy of 0.01 mm.

OTHER WORKSHOP INSTRUMENTS

Many other instruments incorporating a micrometer or a vernier scale will be encountered in the workshop. Examples of these are the height gauge, the depth gauge, and the internal micrometer.

If a higher degree of accuracy is required than is obtainable by these instruments, then various forms of measuring machine may be employed. These machines may be enlarged micrometers in principle, fixed to a bench, and permitting measurement to an accuracy of $\frac{1}{100,000}$ in. or more.

Certain kinds of measuring machine used in gauge and standards checking can read to an accuracy of $\frac{1}{1,000,000}$ in. These machines may incorporate a microscope sighting-head mounted on a micrometer screw-operated travelling mechanism. With such a degree of accuracy, particular attention must be paid to eliminating errors due to temperature; so such machines are often kept in a temperature-controlled room.

10. Slip gauges

For accurate "setting-up" and checking purposes, accurately ground rectangular pieces of steel, each of a specified thickness, may be used. They are known as "slip gauges", one of the best known being the original Swedish Johansson

pattern. They are supplied in sets, so that, when “wrung” together in a suitable combination, any required dimension is obtainable within the range of the set. A standard set of 81 blocks can be used to build up any dimension between 0.2 and 0.5 in. in steps of $\frac{1}{10,000}$ in.

11. Surface plate

In order that parts may be measured accurately it is often necessary to place them on a specially ground flat table or “surface plate”. The various measuring instruments, such as height gauges, or slips, may then be set up in accurate relationship with the parts to be measured.

The art of accurate dimensional measurement is sometimes referred to as **metrology**.

12. Measurement of angle

A plain protractor graduated in degrees can be used for the checking of angles.

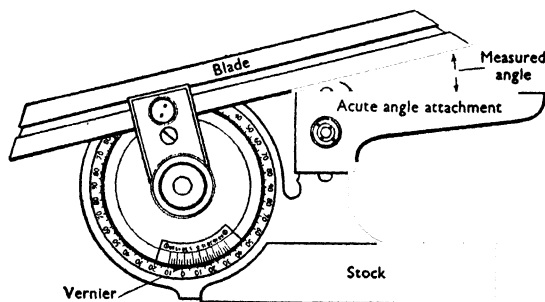


Fig. 5.7a.—Bevel protractor (Brown and Sharpe)

When a greater degree of accuracy than is possible with this is required a “universal bevel protractor” can be used. This is a form of circular vernier. A typical instrument is shown in fig. 5.7a. It consists of a stock, and a blade attached to a disc which can be rotated and clamped in any angular position relative to the stock. The dial is graduated in degrees and forms the main scale. The vernier scale, attached to the other

moving portion, is made by dividing the space occupied by 23° on the main scale into 12 equal parts, i.e.

12 divisions on vernier = 23 divisions on dial.

$$\therefore 1 \text{ division } ,, ,, = \frac{23}{12} ,, ,, = \frac{23}{12}^\circ.$$

Difference between 1 division on vernier and 2 on dial

$$= 2^\circ - \frac{23}{12}^\circ = \frac{1}{12}^\circ \text{ or 5 minutes (5')}.$$

Therefore the instrument can be read to an accuracy of 5 minutes of angle. The method of reading it is shown in fig. 5.7b.

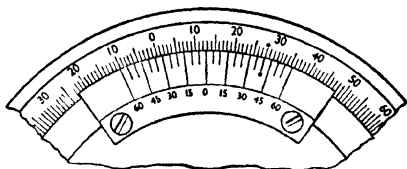


Fig. 5.7b.—Enlargement showing vernier reading $12^\circ 40'$

Method of Reading the Vernier.—Read off directly from the main scale the number of whole degrees between 0 and the 0 of the vernier scale. Then count, in the same direction, the number of spaces from the 0 of the vernier scale to the point of coincidence with the main scale (marked *). Multiply this number by 5, and this will give the number of minutes to add to the whole number of degrees.

13. Workshop gauges

The use of precision measuring instruments alone for the checking of dimensions in the workshop would obviously be a lengthy and therefore costly process, especially if large quantities of a particular product were being made. To reduce the time required for such checking, and to enable less skilled labour to be employed, various forms of production and inspection gauges are employed.

The earliest types of gauge so used were the cylindrical internal and external gauges (see fig. 5.8). These gauges were made each of a definite standard diameter. If, for instance, a 2-in. diameter shaft was being turned, then the workman would take a 2-in. diameter plug gauge, adjust his calipers to just slide over it, and then use these calipers as his standard for checking his work. Similarly, for boring a hole, he would use

a ring gauge and a pair of inside calipers. Note that "the sense of touch" would be used to decide when the calipers just touched the work.

14. Tolerance

Modern methods of mass production make even this process too lengthy, and they require that similar parts should be made interchangeable with each other. In order to secure this, a tolerance on each dimension is stated on the drawing. Thus

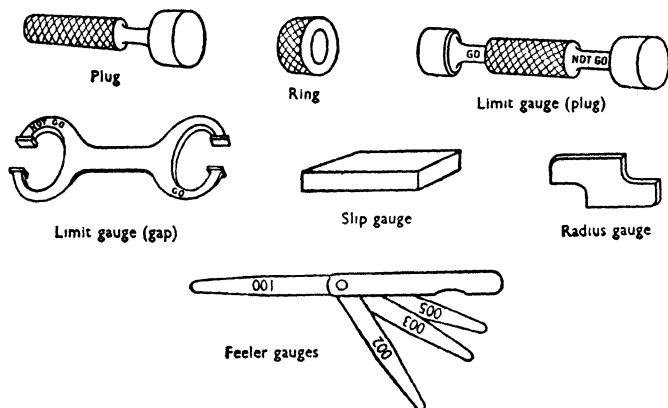


Fig. 5.8.—Various gauges

a certain dimension might be marked $1.55 \pm .01$ in. This means that parts would be acceptable up to a high limit of 1.56 in., and down to a low limit of 1.54 in. A more important dimension might be marked $1.125 \begin{smallmatrix} + .000 \\ - .001 \end{smallmatrix}$ in. This, of course, means that the limits of acceptance allowed are from 1.125 in. down to 1.124 in., and that the tolerance is only .001 in.

Limit gauges are used both in production and on final inspection to ensure that the appropriate tolerances are observed. Examples are shown in fig. 5.8. It will be noticed that one end of the gauge is marked GO and the other NOT GO. If, when offered to the work, one end is accepted (goes in), and the other is not, then it is evident that the work is within the permitted tolerance.

In mass-production work, where fine tolerances are specified, many types of plug, gap, and "form" gauges will be encountered, some for checking the most intricate dimensions at one operation. Some other workshop gauges, relying largely upon the sense of touch for their use, are shown in fig. 5.8, e.g. *feeler gauges* (for checking fine clearances) and *radius gauges* (for checking internal and external radii).

15. The dial or clock gauge. (Dial test indicator)

This instrument is very useful in the workshop for the rapid checking of mass-produced parts. It also finds many applica-

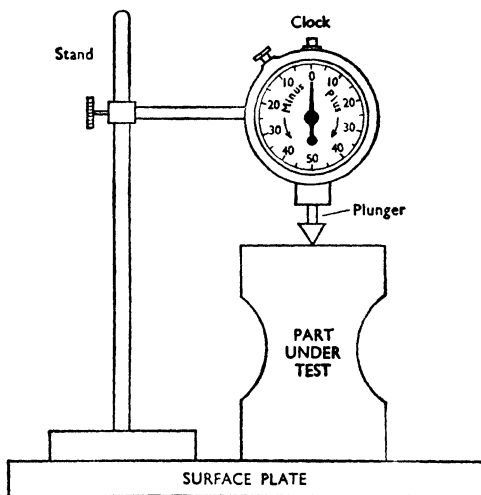


Fig. 5.9.—Dial gauge in use

tions in both workshop and laboratory, when a change of dimension has to be measured. It consists essentially of a plunger free to move up or down, which, when in contact with the work, shows its relative height above or below some datum line by operating a mechanism that moves a pointer round a dial. This dial is usually calibrated to indicate to $\frac{1}{1000}$ in.

One method of setting-up the instrument is shown in fig. 5.9.

Slip gauges of the required nominal dimension are placed on the surface table, the plunger is placed in contact with these gauges, and the dial pointer is then set to zero (or any other convenient point). The slip gauges are now removed, and the parts to be checked are placed on the table one at a time. The plunger is brought into contact with each, and the dial pointer indicates the number of thousandths of an inch each is above or below the nominal dimension. If desired, the scale can be marked with red lines showing the high and low limits, so that rapid checking is facilitated.

16. Measurement of area. The planimeter

An instrument known as a "planimeter" is sometimes encountered in the workshop or laboratory. It is used for the

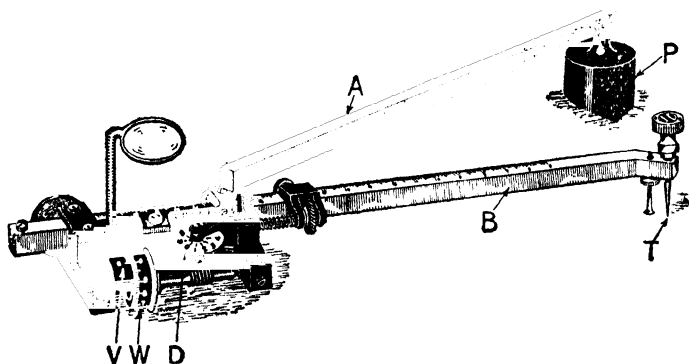


Fig. 5.10.—Planimeter

approximate determination of the area of irregular figures. The commonest form is the Amsler pattern, shown in fig. 5.10. It consists of two arms A and B pivoted together, with a wheel and recording mechanism arranged between. In use the weighted point (P) is stuck in a suitable position outside the figure whose area is to be measured. The tracing point (T) is then carefully traced round the outline of the figure. The wheel will be seen to rotate first one way, then the other, at a rate depending upon the mode of movement of the tracing point. The number of revolutions made by the wheel is indicated on the dial (D),

parts of a revolution (usually in hundredths) on the wheel (W), and fractions of a revolution (usually thousandths) by means of the vernier attachment (V).

Method of using the planimeter

Make sure that the tracing point can be taken round the figure, i.e. that the point (P) is in a suitable position. Then:

1. Start with tracing point on a marked point on the figure.
2. Read the dial, wheel, and vernier.
3. Trace round the figure.
4. Read the dial, wheel, and vernier again.
5. Note the difference between 2 and 4.
6. Repeat tracing in opposite direction, and again note difference in readings.
7. Take the mean difference, and multiply by the constant of the instrument. This will give the area of the figure.

If the constant of the instrument is not known, it can be determined by tracing around a figure of known area, and finding the mean difference of readings. Then

$$\text{constant of instrument} = \frac{\text{area of known figure}}{\text{difference of readings}}.$$

The planimeter has particular applications in Engineering Science, in that it can be used as an alternative to the mid-ordinate rule to determine the area of an indicator diagram (see Chap. XV, § 7), the area under a speed-time graph (see Chap. XXI, § 3), etc.

Care must be taken to see that the wheel does not slip in use, otherwise serious errors may be introduced.

EXERCISES ON CHAPTER V

1. Make diagrams illustrating vernier scales to measure to (a) 1/100 in., (b) 1/64 in. (S.W.E.T.C.)
2. Make a simple sketch of (a) a micrometer screw gauge or (b) a vernier caliper, and give the theory of the working of the instrument. (S.W.E.T.C.)
3. Sketch the setting of a simple 1/100-in. vernier set to read 1.16 in.

4. Sketch the thimble and barrel of a micrometer when set to a reading of 0.459 in.

5. Describe briefly the method of using a planimeter.

6. What do you understand by "limit gauges"?

A dimension on a drawing is marked $2.266 \begin{smallmatrix} + \\ - \end{smallmatrix} \begin{smallmatrix} .003 \\ .000 \end{smallmatrix}$. What does this mean?

7. (a) Describe an instrument for the accurate measurement of angles. (b) Explain the use of the dial gauge.

8. Make diagrams to illustrate:

(a) A micrometer reading of 0.382 in.

(b) A vernier reading of 1.46 in.

(c) A vernier scale reading to $1/80$ in. (S.W.E.T.C.)

CHAPTER VI

Measuring Fundamental Quantities—Mass and Time

MEASUREMENT OF WEIGHT AND MASS

1. Weight

Since the dawn of civilization, the measurement of weight has been a necessity wherever trade has flourished. Most early methods made use of the principle of the lever (see Chap. IX).

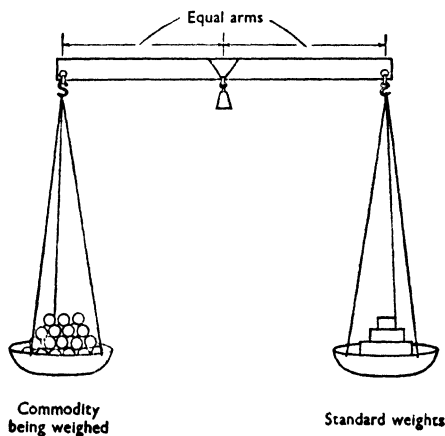


Fig. 6.1.—Lever balance

Fig. 6.1 shows a balance of the general type used by the Romans (and others), and it dates back to well before the Christian era. The commodity to be weighed is placed in one scale-pan, and it is *balanced* by placing suitable weights in the other scale-pan. The arms of the lever being equal, then the value of these

weights will be the same as that of the weight of the commodity. It will be realized that a balance of this type really compares the weight of the commodity with the weights of the set of "weights" used. To be of any commercial use this set of weights must correspond accurately with those in use elsewhere. Consequently, the checking of weights has been an important administrative function since early times.

The type of balance shown above is still the most accurate form available, but certain precautions have to be taken in

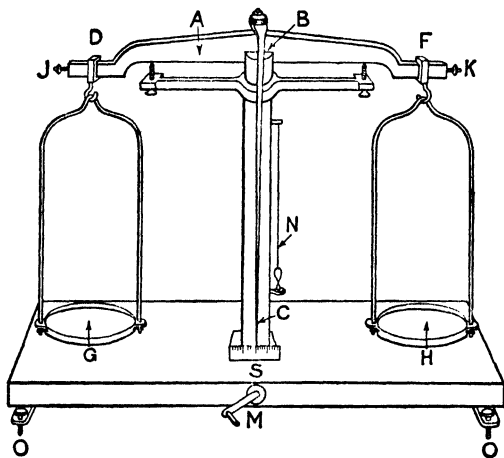


Fig. 6.2—Chemical balance

its design in order to obtain this accuracy. Many forms of tradesmen's scales, etc., making use of the principle of the lever, with either equal or unequal arms, are in use to-day.

For accurate scientific work we use a **chemical balance** (see fig. 6.2), which is an equal-arm balance.

The beam (A) is supported on a knife-edge (at B), and the scale-pans are also hung upon knife-edges (D and F). With the weights and object in the pans, the beam is raised by turning the handle (M) and lifting it off its supports. The pointer (C) indicates on the scale (S) which pan is the heavier, and successive adjustments to the weights are then made until balance is obtained. The beam should be lowered on to its

supports whilst adjusting the weights. Adjusting screws (O) and a plumb bob (N) are provided for levelling the balance. Very often a good chemical balance is enclosed in a glass case to protect it from draughts, etc.

2. The steelyard

With an ordinary equal-armed balance, the weights used must be equal in value to the commodity being weighed. This is inconvenient when dealing with heavy commodities, so various lever weighing machines, making use of unequal arms, were brought into use, enabling weights of only a fraction of that of the commodity to be used. Such machines are called **steelyards**, and various forms were devised by the Romans, Danes and other peoples, and have been used, in some cases with little change in form, for many centuries.

The Roman steelyard is described in more detail in Chap. X.

3. The weighbridge

To avoid the necessity for lifting a heavy object, such as a horse and cart, into the air during the process of weighing, the **weighbridge** was developed and came into use around 1750. The vehicle, or the commodity to be weighed, is run on to a platform flush with the ground, and by means of a system of levers it is counter-balanced by a steelyard (usually placed in a weighhouse). The position of the vehicle on the platform does not affect the result.

A description of a weighbridge is given in Chap. X, § 5.

4. The spring balance

The weight of a body being defined as a measure of the force due to gravity acting on it, it can be determined by finding the extension (or compression) it produces when hung from a calibrated spring. The first spring balance was produced in 1783, and made use of an elliptical spring. Modern spring balances employ spiral springs, and make use of Hooke's law (Chap. XI), which tells us that the load hung from a spring is proportional to the extension it produces.

Two forms of spring balance are shown in fig. 6.3. In one the spring is stretched and in the other it is compressed by the pull of the weight. This extension or compression being, in

accordance with Hooke's law, directly proportional to the load producing it, the scale marked on the balance will be linear.

Sometimes a spring balance is fitted with a rack and pinion mechanism, operated by the pull of the weight, a pointer being

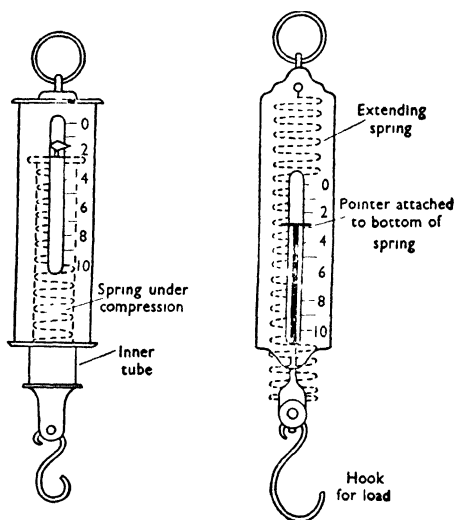


Fig. 6.3.—Spring balances

attached to the pinion to indicate the weight on a circular scale.

5. Measurement of mass

Mass, being the quantity of matter contained in a body, can be measured by comparing that body with some standard mass by means of a lever balance. The force due to gravity acting on the body on one side of the balance will be counteracted by the force due to gravity acting on the standard mass on the other side of the balance. Therefore gravity will not affect the result. Masses compared in this manner will show the same result in any part of the world, or at any height above or below sea level. As the mass of a body is by definition a constant, the lever balance will therefore register true mass.

If we use a spring balance, however, the results obtained will depend upon the local value of g , and will not be constant. Therefore this instrument cannot be used for the measurement of mass.

For measuring weight, spring balances are calibrated for use in the desired locality (if strict accuracy is required). A standard 1 lb. weight which registers 1 lb. on the spring scale near the North Pole would register approximately 99.5 per cent of this value at the equator.

6. Measurement of force

Weight being a force of a particular kind, it follows that a spring balance may be used to record the pulls of forces, irrespective of their direction.

MEASUREMENT OF TIME

7. Standards of time

When dealing with the measurement of time two aspects have to be considered: (a) the time of day, and (b) the interval of time between two successive moments.

The "time of day" is expressed as so many units of time after noon or midnight at some particular place, or as some agreed variation from this. Noon is readily determined, being that instant when the sun reaches its highest point in the heavens on that particular day at that particular location. It can be observed by means of an instrument known as a "sexant", which is much used in navigation.

The agreed standard for fixing the time of day is the time at Greenwich (Greenwich Mean Time—G.M.T.). In the 24-hour clock, or Continental system, the time of day is stated as so many hours, minutes, and seconds after midnight (up to 24 hours), e.g. 18 hours G.M.T. Sometimes it is more convenient to use the 12-hour clock, in which time from midnight to noon (12 noon) is stated as so many hours, etc., *ante meridiem* (a.m.), and from noon to midnight, starting again from zero, as so many hours, etc., *post meridiem* (p.m.), e.g. 18 hours G.M.T. = 6 p.m.

A point on the earth's surface diametrically opposite to Greenwich will be in darkness when it is noon at Greenwich,

i.e. its solar time will be midnight. As we could not use G.M.T. for normal purposes at this spot, and as also it would be very confusing nowadays for every spot to use its own true solar time, resort is made to *local standards of time*. Thus we have Eastern and Central European Time, and, in America, Eastern Standard Time, Mountain Time, Pacific Time, etc., each of these local times being used in a particular area, and approximating to the local solar time. Each area's local time usually differs by 1 hour from its neighbour's local time.

Sometimes in the interest of daylight saving, the local standard time may be advanced by an hour, as in the case of British Summer Time.

The standard unit of time is universally taken to be the second. As far as we are concerned in mechanics, etc., we mean the "mean solar second".

$$\text{One second is } \frac{1}{24} \times \frac{1}{60} \times \frac{1}{60} = \frac{1}{86,400} \text{ of a day.}$$

The length of a day, as measured by the interval between two successive transits of the sun across a particular meridian, varies slightly in length throughout the year, due to various reasons. We therefore take the average length and call it the "mean solar day".

It would be possible to have a day of constant length by reckoning time with reference to the stars, but this is not always convenient. Astronomers, however, use it, and call it a "sidereal day".

8. Time and longitude

When it is noon at Greenwich, it will be midnight by true solar time at a point on the earth's surface diametrically opposite Greenwich, i.e. 180° of longitude from Greenwich.

Therefore 180° difference in longitude represents 12 hours' difference between G.M.T. and local solar time, or

$$1^\circ \text{ difference} = \frac{12}{180} \text{ hr.} = \frac{720}{180} = 4 \text{ min.}$$

Thus, by knowing G.M.T. at the instant of noon (local solar time, as determined by a sextant), the longitude of a place may be determined.

If G.M.T. is ahead of local time, the place is west of Greenwich by 1° of longitude for every 4 min. of time difference; whereas, if G.M.T. is behind the local time, the place is east of Greenwich by the same amount.

This relationship between time and longitude is of great importance in navigation, and it led directly to the development of accurate methods of time measurement.

9. Early methods of measuring time

From the earliest days man has used methods of measuring time. At first, the methods used gave an approximate idea of the hour of the day, which was all that was required. The sundial was used, the position of the shadow of the upright portion giving an indication of the hour on a scale or dial. The sundial was, of course, dependent upon the sun shining. It could not be used during the night, or on sunless days.

To enable this difficulty to be overcome, other devices were employed. One of the earliest of these was the water clock, in which water dripped into or out of a vessel at a steady rate, the amount of water collected or lost by the vessel being a measure of time, and the vessel being graduated accordingly.

From the water clock the hour glass was developed, sand being employed instead of water. This has survived into modern times in the form of the familiar egg-timer, and, in a modified form, as an electrical timing device. This latter device consists of two glass bulbs joined by a constricted tube through which mercury can trickle. After the passage of a specified time the mercury covers a contact, and so completes an electrical circuit.

Another early time-measurer was the candle clock, the burning of a marked length of candle indicating an hour.

Mechanical clocks came into use during the Middle Ages, and made use of a falling weight attached by a rope to a train of wheels, a form of rotating fan being used as a controlling device. The train of wheels was used to drive some form of dial indicator, and also to operate a chiming mechanism at regular intervals.

10. The pendulum

The invention of the pendulum by Galileo around the year 1585 provided an accurate method of controlling the rate of unwinding of the driving mechanism, and hence of the time-

keeping. For all practical purposes the time of swing of a pendulum is constant, and depends upon its length. Variations in the length of the arc through which it swings, therefore, do not affect it. Adjustment of the length of the pendulum, by moving the weight at the bottom up or down, causes the time taken for each swing to be decreased or increased. Thus a definite time period of swing may be set.

The time taken for one swing of a pendulum can be found from the formula

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where t = time (sec.), l = length (ft.), g = acceleration due to gravity (ft./sec.²).

N.B. 1 swing consists of 2 beats.

Modern mechanical clocks

Developments in mechanical clocks after the invention of the pendulum included the use of the energy stored in a spring as a driving device, and the use of an oscillating balance wheel and spring as a controlling device. Portable clocks, or watches, were thus made possible.

Mechanical methods of measuring time reached their highest state of perfection with the development of the *marine chronometer*, an extremely accurate clock used to enable a navigator to determine longitude.

In 1713 the British Admiralty offered a prize of £20,000 for such an instrument. This prize was eventually awarded to John Harrison, who made several models between 1728 and 1759. One of these models on test was reputed to be capable of an accuracy such that its error was only 15 sec. in 5 months. Modern navigators possess a great advantage over their predecessors. They can check their chronometers daily by means of time signals from Greenwich, broadcast by wireless.

11. The electric clock

The true modern electric clock consists essentially of an electromagnet which is used to supply energy to maintain the swing of the pendulum (or other controlling device). Such an arrangement is often used as a *master clock*, a number of *repeater*

clocks being controlled from it by means of electrical impulses sent at short intervals along a connecting circuit.

The *synchronous electric clock*, so widely used in the home and elsewhere to-day, is not really a clock at all. It is merely a synchronous electric motor, driven by alternating-current electricity obtained from the mains supply. Provided that the frequency of this a.c. supply is kept constant, the synchronous motor will turn at a constant speed, and will move the hands of the clock through gearing designed to indicate the correct time. The frequency of the a.c. supply (50 cycles per second in Great Britain) is normally kept constant at the power station by means of suitable speed-governing mechanism on the machines driving the alternators.

12. The quartz crystal clock

The quartz crystal clock is the most modern development in timekeeping. By its use, fractions of a microsecond (millionth of a second) can be determined accurately.

It depends for its action upon what is known as the "piezo-electric effect". A thin slice of quartz crystal can be set vibrating by electrical means, and, provided that certain precautions are taken, its rate of vibration can be kept constant. This rate of vibration is known as the natural frequency of oscillation of the crystal, and it depends upon the crystal's dimensions. It is a very high frequency, of the order of many thousands of cycles per second.

These oscillations are transmitted electrically to a thermionic-valve frequency divider, which in effect steps the frequency down to some such value as 1000 c./s., which is used to operate a suitable synchronous-motor clock.

EXERCISES ON CHAPTER VI

1. Sketch and describe the method of using a chemical balance.
2. A lever balance and a spring balance are used to weigh a certain object, first at the equator, and then in the polar regions. What results would you expect? Explain your answer.
3. Calculate the length of a simple pendulum to give a beat of one second, (a) at a location where g is 32.2 ft./sec.², (b) at the equator, where g is 32.086 ft./sec.²

CHAPTER VII

Density and Specific Gravity

1. Density

It is apparent that some materials are heavier than others. For instance, an inch cube of lead weighs more than an inch cube of cork. In order to compare the relative masses of different materials we use the term **density**. It is defined as the mass of a body per unit volume, and its symbol is the Greek letter rho (ρ).

$$\text{Density} = \frac{\text{mass}}{\text{volume}},$$

or
$$\rho = \frac{m}{V}.$$

The units employed for measuring mass and volume must be stated when referring to density. Thus "pounds per cubic foot" may be used commercially, and "grams per cubic centimetre" may be used scientifically. In modern scientific work, however, it has become the practice to use the litre as the unit of volume, density then being expressed in grams per millilitre (gm./ml.). As the difference between a volume of 1 c.c. and one of 1 ml. is very small (see Chap. II, § 4), and as the older expression for density is still much employed, any further reference to gm./c.c. in what follows may also be read as gm./ml., and vice versa, without any great sacrifice of accuracy.

Owing to the changes in volume which occur when most materials are heated, an accurate statement of density should also include reference to temperature. Thus, whereas the density of copper might be given commercially as 546 lb./cu. ft., a more accurate statement would be 8.76 gm./c.c. at 4° C. Again, the density of fresh water is given roughly as 62.5 lb./cu. ft., or 1000 oz./cu. ft. A more accurate statement would be 62.4 lb./cu. ft. at 40° F., or 62 lb./cu. ft. at 100° F.;

or, in metric measure, 1 gm./c.c. at 4° C. (its point of maximum density), and, more accurately still, 1 gm./ml. at 4° C.

2. Specific gravity

As the numerical value of the density of a substance thus depends upon the units of mass and volume employed, it is usually more convenient to refer to the **relative density** or **specific gravity** (sp. gr.) of the substance. Specific gravity is a *pure number*, no units being necessary, and it gives us a measure of the density of a substance relative to that of water.

The specific gravity of a substance may be defined as the ratio of the weight of any volume of the substance to the weight of an equal volume of water.

$$\text{Specific gravity} = \frac{m/V \text{ of substance}}{m/V \text{ of water}} = \frac{\text{density of substance}}{\text{density of water}}.$$

Thus, if we are told that the specific gravity of a certain iron is 7.89, it means that this iron weighs 7.89 times as much as the same volume of water.

It should be noted that, as 1 c.c. of water is taken as weighing 1 gram (at 4° C.), its density in C.G.S. units and its specific gravity are both unity (1.0000). Consequently the density of a substance expressed in gm./c.c. is numerically the same as its specific gravity.

The densities and specific gravities of some common materials are listed in Table 4 (p. 70).

Example 1.—Find the volume of a piece of copper weighing 80 lb., given that the density of copper is 546 lb. per cubic foot.

Determine also the specific gravity of copper, the density of water being 62.4 lb./cu. ft.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}.$$

$$\therefore \text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{80}{546} = 0.1465 \text{ cu. ft.}$$

$$= 0.1465 \times 1728 = 253 \text{ cu. in.}$$

$$\text{Specific gravity} = \frac{\text{density of substance}}{\text{density of water}} = \frac{546}{62.4} = 8.76.$$

Example 2.—What saving in weight could be made by replacing a 1-ton iron casting by one made of aluminium?

The weights of the two castings will be in the ratio of their respective specific gravities.

$$\frac{\text{S.G. of aluminium}}{\text{S.G. of cast iron}} = \frac{2.6}{7.5} = 0.347.$$

$$\therefore \text{Weight of aluminium casting} = 0.347 \times 20 = 6.94 \text{ cwt.}$$

$$\therefore \text{Saving in weight} = 20 - 6.94 = 13.06 \text{ cwt.}$$

TABLE 4

DENSITIES AND SPECIFIC GRAVITIES OF
SOME COMMON SUBSTANCES

Substance	Specific Gravity	Density, lb. per cu. ft.
Aluminium	2.6	161.7
Brick	2.0	124.8
Coal (anthracite)	1.43	88.7
Copper	8.76	546
Glass	2.89	180.7
Iron (cast)	7.5	465
Lead	11.3	710
Mercury	13.59	848
Oil (heavy fuel)	0.88	55
Petrol	0.72	45
Steel	7.83	486
Water, distilled (at 4° C.)	1.0	62.42
„ sea (average)	1.025	64
Cork	0.25	15.6
Oak	0.8	54

Some variation in these values may be expected owing to variations in the composition, preparation, etc., of the samples

3. Determination of specific gravity

A. SOLIDS

(a) *Direct measurement.*—If of suitable shape, the volume of a solid may be calculated by measuring its dimensions, and

then using the appropriate formula. Its mass is found by weighing on a suitable balance. Then

$$\text{S.G.} = \frac{\text{mass (gm.)}}{\text{volume (c.c.)}}$$

(b) *Determination by means of Archimedes' Principle.*—The mass of the solid is first found by weighing. A suitable vessel is then filled to the brim with water, and the solid is immersed in it. A quantity of the water equal in volume to the solid will overflow. This displaced water is caught and weighed.

Then

$$\text{S.G.} = \frac{\text{mass of solid}}{\text{mass of equal volume of water displaced}}$$

or

$$\frac{\text{mass of solid (gm.)}}{\text{volume of water displaced (c.c.)}}$$

The volume of water displaced can be determined directly by using a suitably graduated measuring cylinder.

(c) *Alternative method using the hydrostatic balance.*—The hydrostatic balance is an ordinary balance with one scale-pan arranged so that a beaker of fluid can be placed under it. Solids suspended from this scale-pan can then be weighed when immersed in the fluid.

The weight of the solid is first found in air, and then when immersed in water. The difference between these weights (i.e. the apparent loss of weight in water) is the weight of a quantity of water equal in volume to the solid. Then

$$\begin{aligned} \text{S.G.} &= \frac{\text{weight of solid}}{\text{weight of equal volume of water}} \\ &= \frac{\text{weight of solid}}{\text{weight of solid in air} - \text{weight in water}} \end{aligned}$$

Method (b) is the original method used by Archimedes around 250 B.C. He discovered that a solid immersed in a liquid displaces an equal volume of the liquid. He used this discovery to find the specific gravity of a golden crown of which the quality was in doubt.

Archimedes' Principle may be summarized as follows:

"A body immersed in a fluid loses as much of its weight as is equal to the weight of the fluid it displaces."

This apparent loss of weight is really an upward force on the body. If this upward force exceeds the weight of the body, then the body will float. Consequently this upward force is called **buoyancy**.

Example 3.—A sample of gold weighs 199.4 gm. in air, and 189.1 gm. immersed in water. Determine its volume and its specific gravity.

$$\begin{aligned}\text{Apparent loss of weight} &= \text{weight of water displaced} \\ &= \text{volume of solid.}\end{aligned}$$

$$\therefore \text{Volume of sample of gold} = 199.4 - 189.1 = 10.3 \text{ c.c.}$$

$$\text{Specific gravity of sample} = \frac{\text{mass}}{\text{volume}} = \frac{199.4}{10.3} = 19.35.$$

B. LIQUIDS

(a) *Use of a density or specific-gravity bottle.*—A specific-gravity bottle is a vessel which, when fitted with its stopper, has an accurately known volume. Therefore, when exactly filled, it can be used for comparing the weights of precisely equal volumes of different liquids. As its volume will vary with temperature, its value (in c.c. or ml., or the weight of water [gm.] it will hold) at some specified temperature (known as the standard temperature) is marked on the bottle.

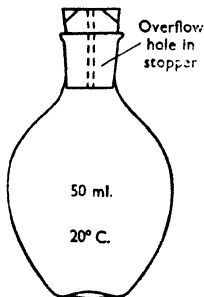


Fig. 7.1.—Density bottle

To find the specific gravity of any liquid, weigh the bottle filled with that liquid, and then subtract the weight of the empty bottle, thus getting the weight of the liquid. Then

$$\text{S.G.} = \frac{\text{mass of liquid (gm.)}}{\text{volume of bottle (c.c.)}}$$

The above method is only strictly accurate if carried out when the temperature of the liquid is the same as the standard temperature marked on the bottle. To remove this small source of error, we would proceed as follows:

1. Find the mass of the liquid as before.
2. Whilst the room temperature remains the same, fill the bottle with water, and find its mass at this temperature.

Then

$$\text{S.G.} = \frac{\text{mass of liquid (gm.)}}{\text{mass of equal volume of water (gm.)}}$$

N.B. The standard temperature of density bottles has been fixed as 20° C., and they are made in sizes having capacities of 10, 25, 50 and 100 ml. at this temperature. In practice, using standard density bottles, the first-mentioned method may be used at any temperature, a correction factor being applied to give the true specific gravity.

(b) *The hydrometer.*—The specific gravity of a liquid can be readily found by means of a suitable **hydrometer**. This method is used extensively in commerce, for such purposes as testing the quality of milk, alcoholic liquors, acids, etc. A suitable type is usually designed for each purpose. A typical example is shown in fig. 7.2 and represents the type used for testing the specific gravity of the sulphuric acid used in accumulators, in order to ascertain the state of charge of the accumulators. It consists of a narrow glass tube with a bulb at its lower end. A suitable quantity of lead shot is fixed in this bulb, so that the hydrometer floats vertically. The amount of tube projecting above the surface indicates the specific gravity of the liquid on the graduated scale, the hydrometer displacing its own weight of liquid.

A hydrometer usually covers only a small range of specific gravities, sufficient for the particular purpose required. Note that the greater the specific gravity of the liquid, the higher the hydrometer floats in it.

If the hydrometer scale is not graduated in terms of specific gravity, this can be determined if we know:

1. The weight of the hydrometer.
2. The volume of liquid displaced when the hydrometer floats in it.

The hydrometer displaces its own weight of liquid, and the volume of liquid displaced (the immersed volume) can be determined if we know the volume of the bulb, the cross-sectional area, and the immersed length of the cylindrical glass tube.

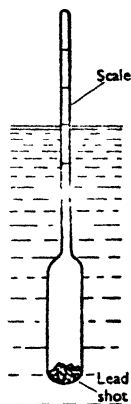


Fig. 7.2.—
Common
hydrometer

Then

$$\text{S.G.} = \frac{\text{wt. of hydrometer, or wt. of displaced liquid (gm.)}}{\text{immersed volume, or volume of liquid displaced (c.c.)}}$$

Example 4.—A common hydrometer weighs 15.3 gm. and sinks into sulphuric acid until 12 c.c. are immersed. Find the specific gravity of the acid.

$$\text{S.G. of acid} = \frac{\text{wt. of hydrometer}}{\text{immersed volume}} = \frac{15.3}{12} = 1.275$$

(c) *Use of a U-tube.*—The specific gravity of a liquid can be found by comparing the height of a column of the liquid required to balance a known column of water.

In the U-tube shown in fig. 7.3, the mercury was initially at the same height in each limb (i.e. it was in equilibrium). If water is now poured into one limb, its weight ($= h \times \text{cross-sectional area of tube} \times \text{density}$) will cause the mercury in the other limb to rise to preserve equilibrium. Balance will be obtained when the weight of the displaced mercury ($H \times \text{cross-sectional area} \times \text{density of mercury}$) is equal to the weight of the water.

If D and d are the densities of mercury and water respectively and A is the cross-sectional area of the tube, then

$$D = \frac{h \times A \times d}{H \times A} = \frac{hd}{H}$$

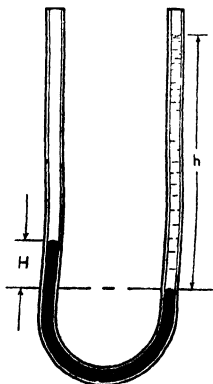


Fig. 7.3

By pouring another liquid into the left-hand limb, until the level of the mercury returns to the same height in each limb, the density of this liquid can be compared with that of water. If the liquids balance when the heights of the columns are h_1 and h_2 (for water column), and if d' is the density of the other liquid, then

$$d'h_1 = dh_2 \quad \text{and} \quad d' = \frac{dh_2}{h_1};$$

also specific gravity of other liquid $= \frac{\text{its density } (d')}{\text{density of water } (d)} = \frac{h_2}{h_1}$.

Example 5.—A U-tube contains mercury to within 5 in. of the top of each limb (the bore of which is uniform). Find the height of the column of water which can be poured in to just fill one limb. Take the specific gravity of mercury as 13·6.

The original free surface of mercury is 5 in. from top. Let x be the drop in level of this free surface when one limb is filled with water. Then x is also the rise in mercury level in the other limb.

Then height (h) of water column = $5 + x$, and height (H) of mercury balancing column = $2x$. Balance is obtained when $hd = HD$,

$$\text{i.e.} \quad (5 + x) \times 1 = 2x \times 13\cdot6,$$

$$\text{i.e.} \quad 5 + x = 27\cdot2x.$$

$$\therefore 26\cdot2x = 5,$$

$$\therefore x = \frac{5}{26\cdot2} = 0\cdot19 \text{ in.}$$

$$\therefore \text{Height of column of water} = 5\cdot19 \text{ in.}$$

C. GASES

The densities of gases are generally stated in pounds per cubic foot, or in grams per litre, under specified temperature and pressure conditions.

The density of a gas is not compared with that of water when specifying its relative density or specific gravity, as is the case with solids and liquids, because the value obtained would be very small. The standard of comparison when dealing with gases is the density of hydrogen,* the lightest element. The relative densities of other gases will therefore be greater than unity.

In order to determine the appropriate value we can weigh a flask containing hydrogen *under certain conditions of temperature and pressure*, and then weigh it again when filled with the gas whose relative density is required. The *same* conditions of temperature and pressure must, however, exist in the flask. The ratio of the two weights will represent the relative density of the gas.

EXERCISES ON CHAPTER VII

1. A metal lamina has an area of 15·35 sq. in. and a weight of 5 oz. Its density is 0·28 lb. per cu. in. What is its thickness?

(S.W.E.T.C.)

2. Lead piping is 4 in. in external diameter, the thickness of the metal being $\frac{1}{4}$ in. Find (a) its volume, and (b) its weight per foot length.

* The density of air is sometimes used as the standard.

3. A wooden pattern weighs 6.5 lb. Find the weight of an iron casting taken from it (take the S.G. of the wood as 0.74).

4. An object is built up of two semicircular plates, spaced one on either side of a centre line. The first plate is $2\frac{3}{8}$ in. diameter, and the second $1\frac{1}{8}$ in. Find (a) the area of one face, (b) the density in ounces per cubic inch, (c) the specific gravity, if the plates are $\frac{1}{4}$ in. thick and the object weighs 2.39 lbs.

5. Describe a hydrometer and the principle of its action. A hollow cylindrical tube, closed at one end and weighted with lead shot, has an area of cross-section of 2 sq. cm. and weighs 30.6 gm. Find the depths to which it sinks, (a) in water, and (b) in liquid of specific gravity 1.5. Explain the calculations involved. (C.P.E.)

6. Define "specific gravity".

A block of metal 3 in. $\times$ 2.5 in. $\times$ 1.5 in. is found to weigh 2.92 lb.

(a) What is the density of the metal?

(b) Given that 1 cu. ft. of water weighs 62.4 lb., what is the specific gravity of the metal?

(c) A casting of the same material weighs 10 lb. What is the volume of metal in the casting? (Poly.)

7. The following results were obtained in a test using a specific-gravity bottle: mass of empty bottle, 44.4 gm.; mass filled with petrol, 134.4 gm.; mass filled with water, 169.4 gm. Find the S.G. of petrol.

8. In an experiment to determine the density of a metal lamina, the following results were obtained: weight of metal lamina, 100 gm.; area of metal lamina, 10 sq. in.; thickness of metal lamina, 0.25 in. Find the density of the metal in lb. per cubic foot. (S.W.E.T.C.)

9. Water is poured into a U-tube until its limbs, 8 in. long, are half full. Oil is then poured into one limb until it commences to overflow. Find the height of this column of oil. The specific gravity of the oil is 0.666.

10. A U-tube containing mercury has water poured into one limb; this depresses the mercury level in this limb by 1 cm. Find the height of the water column.

If paraffin is now poured into the other limb until the mercury levels are the same in each, and the height of this column of paraffin is 34 cm., determine the specific gravity of paraffin. (Specific gravity of mercury 13.6.)

11. A piece of paraffin wax, specific gravity 0.9, has embedded in it a piece of metal of specific gravity 2.5. When weighed in air the whole weighs 122 gm. and when immersed in water weighs 22 gm. Find the volumes of wax and metal. (C.P.E.)

CHAPTER VIII

Vectors

1. Scalar and vector quantities

It is often necessary to represent certain physical quantities graphically. This can be done in two ways—scalar or vector.

A **scalar quantity** is one which is completely specified by its numerical value expressed in the appropriate units, e.g. 10 tons, 12 inches, 1 microsecond, etc. Thus a scalar quantity possesses *magnitude* only. It may be shown graphically as in fig. 8.1.

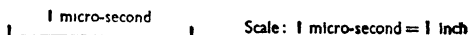


Fig. 8.1

A **vector quantity** is one which possesses both *magnitude* and *direction*, e.g. a ship steaming at 20 knots in a north-easterly direction; a force of 10 lb. wt. pulling vertically upwards.

These two examples may be shown graphically as in figs. 8.2 and 8.3.

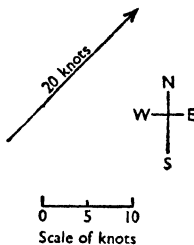


Fig. 8.2

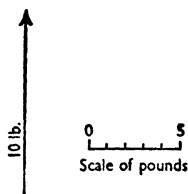


Fig. 8.3

It will be seen that the length of the lines represent magnitude to some specified scale, and that the arrowheads and inclination of the lines represent direction.

To represent a quantity vectorially we should know:

1. Its magnitude and *point of application*.
2. Its direction and *sense of direction*.

For example, we could not draw the vector of the ship mentioned above if we did not know at which *point* (on a map) to start, and whether the ship was proceeding along the line of inclination in a N.E. or a S.W. *sense of direction*.

2. Addition and subtraction of vectors

Scalar quantities may be added and subtracted arithmetically, e.g.

$$5 \text{ lb.} + 5 \text{ lb.} = 10 \text{ lb.}$$

$$5 \text{ lb.} - 3 \text{ lb.} = 2 \text{ lb.}$$

The sum of 2 *vectors* of 5 lb. is not, however, necessarily 10 lb. Vectors must be added together in such a manner as to take into account their direction.

If the vectors are acting in the same straight line (see fig. 8.4), and both in the same direction, this means that we simply add

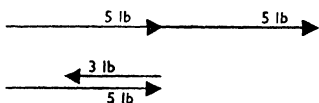


Fig. 8.4

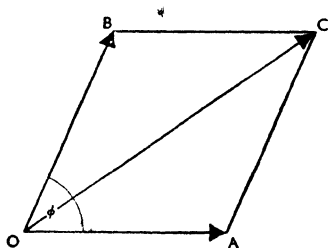


Fig. 8.5

(5 lb. + 5 lb. = 10 lb.). If they act in opposite directions, the addition is the algebraic sum of $5 + (-3) = 5 - 3 = 2 \text{ lb.}$

To subtract in the above two cases the operation may be carried out algebraically:

In the first case: $5 \text{ lb.} - 5 \text{ lb.} = 0$.

In the second case: $5 - (-3) = 5 + 3 = 8 \text{ lb.}$

If the vectors are not in the same straight line, the addition can be carried out by drawing a vector diagram. Fig. 8.5 shows two vectors OA and OB acting at an angle phi (ϕ) to each

other. Lines are drawn through B parallel to OA, and through A parallel to OB, meeting at C, and so forming a parallelogram. The diagonal OC is then drawn from O, and this represents the **vector sum** of OA and OB. Its value can be determined by measuring it to the same scale as was used for drawing OA and OB (or it can be calculated by trigonometry). This diagonal OC is termed the **resultant** of OA and OB.

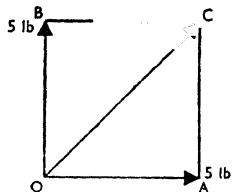


Fig. 8.6

In the special case where the vectors are at right angles to each other, the same construction can be followed (see fig. 8.6), or the vector sum may be calculated using the theorem of Pythagoras: square on the hypotenuse = sum of squares on other two sides, i.e.

$$(OC)^2 = (OA)^2 + (OB)^2.$$

$$(OC)^2 = 5^2 + 5^2 = 25 + 25 = 50.$$

$$\therefore OC = \sqrt{50} = 7.07 \text{ lb.}$$

Take the case of a ship steaming at 15 knots (i.e. 15 nautical miles per hour), and making successive alterations of course. This can be plotted by means of vectors, each vector representing the actual distance steamed on each particular course, and the final position of the ship determined (fig. 8.7). It will be seen that the net result, as far as the final position of the ship is concerned, could have been achieved by steaming from O to F, i.e. OF represents the vector sum or **resultant** of all the vectors.

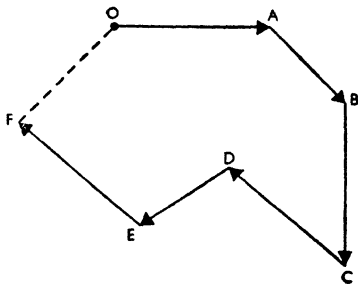


Fig. 8.7

When subtracting two vectors, whether they are acting in the same straight line or not, the operation may be carried out by drawing the vectors, changing the sign (direction) of one of them, and then adding vectorially.

Vectors can be used for the solution of problems in many different types of work, such as navigation, electrical engineering, and bridge-building. We will now consider some important applications of vectors in mechanics. First we shall deal with *forces*.

3. The parallelogram of forces

If two forces meet at a point at the same instant, their resultant may be determined by completing the parallelogram and drawing its diagonal. This resultant OC will be the vector sum of OA and OB, and may be determined by direct measurement from the vector diagram (see fig. 8.8).

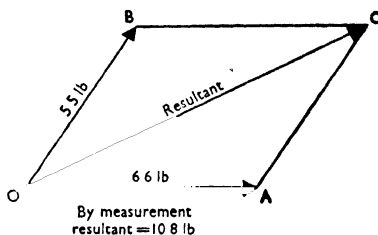


Fig 8.8

The law of the parallelogram of forces may be stated as follows:

"If two forces in one plane, acting simultaneously at a point, are represented in magnitude and direction by two straight lines drawn from the point, then the diagonal of a parallelogram drawn with these two forces as adjacent sides will represent in magnitude and direction their resultant."

4. The triangle of forces

The triangle of forces deals with the case of three forces acting at a point. Its law may be stated as follows:

"If three forces in one plane act at a point and *produce equilibrium*, then these forces may be represented in magnitude and direction by the three sides of a triangle, *taken in order*."

By hanging weights on 3 strings as shown (fig. 8.9), an experiment may be performed to prove this law. The **space**

diagram is first drawn, and from this the **vector diagram** is constructed, each vector representing in magnitude and direction one of the forces (see fig. 8.10). Note that the forces must be taken in order (i.e. the arrowheads must follow each other round, either clockwise or anticlockwise). The force between A and B is represented by vector ab . The force BC is then drawn from b to c , and the force CA from c to a . It will be found that the triangle closes if the forces are in equilibrium.

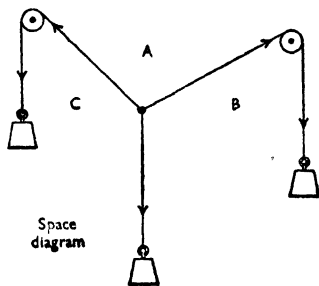


Fig. 8.9

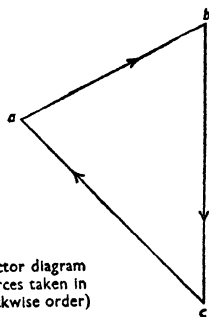


Fig. 8.10

The converse of the triangle of forces should also be realized. It can be stated as follows:

“If three non-parallel forces are in equilibrium, then their lines of action must intersect at one point.”

5. The polygon of forces

“If any number of forces in one plane act at a point, and produce equilibrium, then these forces may be represented in magnitude and direction by the sides of a closed polygon taken in order.”

Take the case of five forces meeting at a point, and producing equilibrium (see space diagram, fig. 8.11). The forces A, B, C, D should be taken in order (in this case they have been taken anticlockwise) and represented by straight lines ab, bc, cd, de in the vector diagram. They will form a polygon, the closing line of which (ea) will be found to represent in magnitude and direction the fifth force E .

Note particularly that the direction of this closing line (from e to a in follow-my-leader fashion) is the same as that of E in the space diagram, and that it maintains the balance or equilibrium of the forces. Therefore it is called the **equilibrant**.

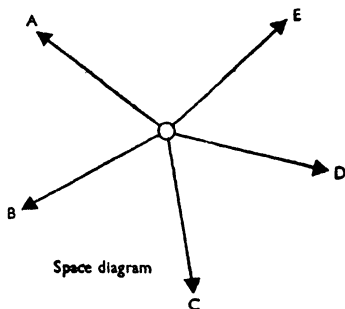
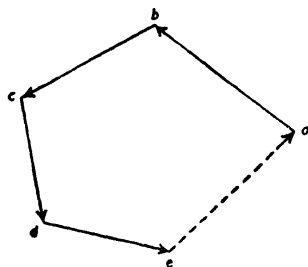


Fig. 8.11



Vector diagram

Fig. 8.12

N.B. Forces **must** be taken in order, the vectors (and arrow-heads) following each other round in one continuous line. Clockwise or anticlockwise order will, however, give the same result.

6. Resultant and equilibrant

It will be seen that the **resultant** is that single force or quantity which can be used to *replace* two or more forces or quantities *in every respect*.

The **equilibrant**, however, is that single force or quantity which will just *balance* any number of forces or quantities. It produces equilibrium.

In any system of forces the resultant and equilibrant are equal in magnitude, but *opposite in sense of direction*. They form another example of the law that action and reaction are equal and opposite.

The various laws concerning parallelogram, triangle, and polygon of forces enumerated above are often referred to as the *laws of concurrent coplanar forces*—"concurrent" meaning "acting together through the same point", and "coplanar" meaning "acting in the same plane surface".

7. The parallelogram, triangle, and polygon of velocities

Another important application of vectors in mechanics is concerned with **velocities**.

When a body is changing its position, it is said to be "in motion", or to be "undergoing displacement".

In order to specify this movement we must state:

1. How fast it is moving, i.e. its speed (displacement $\div$ time).
2. The direction in which it moves.

These two factors constitute the **velocity** of the body, and it will be seen that they correspond to a vector quantity. Should either factor change, then the velocity changes. In common language we often use the terms "speed" and "velocity" indiscriminately, but the difference should be remembered.

We can consequently often solve problems on velocities by drawing a vector diagram. Similar constructions to the parallelogram, triangle, and polygon of forces are used as appropriate.

The following example shows the method.

Example 1.—An aeroplane is steered in a N.E. direction at a speed of 250 m.p.h. The wind velocity is 50 m.p.h. due east. Find the true (resultant) velocity of the aeroplane over the ground.

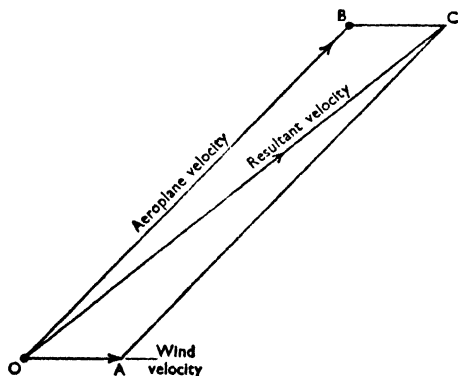


Fig. 8.13

Draw vectors OB and OA, representing the aeroplane and wind velocities respectively (see fig. 8.13). Then the resultant velocity of the aeroplane over the ground is represented by the diagonal OC.

By measurement.—OC = 290 m.p.h. in a direction 39° (approx.) from east, or 51° (approx.) east of north.

8. The parallelogram, etc., of accelerations

The parallelogram, etc., of velocities may be extended to include the *parallelogram, etc., of accelerations*.

The rate at which velocity changes is termed the “acceleration” of the body, or simply “acceleration”. This change in velocity may be in speed (an increase or decrease), or in direction, or both. It is thus a special case of velocity, and the same type of vector construction may be used when dealing with acceleration problems.

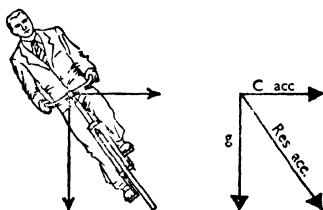


Fig. 8.14

Acceleration may be either positive or negative, according to whether the speed is increasing or decreasing. Negative acceleration is referred to as “retardation”.

The unit of acceleration in the F.P.S. system of units is a velocity of one foot per second added every second. This is abbreviated to 1 ft./sec./sec. or 1 ft./sec.² An example of the use of the parallelogram of accelerations is shown in fig. 8.14.

A cyclist is racing around a circular track. As his direction is continually changing, his velocity is also changing, and hence he is subjected to an acceleration, known as “centrifugal acceleration”, which acts outwards from the centre of the track. The weight of the cyclist and his machine is also subjected to the acceleration due to gravity, and this acts vertically downwards. In order to maintain his balance, the cyclist must lean over inwards, so that the angle he makes coincides with that of the resultant acceleration vector.

A more detailed consideration of motion must be left to a later stage.

9. Composition and resolution of vectors

As a vector may be the sum of several other vectors, it is obvious that it may be broken down, or resolved, into a number of components. For instance, in fig. 8.8 it was seen that the resultant vector OC was built up from the addition of vectors OA and OB . Therefore, given OC only, it is easy to see that it may be broken down, or resolved, if required, into two component parts OA and OB . OA and OB are therefore termed the **components** of OC .

It is sometimes desired to resolve a vector into two components at right angles to each other. By so doing visualization and calculation of a problem is often facilitated. Thus in fig. 8.15 the vector OC is shown together with its **horizontal** and **vertical** components. These components may be determined by drawing and measuring, by the Theorem of Pythagoras, or by trigonometry.

$$\text{Vertical component} = OC \sin \phi.$$

$$\text{Horizontal component} = OC \cos \phi.$$

This process of resolving into components at right angles to each other is termed resolution into **rectangular components**.

Vector problems are capable of solution by means of trigonometry, but for most cases it is reasonably easy to draw a vector diagram and obtain the result by measurement.

Example 2.—A ship steams 120 miles in a N.N.E. direction. Express this distance in northerly and easterly components.

As N.N.E. is $22\frac{1}{2}^\circ$ E. of N., set out the vector OA to represent 120 miles to scale, and at an angle of $22\frac{1}{2}^\circ$ E. of N. (see fig. 8.16). Project from this to give the vertical and horizontal components OB and OC at right angles to each other, north and east. Then by measurement:

$$\text{Northerly component} = OB = 110 \text{ miles.}$$

$$\text{Easterly component} = OC = 46 \text{ miles.}$$

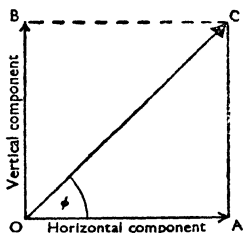


Fig. 8.15

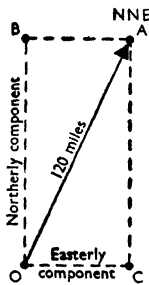


Fig. 8.16

Alternatively, by trigonometry,

Northerly component = $120 \sin AOC = 120 \sin 67\frac{1}{2}^\circ = 110.9$ miles.

Easterly component = $120 \cos 67\frac{1}{2}^\circ = 45.9$ miles.

The resultant of two or more vectors can be calculated by first resolving them into horizontal and vertical components, adding all the horizontal and then all the vertical components, and then making use of the Theorem of Pythagoras.

Example 3.—A ship steaming at constant speed travels a distance of 50 miles in a N.E. direction. It then alters course by 30° to starboard and steams a further 30 miles. Find its resultant course.

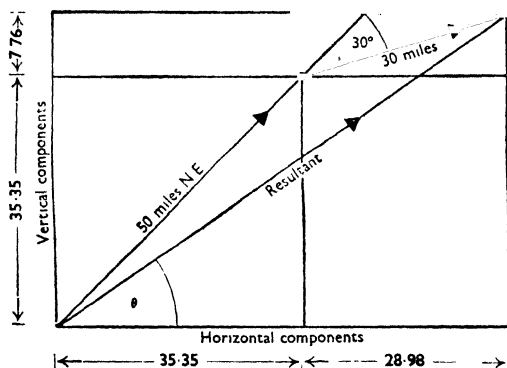


Fig. 8.17

A vector diagram showing the ship's course is given in fig. 8.17.

Horizontal component of 50 miles N.E. = $50 \cos 45^\circ$
 $= 50 \times 0.7071 = 35.35$.

Vertical " " " " = $50 \sin 45^\circ$
 $= 50 \times 0.7071 = 35.35$.

The change of course brings the ship's head 15° N. of E.

Horizontal component of new course = $30 \cos 15^\circ = 30 \times 0.9659$
 $= 28.98$.

Vertical " " " " = $30 \sin 15^\circ = 30 \times 0.2588$
 $= 7.76$.

Total of horizontal components = $35.35 + 28.98 = 64.33$.

Total of vertical " " = $35.35 + 7.76 = 43.11$.

Resultant = $\sqrt{\{(\text{total H. comp.})^2 + (\text{total V. comp.})^2\}}$
 $= \sqrt{(64.33^2 + 43.11^2)} = 77.5$ miles.

The direction of the resultant course can be found by trigonometry. Calling this direction θ degrees N. of E., then

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{total vertical component}}{\text{total horizontal component}} = \frac{43.11}{64.33} = 0.67.$$

$$\therefore \theta \text{ (from tables)} = 34^\circ \text{ (approx.)}$$

These results may be checked graphically.

N.B. When adding components, due regard must be paid to their sign. Thus, if the alteration of course had been 60° , the total vertical component would then be $35.35 - 7.76 = 27.59$.

EXERCISES ON CHAPTER VIII

1. What do you understand by (a) a vector quantity, (b) a scalar quantity? Give two examples of each.

A man walks 3 miles due N., then 2 miles S. 36° E., then 1 mile E. 20° N., and finally 4 miles due E. Find the distance and bearing of his final position from the starting point. (S.W.E.T.C.)

2. (a) Define the terms "equilibrant" and "resultant" of a number of forces.

(b) What is the angle between the lines of direction of two forces of 5 lb. and 12 lb., if their resultant is 15 lb.? (S.W.E.T.C.)

3. A lathe weighing 2000 lb. is suspended in the air by means of two chains. One chain OA makes an angle of 35° and the other OB an angle of 20° with the horizontal. Find the tension in each chain. (S.W.E.T.C.)

4. A force AB of 15 lb. acts at an angle of 30° from a force AC of 12 lb. Find the magnitude and direction of their resultant.

5. Two tugs are towing a ship at constant speed. The angles made by the tow ropes with the direction of motion of the ship are 60° and 30° respectively. The force opposing motion is 6 tons, acting along the line of motion. Find the pull exerted by each tug.

6. State the theorem of the triangle of forces and describe an experiment to verify it. A piece of elastic is stretched between two nails 112 cm. apart at the same horizontal level. A weight of 500 gm. is attached to the mid-point of the elastic with the result that X, the mid-point, is 42 cm. below its original position. Calculate the tension in the elastic. (C.P.E.)

7. Four forces acting at a point have magnitudes and directions as follows: 15 lb. easterly, 20 lb. northerly, 25 lb. north-westerly, and 10 lb. south-westerly. Find the magnitude and direction of the resultant force. (C.P.E.)

8. An aeroplane, whose speed in still air is 150 m.p.h., is flying against a wind blowing from the north at 30 m.p.h. Find the direction in which the aeroplane must be steered to travel in a north-easterly direction, and determine the distance covered in 1 hour.

(C.P.E.)

9. Explain the principle of the parallelogram of forces and describe an experiment to verify it. Find the resultant of two forces of 10 lb. wt. and 6 lb. wt. acting at right angles to one another.

(C.P.E.)

10. The base of a machine exerts a thrust of 5 tons, inclined at an angle of 10° to the vertical, upon its horizontal bedplate. Determine the vertical and horizontal components of the thrust, and state what the component forces tend to do.

(U.L.C.I.)

11. The horizontal and vertical components of a force are 5 and 12 lb. respectively. Specify this force.

12. A weight of 12 lb. is suspended from the junction of two strings 25 in. and 15 in. long respectively. The two strings are attached to two points A and B 30 in. apart. AB is horizontal.

(a) By drawing diagrams to scale, find the tensions in the strings.

(b) State the general effect on the tensions in the strings if A and B are moved farther apart.

(U.E.I.)

CHAPTER IX

Moments and Parallel Forces

The effect of forces which are parallel to each other will now be considered. Such forces do not meet at any point, and consequently cannot be dealt with by vectors as in Chap. VIII.

1. Turning effect of a force

The effect of a force at a point not in its line of action is spoken of as its **moment** about that point.

Consider a hinged door (fig. 9.1). A given push of say 10 lb. applied at A will have more effect on turning the door than 10 lb. applied at B.

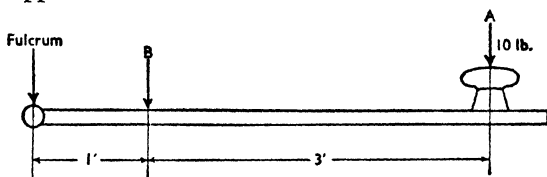


Fig. 9.1

We say that the “moment of the force” (meaning its turning effect) when applied at A is greater than its moment when applied at B.

“Moment” therefore depends upon “force” and “distance”, and may be measured in terms of derived units such as pound-feet (lb.-ft.), pound-inches, ounce-inches.

Taking the example of the door, the moment of the force of 10 lb. at A (when applied to the knob) about the hinge is

$$10 \text{ lb.} \times 4 \text{ ft.} = 40 \text{ lb.-ft.}$$

Its moment when applied at B is

$$10 \text{ lb.} \times 1 \text{ ft.} = 10 \text{ lb.-ft.}$$

Thus the force has four times greater moment when applied at the knob (here moment can be considered as meaning "importance" in the same way as we speak of "events of great moment").

The name **fulcrum** is given to that point about which rotation would occur. (The hinge is the fulcrum in the case of the door.)

2. Consideration of the effect of distance on moment

Consider an angler landing a fish of 5 lb. wt., using a 10-ft. rod (see fig. 9.2).

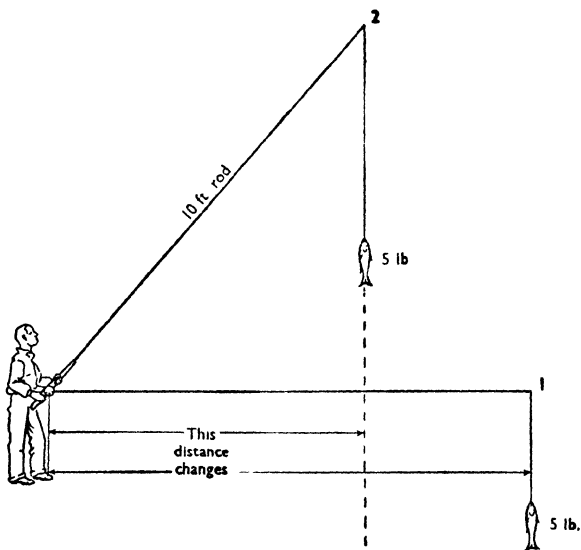


Fig. 9.2.—Showing the effect of distance on moment

It is obvious that he will have a more difficult task if he attempts the operation whilst in position 1 than if he does so in position 2.

This is because the moment of the weight of the fish about the fulcrum (the angler's arm) is greater in position 1 than it is in position 2. The weight of the fish has not changed, nor has

the length of the rod; but the distance between the angler and the fish changes, so changing the moment.

It is important to realize the correct distance to be used when calculating moments. It is the perpendicular distance between the line along which the force acts and the fulcrum.

N.B. "Perpendicular" distance means the distance measured at right angles to this line.

$\therefore$ **Moment of a force = force $\times$ perpendicular distance between line of action of force and the fulcrum.**

3. Turning moment of a jib crane

Consider the jib crane, with a jib of length 50 ft., designed to take loads up to a nominal value of 5 tons, and illustrated in fig. 9.3.

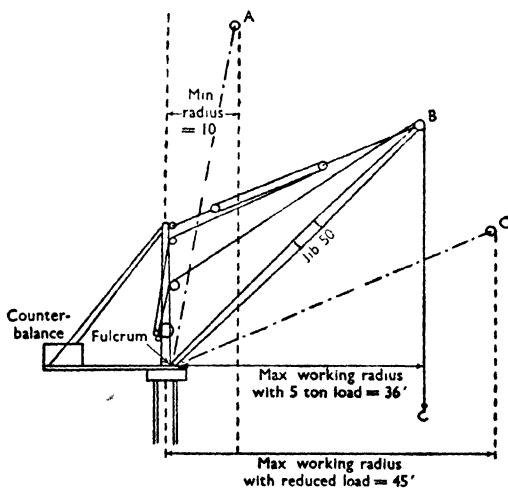


Fig. 9.3.—Turning moment of jib crane

The angle of inclination of the jib can be varied between position A and position C, by paying in or out on the pulley-tackle luffing gear. In position A, the crane is said to be operating at its "minimum working radius", and in position C it is at its "maximum working radius". The line of action of the load being vertically downwards, it will be seen that the per-

pendicular distance between this line and the fulcrum is the actual working radius.

$\therefore$ Turning moment in position A

$$= \text{load} \times \text{working radius} = 10L,$$

and

Turning moment in position C

$$= \text{load} \times \text{working radius} = 45L,$$

where L is the load. Consequently with a given load the force tending to overturn the crane (i.e. the turning moment) is $4\frac{1}{2}$ times greater in position C than in position A.

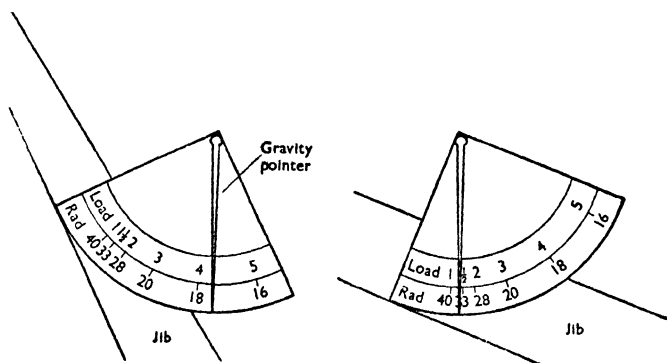


Fig. 9.4.—Simple safe-load indicator

This crane is designed to carry a load of 5 tons at a maximum radius of 36 ft. (position B). The maximum permissible turning moment is therefore $5 \times 36 = 180$ ton-ft. Should this value be exceeded, as it would be if the 5-ton load was swung out to the maximum working radius of 45 ft., then disaster might occur, and the crane might topple over. Hence, only restricted loads can be carried beyond a 36-ft. radius.

In practice a "safe load indicator" must be provided to indicate or warn the crane-driver when he approaches the danger point for the radius at which he is working. The simplest form of indicator is shown in fig. 9.4. It consists of a pendulum pointer capable of movement over a scale quadrant fixed to the jib. As the jib is raised or lowered, the pointer remains per-

pendicular, and indicates on the scale on the quadrant the working radius and the corresponding safe load. As the crane-driver may not always notice the indication, and as he does not always know the value of the load he is lifting, more complex warning devices are often fitted. These are arranged to weigh the load and to measure the angle of the jib (and hence its working radius). Should the combination of these two quantities (i.e. the moment of the load) exceed the permitted value for the angle in question, then a warning bell or light operates.

4. Torque

The turning effect, or turning moment of a force about a fulcrum, can produce rotation, and is often referred to as **torque**.

Example 1.—Find the turning moment, or torque, produced by a force of 150 lb. applied to a crank, at each of the points marked 1, 2, 3, in fig. 9.5.

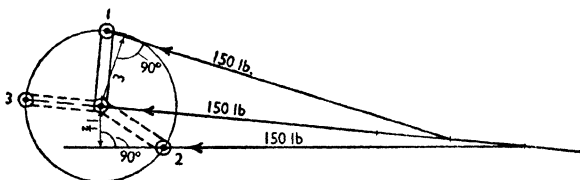


Fig 9.5

Turning moment =

force $\times$ perpendicular distance of line of action from fulcrum.

The fulcrum is the wheel centre.

At point 1: Torque = $150 \times 3 = 450$ lb.-ft.

At point 2: Torque = $150 \times 1\frac{1}{2} = 225$ lb.-ft.

At point 3: Torque = $150 \times 0 = \text{zero}$.

From this example it will be seen that the torque produced by the action of the piston and connecting rod in an engine varies according to the relative position of the crank. In two diametrically opposite positions, when the rod is acting through the fulcrum, the torque is zero. These positions are known as "dead-centre" positions, and the engine cannot be started in these positions.

5. Kinds of moments

For convenience, moments may be divided into two classes:

1. Those producing, or tending to produce, **clockwise** rotation.
2. Those producing, or tending to produce, **anticlockwise** rotation.

These are sometimes called negative and positive moments respectively.

The net turning moment in any given case may be found by calculating these clockwise and anticlockwise moments, and then subtracting one from the other. The direction of rotation will, of course, be that of the predominating moment.

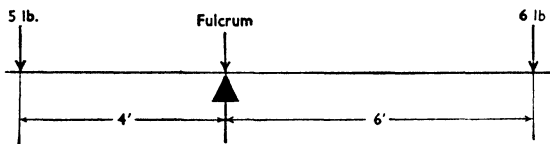


Fig. 9.6

Example 2.—Find the net turning moment and its direction in fig. 9.6.

Take moments about the fulcrum.

Clockwise moments: $6 \text{ lb.} \times 6 \text{ ft.} = 36 \text{ lb.-ft.}$

Anticlockwise moments: $5 \text{ lb.} \times 4 \text{ ft.} = 20 \text{ lb.-ft.}$

Net moment = 16 lb.-ft.

As the clockwise moments predominate, then the plank would turn clockwise with a moment of 16 lb.-ft.

6. The principle of moments

From the above example it will readily be seen that the net turning moment will be zero if the clockwise moments are so arranged as to equal the anticlockwise moments. (The plank will then be balanced, or in equilibrium.)

This is stated by the **Principle of Moments**:

“A body free to rotate about a point is in equilibrium when the sum of the anticlockwise moments is equal to the sum of the clockwise moments, taken about the fulcrum.”

Example 3.—Find the force required in lieu of the 5-lb. weight in the previous example in order to produce equilibrium.

Call this required force E lb.

Then clockwise moments about fulcrum = 6 lb. $\times$ 6 ft. = 36 lb.-ft.
and anticlockwise „ „ „ = $4E$.

In order to produce equilibrium, clockwise and anticlockwise moments must be equal.

$$\therefore 4E = 36,$$

and
$$E = \frac{36}{4} = 9 \text{ lb.}$$

7. Levers

The Principle of Moments, or the Principle of the Lever, as it is sometimes called, has been applied to lifting heavy articles, comparing weights, etc., since ancient times. By applying a small force (the **effort**) to one end of a lever, a much greater force can be exerted at the other end. However, the small force must be moved through a large distance in order to raise, say, a heavy weight (the **load**) through a small distance. One early philosopher, Archimedes, enthusiastically describing the use of levers, said, "Give me a lever long enough and I will move the earth." He would, of course, have needed some suitable fulcrum for his lever to work against.

To-day the lever is encountered in every walk of life. Common examples are the crowbar, a human arm, nutcrackers, shop scales, scissors, signal levers, and control arms in many forms of machinery.

8. Types of levers

Levers may conveniently be classified into three types, according to the relative positions of load, effort, and fulcrum. Examples are given below.

Lever of the first order.—Fulcrum between load and effort.

Example 4.—Determine the force which a man must exert at E in order to raise a body of weight $\frac{1}{2}$ ton at L , by means of the crowbar shown (fig. 9.7). (Note that the force applied to a lever is known as the "effort" and the force overcome, or balanced, is known as the "load".)

Take moments about the fulcrum.

Clockwise moments = $\frac{1}{2}$ ton $\times$ 1 ft. = 1120 lb.-ft.

Anticlockwise moments = $E \times 5$.

To produce equilibrium $5E$ must equal 1120.

$$\therefore E = \frac{1120}{5} = 224 \text{ lb.}$$

$\therefore$ The man must exert something over 224 lb. to raise the load.

N.B. This is the force required if applied vertically downwards. If applied at right angles to crowbar, slightly less force would be needed.

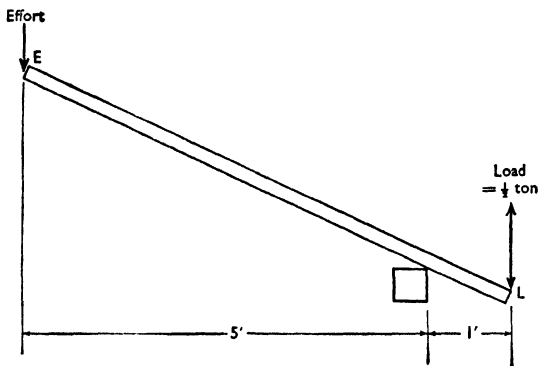


Fig. 9.7

Example 5.—In the case of the jib crane in fig. 9.3, find the minimum weight required to just counterbalance a 5-ton load at the maximum working radius of 36 ft., if the distance from the kingpost (fulcrum) to the centre of the counterbalancing weight is 15 ft.

Let x = required weight. Then, taking moments about fulcrum,

Clockwise moments = $5 \times 36 = 180$ ton-ft.

Anticlockwise moments = $x \times 15$.

$$15x = 180.$$

$$\therefore x = \frac{180}{15} = 12 \text{ tons.}$$

Lever of the second order.—Load (L) between fulcrum and effort (E).

In this class the effort is always less than the load.

Example 6.—A porter's trolley is being used to lift a box of 56 lb. wt. Find the effort required to balance this load if the dimensions are as shown in fig. 9.8.

Take moments about the fulcrum.

$$\text{Clockwise moments} = 3\frac{1}{2}E.$$

$$\text{Anticlockwise moments} = 56 \times \frac{3}{4} = 42.$$

$$\therefore 3\frac{1}{2}E = 42.$$

$$\therefore E = 42 \div 3\frac{1}{2} = 12 \text{ lb.}$$

Note in this case that half the effort is applied at each handle.

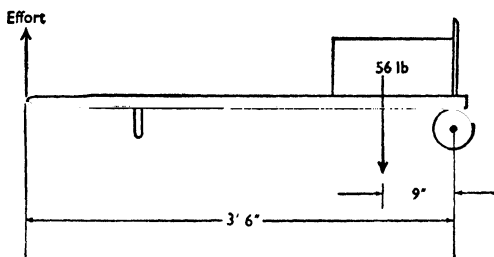


Fig. 9.8

Lever of the third order.—Effort applied between fulcrum and load.

In this class of lever the effort is greater than the load, but the load moves through a greater distance than the effort. This may be very convenient in some cases, as in the lever safety valve illustrated. This type of safety valve is sometimes fitted to stationary boilers to prevent the steam pressure rising above the safe limit and thereby causing the boiler to explode. As the steam pressure rises, it increases the force on the valve (V), and thereby increases its moment about the fulcrum. When this moment exceeds the moment of the weight (W), the valve will lift and so relieve the pressure.

Example 7.—Find the weight needed in order that steam may “blow off” at 100 lb. per sq. in. in the valve illustrated in fig. 9.9.

$$\text{Area of valve} = \pi r^2 = 3.142 \times 1^2 = 3.142 \text{ sq. in.}$$

$$\text{Pressure on valve} = 3.142 \times 100 = 314.2 \text{ lb.}$$

Take moments about fulcrum.

$$\text{Clockwise moments} = 30W$$

$$\text{Anticlockwise moments} = 314.2 \times 5 = 1571 \text{ lb.-in.}$$

$$30W = 1571, \quad \therefore W = 1571 \div 30 = 52.36 \text{ lb.}$$

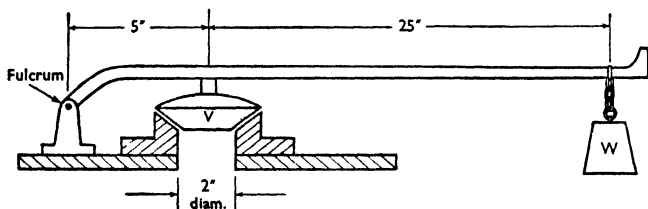


Fig. 9.9.—Lever safety valve

9. Reaction at the fulcrum of a lever

As action and reaction are equal and opposite, then, in any form of lever, the thrust of the forces on it must be balanced by a reaction between the fulcrum and its point of contact with the lever.

Example 8.—Find the force at E to produce equilibrium, and then determine the reaction at the fulcrum (see fig. 9.10).

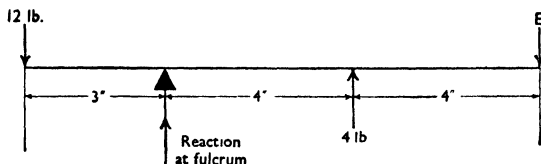


Fig. 9.10

$$\text{Clockwise moments} = 8E.$$

$$\text{Anticlockwise moments} = 12 \times 3 + 4 \times 4 = 36 + 16 = 52.$$

$$8E = 52.$$

$$\therefore E = 6\frac{1}{2} \text{ lb.}$$

$$\text{Pressure on fulcrum} = 12 + 6\frac{1}{2} - 4 = 14\frac{1}{2} \text{ lb.}$$

From this it will be seen that the reaction at the fulcrum is equal to the *algebraic* sum of the forces, and that its direction is opposite to that of this sum, *if the forces are parallel*.

10. The resultant of a number of parallel forces

The resultant of a number of parallel forces may be found in like manner. Its value will be the algebraic sum of the forces, and its position such that the clockwise moments taken about it equal the anticlockwise moments. (Or, moment of resultant about any point equals algebraic sum of moments of component forces about same point.)

Example 9.—Find the resultant of the forces shown in fig. 9.11 (same forces as in previous example).

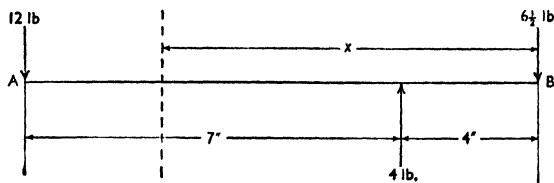


Fig 9 11

Magnitude of resultant $= 12 + 6\frac{1}{2} - 4 = 14\frac{1}{2}$ lb. (acting downwards). To find position of resultant, take moments about B, calling the position of the resultant x from B.

$$\text{Moment of resultant} = 14\frac{1}{2} \times x.$$

$$\text{Algebraic sum of component forces} = (12 \times 11) - (4 \times 4).$$

$$\therefore 14\frac{1}{2}x = 132 - 16 = 116.$$

$$\therefore x = 8.$$

Resultant is $14\frac{1}{2}$ lb. acting downwards 8" from B. (Compare with previous example.)

N.B. The equilbrant is equal but opposite to the resultant, and corresponds to the reaction at the fulcrum in the previous example. Check that the clockwise moments taken about the position of the resultant are equal to the anticlockwise moments:

$$\text{Clockwise moments} = 6\frac{1}{2} \times 8 = 52.$$

$$\text{Anticlockwise moments} = (12 \times 3) + (4 \times 4) = 52.$$

Example 10.—A beam AB of negligible weight carries loads of 3, 5, 6 and 4 lb. situated at 8-in. intervals from A (the 3-lb. load being at A). Determine the position about which the beam can be balanced.

$$\text{Resultant load} = 3 + 5 + 6 + 4 = 18 \text{ lb.}$$

The algebraic sum of the moments of the loads about any point in the beam will be equal to the moment of the resultant load about that point.

Take moments about A, and let x be distance of resultant from A.

$$\begin{aligned}\text{Clockwise moments} &= 3 \times 0 + 5 \times 8 + 6 \times 16 + 4 \times 24 \\ &= 232 \text{ lb.-ft.}\end{aligned}$$

$$\text{Anticlockwise moments} = 18x.$$

$$\therefore x = \frac{232}{18} = 12.9 \text{ in.}$$

11. Couple

Two equal and parallel forces of opposing sense of direction produce what is termed a **couple**. They produce no resultant force, as one balances the other, but they do produce a resultant

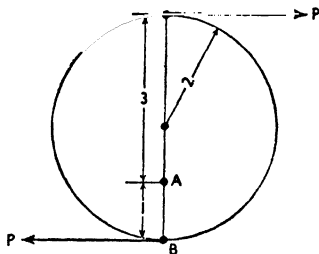


Fig 9.12 — A couple

moment. This causes rotation to take place, owing to the moment of each force taken about *any* point in the perpendicular line joining them.

If we imagine rotation to occur about the mid-point (fig. 9.12), then $\text{Moment of couple} = P \times 2 + P \times 2 = 4P.$

If moments are taken about point A, then

$$\text{Moment of couple} = P \times 3 + P \times 1 = 4P.$$

If moments are taken about point B, then

$$\text{Moment of couple} = P \times 4 + P \times 0 = 4P.$$

Thus, the same result ($4P$) is obtained in each case.

Therefore the moment of any couple = one of the forces $\times$ perpendicular distance between the forces,

i.e. $\text{Moment} = PD$,

where P = one of the forces, and D = perpendicular distance between them. The distance D is often called the "arm" of the couple.

An everyday example of a couple is given by the action of winding up a clock. A couple is applied to the key and results in the production of a turning moment (torque).

Example 11.—In a certain electrical instrument two conductors placed diametrically opposite on the circumference of a rotor of 0.6 in. diameter are acted upon by two equal and opposite forces of 2.2 oz. each. Find the torque developed.

Arm of couple = $D = 0.6$ in. One force = $P = 2.2$ oz.

Moment of couple (torque) = $PD = 2.2 \times 0.6 = 1.32$ oz.-in.

12. Application of the principle of moments to the determination of the reactions at supports

It is often desired to determine the loadings which the supports of a bridge, roof, or similar structure have to withstand under given conditions. For instance, a heavy vehicle would impose different loadings on the supports of a bridge according to its position; again, two men carrying a load not positioned centrally between them would share this load unevenly.

To produce equilibrium, the sum of the forces acting downwards must equal those acting upwards.

$\therefore$ Total loading

= reaction at support R + reaction at support S.

Example 12.—Determine each man's share of the load when carrying 150 lb. arranged as in fig. 9.13.

Reaction at R + Reaction at S = 150 lb. Now suppose the man at S were to let go. The plank would then rotate about R. Take moments about R as fulcrum.

Clockwise moments = $150 \times 2\frac{1}{2}$ ft. = 375 lb.-ft.

In order to support this, the man S must produce an equal moment, i.e.

$$\text{Reaction at S} \times 7\frac{1}{2} \text{ ft.} = 375 \text{ lb.-ft.}$$

$$\therefore \text{Reaction at S} = 375 \div 7\frac{1}{2} = 50 \text{ lb.}$$

Man at R has a share of the load which will be given by

$$\text{Total loading} - \text{reaction at S} = 150 - 50 = 100 \text{ lb.}$$

A similar result would be obtained if moments were taken about S as fulcrum.

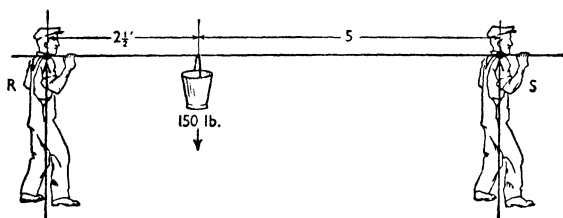


Fig. 9.13

Example 13.—A locomotive is standing on a bridge. The axle loadings and their distribution are as shown in fig. 9.14. Find the division of the total load between the supports of the bridge.

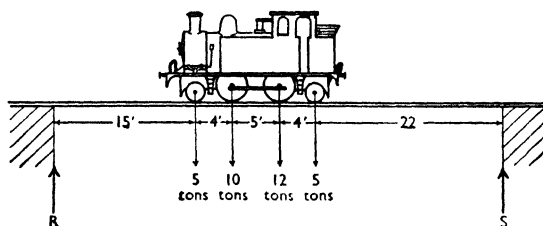


Fig. 9.14

The total loading must equal (reaction at R + reaction at S). Take moments about R.

Clockwise moments

$$= (5 \times 15) + (10 \times 19) + (12 \times 24) + (5 \times 28)$$

$$= 693 \text{ ton-ft.}$$

Anticlockwise moments = $50S$.

$$50S = 693$$

$$\therefore S = \frac{693}{50} = 13.86 \text{ tons.}$$

Reaction at R must equal (total loading - S),

$$\therefore R = (5 + 10 + 12 + 5) - 13.86 = 18.14 \text{ tons.}$$

The loadings are 13.86 tons at S and 18.14 tons at R.

It will be noticed that the weight of the bridge has been ignored in the above calculation, the loading found being that due to the locomotive only. If the bridge is symmetrical, then the supports will each carry half of the weight of the bridge. For instance, if the bridge weighs 80 tons, then the total loading of each support (with locomotive on the bridge) will be

$$R = 18.14 + 40 = 58.14 \text{ tons,}$$

$$S = 13.86 + 40 = 53.86 \text{ tons.}$$

Example 14.—Load overhanging the supports. Determine the reactions at the supports of the beam shown in fig. 9.15.

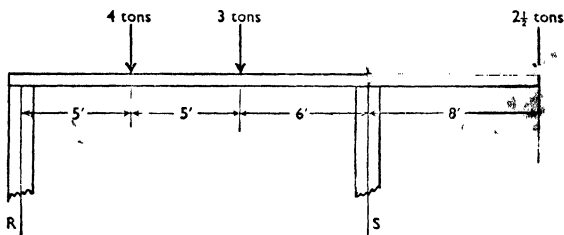


Fig. 9.15

Take moments about R.

$$\begin{aligned} \text{Clockwise moments} &= (4 \times 5) + (3 \times 10) + (2\frac{1}{2} \times 24) \\ &= 20 + 30 + 60 = 110 \text{ ton-ft.} \end{aligned}$$

Anticlockwise moments = $16S$.

$$16S = 110,$$

$$\therefore \text{reaction at S} = \frac{110}{16} = 6\frac{7}{8} \text{ tons.}$$

Reaction at R = total loading - reaction at S

$$= (4 + 3 + 2\frac{1}{2}) - 6\frac{7}{8} = 9\frac{1}{2} - 6\frac{7}{8} = 2\frac{5}{8} \text{ tons.}$$

Again, notice that the weight of the beam has been ignored, as this involves a knowledge of centres of gravity. This will be considered in the next chapter.

13. The weighbridge

Modern railway and platform-type weighbridges were evolved from the design produced by Wyatt in 1741. Prior to this, in order to weigh a heavy vehicle it had been necessary to lift it off the ground. With a weighbridge, however, the vehicle is simply driven on to a platform flush with ground level, and a system of levers does the rest.

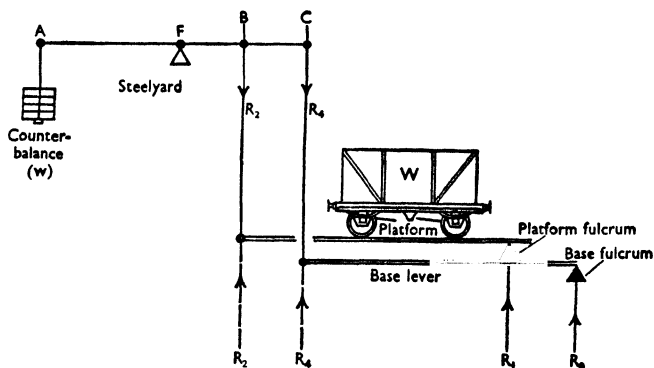


Fig. 9.16.—Weighbridge

A skeleton lever-diagram showing the principle of the weighbridge is given in fig. 9.16. The position of the load (W) will not affect the results. For any position of W , platform reactions

$$R_1 + R_2 = W.$$

R_2 is communicated to the steelyard. R_1 is transmitted to the base lever. (N.B. It has no moment about platform fulcrum.) There it sets up reactions

$$R_3 + R_4 = R_1.$$

R_3 has no moment about base-lever fulcrum. R_4 is communicated to the steelyard. Therefore reactions R_2 and R_4 act on steelyard and are counterbalanced by w .

Taking moments about steelyard fulcrum,

$$\text{Clockwise moments} = R_2 \times BF + R_4 \times CF.$$

$$\text{Anticlockwise moments} = w \times AF.$$

$$\therefore w = \frac{R_2 \times BF + R_4 \times CF}{AF}.$$

Usually the dimensions are designed so that the counterbalance w required is one-tenth of the load W being weighed, i.e. $w = \frac{1}{10}W$.

EXERCISES ON CHAPTER IX

1. A lever 11 ft. 6 in. long operates about a fulcrum 3 ft. 6 in. from one end. Find the greatest weight which can be moved at this end if an effort of 65 lb. is applied at the other end.

2. A lever ABCD is 17 in. long and is pivoted at B. Loads of 100 lb. and 175 lb. are placed at C and D respectively. Find the load necessary at A in order to produce equilibrium, given that AB = 5 in. and CD = 6 in.

3. The axles of a road roller are 8 ft. apart, and carry loads of 20 tons and 5 tons respectively. If the roller is placed on a bridge of 40 ft. span, with its front axle (carrying the 5-ton load) 12 ft. from the right-hand support, determine the distribution of the load between the supports.

4. A wheelbarrow carries a load of 1 cwt. which can be regarded as concentrated at a point 18 in. from the wheel centre. Find the tension in each one of a man's arms when lifting the handles, if his hands are 4 ft. from the wheel centre.

5. The arm of a lever safety valve is 18 in. long. It carries a valve of weight $1\frac{1}{2}$ lb. and area $3\frac{1}{2}$ sq. in. fixed at a point 3 in. from the fulcrum. The valve is required to blow off at a pressure of 100 lb./in.² Find the weight required at the end of the lever to secure this.

6. A beam ABCDE is 17 ft. long and is supported at A and E. It carries loads of 112, 56, and 28 lb. at points B, C, and D respectively. Given that AB = 5 ft., CD = 4 ft., and DE = 2 ft., find (a) the reactions at the supports, and (b) the magnitude and position of the resultant load.

7. What is meant by the moment of a force?

A beam rests horizontally with a support at each end. The distance between the supports is 15 ft. At 7 ft. from the left end is a concentrated load of 7 tons. Find the reactions at the supports. Use decimals. (S.W.E.T.C.)

8. A cantilever is 8 ft. long and carries weights of 10 lb., 6 lb., and 5 lb. at distances of 3 ft., 5 ft., and 7 ft. respectively from the

fixed end. What is the total turning effect of these weights about the fixed end? Where in the cantilever would you place a weight of 20 lb. which would give the same turning effect as the separate weights? (Neglect the weight of the beam.) (S.W.E.T.C.)

9. A light rod AB 20 in. long is pivoted at A, and hangs vertically. Forces are applied simultaneously by means of strings attached to the rod. A force CD of 4 lb. is applied at C, 16 in. below A, the angle BCD being 45° , D being to the left of the rod; and a horizontal force BE of 3 lb. is applied at B, E being to the right of the rod. Find the resultant turning moment on the rod. Where must a force of 2 lb. be applied to prevent rotation of the rod? (U.E.I.)

10. Draw a line horizontally 5 in. long. Mark it AE. On it mark off AB, BC, and CD, 2 in., 1 in., and $\frac{1}{2}$ in. respectively. AE is a lever pivoted at C. Vertical forces of 2, 3, 4, and 4 lb. act at A, B, D, and E respectively. State what will happen to the lever under the action of these forces and why. What additional force will be required at A to produce equilibrium? Neglect the weight of the lever. (U.E.I.)

11. A cranked lever is shown in which a pulling force P just balances the weight W . If W weighs 10 lb., what is the value of P ,

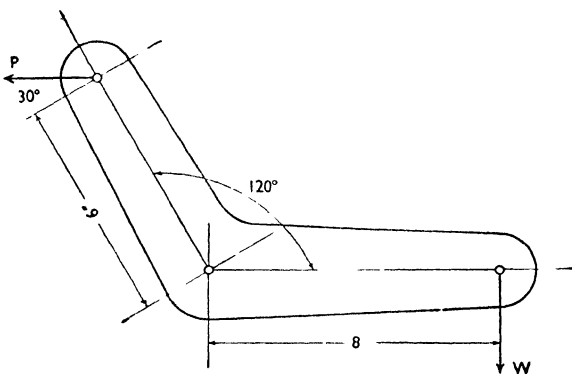


FIG 9.17

(a) when its line of direction is horizontal, (b) when its line of direction is 30° below the horizontal? (S.W.E.T.C.)

12. A beam AB is 40 in. long, and rests on supports at A and B. The reaction at A is 8 lb., and the reaction at B is 12 lb. Two weights are suspended from the beam, one of 10 lb. at a point 16 in. from A, and the other W lb. Find W and its position from A on the beam. (S.W.E.T.C.)

13. A man and a boy carry a uniform plank 12 ft. long on which rests a box, weighing 54 lb. The plank itself weighs 30 lb. If they each hold one end of the plank, at what distance from the man must the weight of the box be concentrated so that the man shall be supporting twice the weight supported by the boy? (U.L.C.I.)

14. A horizontal beam 8 ft. long rests symmetrically on two supports 5 ft. apart. Concentrated loads of 500, 300 and 600 lb. are carried on the beam at points 1, 4 and 7 ft. respectively from one end. Neglecting the weight of the beam, find the reaction at each support.

To what position should the 300-lb. load be moved to make the reactions equal? (C.P.E.)

15. Obtain the turning moment on the crankshaft of an engine when the crank has moved through an angle of 45° from the inner dead centre in a clockwise direction, the total pressure on the piston being 1000 lb. The radius of the crank is 6 in., and the distance between the connecting-rod centres is 2 ft. 6 in. (U.E.I.)

16. What do you understand by a "couple"?

Show that the moment of a couple is the same about all points in its plane.

17. Explain what is meant by the "moment of a force about a point". Give two examples of a lever and explain their functions by the aid of suitable diagrams. A bridge 200 ft. long weighs 100 tons and is supported at each end. What are the reactions of the supports when a bus weighing 10 tons is 40 ft. from one end? Give reasons for your answer. (C.P.E.)

CHAPTER X

Centres of Gravity

1. Centre of gravity

Since gravity acts on all matter, then every particle of a body will experience a force tending to pull it towards the earth's centre. For a body of normal size, i.e. small in comparison to the earth, the forces on these particles can be considered as parallel. Therefore they can be replaced by a resultant force acting at a point which is called the **centre of gravity** (or **centre of mass**) of the body.

For purposes of calculation, then, the weight of a body is considered to be concentrated at its centre of gravity (often abbreviated to c.g.). Many problems, especially those concerning equilibrium, make use of this fact.

In many cases, when a body is symmetrical and of uniform composition, the position of its centre of gravity will be obvious, as it coincides with its geometrical centre. Thus, the centre of gravity of a sphere will be at its centre; of a long uniform rod, at the mid-point of its centre line; of a parallelepiped (parallel-sided figure), at the intersection of its diagonals.

2. Centroid

The centre of gravity of a flat surface (such as a thin uniform sheet) is sometimes referred to as the **centroid**, or **centre of area**, of that figure. The centroids of **laminæ** (thin sheets) of various shapes may readily be found by experiment. The lamina is suspended from a point, preferably near its edge, and a plumb-line is then used to draw a vertical line through this point. This process is repeated with the lamina suspended from various other points. It will be found that these lines intersect, the point at which they do so being the centroid (see fig. 10.1). This may be proved by making a small hole at

the point and pinning the lamina from it. The lamina will then remain at rest in whatever position it is placed.

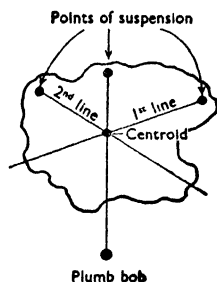


Fig. 10.1.—Method of finding centroid of an irregular figure

The centroid of a triangular lamina may be found in this manner, and it will be seen to lie at the point of intersection of the medians. (A median is a line joining the mid-point of a side of a triangle to the opposite vertex.) This point will be found to be one-third of the distance along a median. The centres of gravity of various figures are shown in fig. 10.2.

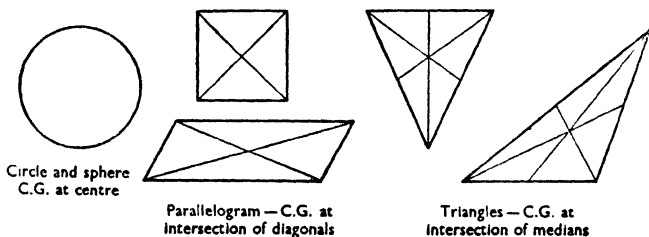


Fig. 10.2

3. Determination of the centre of gravity by moments

The position of the centre of gravity of a figure built up of several areas can be determined by taking moments of these areas about some convenient point, i.e. by treating the areas as if they were forces. The sum of these moments will then be

equal to the moment of the total area, acting through its centroid, considered about the same point.

Example 1.—Find the centre of gravity of the section of the girder shown in fig. 10.3.

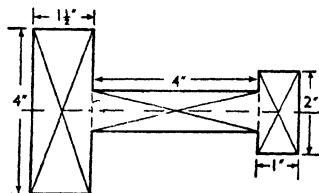


FIG 10.3

The c.g. of each section is first found by drawing diagonals.

Area of section 1 is 6 sq. in.

Area of section 2 is 4 sq. in.

Area of section 3 is 2 sq. in.

Treat areas as forces, and take moments about a point A situated on centre-line at extreme left of section.

Moment of section 1 about A = $6 \times \frac{3}{2} = 4\frac{1}{2}$.

Moment of section 2 about A = $4 \times 3\frac{1}{2} = 14$.

Moment of section 3 about A = $2 \times 6 = 12$.

The "force" of the total area will act through the c.g. of the complete figure. Call position of this c.g. x in. from A. Then moment of total area = $(6 + 4 + 2) \times x = 12x$. This must equal the sum of the component section moments,

$$\therefore 12x = 4\frac{1}{2} + 14 + 12 = 30\frac{1}{2}.$$

$$\therefore x = 2.54 \text{ in.}$$

The centre of gravity of an area or object from which parts have been removed can be found by subtracting the moments of the removed parts from that of the original total moment, all moments being considered about the same point.

Example 2.—A rectangular steel bar, 3 in. wide $\times$ 1 in. thick $\times$ 10 in. long, has a 2-in. diameter hole drilled through it, the centre of the hole being 4 in. from one end of the bar. Find the position of the centre of gravity of the bar.
(S.W.E.T.C.)

The arrangement of the bar is shown in fig. 10.4. The c.g. of the bar without the hole will be 5 in. from X along the centre line.

Area of the face of the bar without the hole is $10 \times 3 = 30$ sq. in.

Area of the hole $= \pi r^2 = \pi \times 1^2 = 3.142$ sq. in.

Area of the face of the bar with hole $= 30 - 3.142 = 26.858$ sq. in.

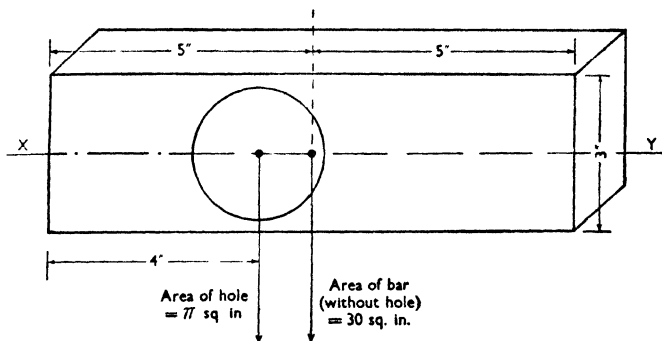


Fig. 10.4

Let d be the distance of the centroid of face of the bar (with hole in it) from X. Take moments about X.

$$30 \times 5 - 3.142 \times 4 = 26.86d.$$

$$26.86d = 137.43$$

$$\therefore d = \frac{137.43}{26.86} = 5.115 \text{ in.}$$

c.g. is 5.115 in. along XY from X, and at a depth of $\frac{1}{2}$ in.

Often the centre of gravity of a body can be determined by taking simultaneous readings of the weights at its supports, then using moments to find the line of action of its resultant weight. If the body is symmetrical the position of its centre of gravity will then be obvious.

When lifting a heavy or cumbersome load by means of a crane, it should be suspended from a point vertically above its centre of gravity. This will ensure that the load is lifted on "an even keel".

Large electric motors, turbine casings, and similar types of heavy machinery are often fitted with an eyebolt placed above the centre of gravity for this very purpose.

Example 3.—From the axle loadings and dimensions of the tank locomotive shown in fig. 9.14, determine the line along which its c.g. lies.

Let x = horizontal distance from leading-wheel centre to the vertical line through the c.g.

Taking moments about leading-wheel centre,

Clockwise moments = $10 \times 4 + 12 \times 9 + 5 \times 13 = 213$ ton-ft.

Anticlockwise moments = (total weight) $\times x = 32x$ ton-ft.

$$32x = 213,$$

$$\therefore x = \frac{213}{32} = 6\frac{21}{32} \text{ ft.}$$

Centre of gravity is on a vertical line $6\frac{21}{32}$ ft., or 6 ft. $7\frac{7}{8}$ in. measured horizontally from and to the rear of the leading axle.

4. Moment of the weight of a body

As the weight of a body acts through its centre of gravity, then this weight will have a moment about a fulcrum. This moment should be considered in any problem dealt with, unless the weight of the body is small in comparison to the other forces involved.

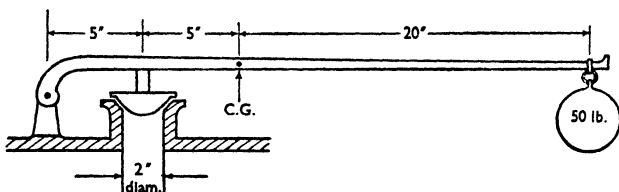


Fig. 10.5

Example 4.—Fig. 10.5 shows a lever safety valve (similar to that considered in the previous chapter, para. 8). The weight of the lever is 10 lb., and its c.g. is 10 in. from the fulcrum. If the weight on the end of the lever is 50 lb., determine the pressure at which the steam will “blow off”.

Let x = total force (lb.) of steam acting on valve.

Then Clockwise moments = $(10 \times 10) + (50 \times 30) = 1600$,

and Anticlockwise moments = $5x$.

$$5x = 1600.$$

$$\therefore x = 320 \text{ lb.}$$

$$\text{Area of valve} = \pi \times 1^2 = 3.142 \text{ sq. in.}$$

Blow off steam pressure will be $\frac{320}{3.142} = 101.8$ lb. per sq. in.

Example 5.—A uniform beam 30 ft. long weighs $\frac{1}{2}$ ton. It is supported at two points B and C. The reaction at C is 2 tons. The position of point B is not specified. It is loaded at the end remote from C, and by means of a truck. Loadings and position are shown in fig. 10.6. Determine the reaction at B, and its position.

Total load = reaction at B + reaction at C.

$$\begin{aligned}\therefore \text{Reaction at B} &= \text{total load} - \text{reaction at C} \\ &= (2 + 2 + 1 + \text{wt. of beam}) - 2 \\ &= 5\frac{1}{2} - 2 = 3\frac{1}{2} \text{ tons.}\end{aligned}$$

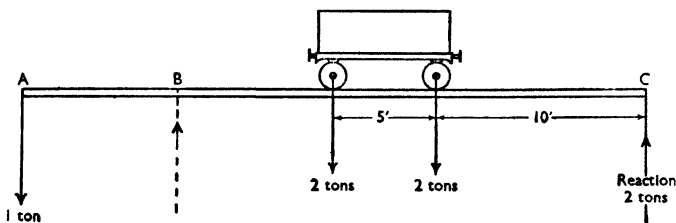


Fig. 10.6

Take moments about C.

$$\begin{aligned}\text{Clockwise moments} &= \text{reaction at B} \times \text{distance BC} \\ &= 3\frac{1}{2} \times \text{BC ton-ft.}\end{aligned}$$

$$\begin{aligned}\text{Anticlockwise moments} &= (2 \times 10) + (2 \times 15) + (1 \times 30) \\ &\quad + (\text{wt. of beam} \times 15) \\ &= 20 + 30 + 30 + 7\frac{1}{2} = 87\frac{1}{2} \text{ ton-ft.}\end{aligned}$$

To produce equilibrium

$$\begin{aligned}3\frac{1}{2} \times \text{BC} &= 87\frac{1}{2}, \\ \therefore \text{BC} &= \frac{87\frac{1}{2}}{3\frac{1}{2}} = 25 \text{ ft.}\end{aligned}$$

5. The common or Roman steelyard

A steelyard is a device which makes use of the Principle of Moments to determine the weight of a body. The common steelyard (sometimes made up in the form of jockey scales) is

shown in fig. 10.7. It consists of a lever pivoted on a fulcrum (F). The body to be weighed (W) is placed in the scale-pan and is balanced by a movable weight or bob (E), which can be adjusted to secure balance by sliding it along the graduated scale 0, 1, 2, 3, etc. The position of this weight indicates the weight of the body in the appropriate units. The centre of

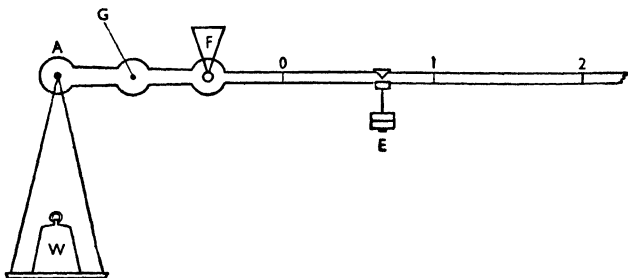


Fig. 10.7.—Roman steelyard

gravity of the steelyard is situated at G, and therefore it is necessary to counterbalance the moment of the weight of the steelyard about its centre of gravity by applying an equal and opposite moment by means of the movable bob (E). When balance is secured, the position of the bob on the scale will be the zero point (O).

Example 6.—(a) The weight of a steelyard is 10 lb. and the position of its c.g. is $1\frac{1}{2}$ in. from the fulcrum. The weight of the movable bob is 10 lb. Find the position of the zero point on the graduated scale.

Take moments about F (fig. 10.7).

Clockwise moments = 10 lb. $\times$ OF.

Anticlockwise moments = 10 lb. $\times$ $1\frac{1}{2}$ in. = 15 lb.-in.

When in equilibrium, $10 \times \text{OF} = 15$,

therefore the position of the zero point (O) will be $\frac{15}{10} = 1\frac{1}{2}$ in. from fulcrum.

(b) If a weight of 14 lb. is placed in the scale-pan, find the position of the bob on the scale to produce balance. Distance AF is 3 in.

Clockwise moments = 10 lb. $\times$ distance of bob from fulcrum.

$$\begin{aligned}\text{Anticlockwise moments} &= (10 \times 1\frac{1}{2}) + (14 \times 3) = 15 + 42 \\ &= 57 \text{ lb.-in.}\end{aligned}$$

$$\therefore \text{Distance of bob from fulcrum} = \frac{57}{10} = 5.7 \text{ in. (or } 5.7 - 1.5 = 4.2 \text{ in. from O).}$$

(c) If the weight in the scale-pan is doubled, find the distance of the bob from O to produce balance.

Clockwise moments = 10 lb. $\times$ distance of bob from fulcrum.

$$\text{Anticlockwise moments} = (10 \times 1\frac{1}{2}) + (28 \times 3) = 15 + 84 = 99 \text{ lb.-in.}$$

$$\therefore \text{Distance of bob from fulcrum} = \frac{99}{10} = 9.9 \text{ in., and distance from O is } 8.4 \text{ in.}$$

It will be noticed from this example that doubling the weight of the body in the scale-pan (increasing it from 14 lb. to 28 lb.) results in the doubling of the distance of the bob from the zero point on the scale (increasing it from 4.2 in. to 8.4 in.). Therefore the length of the scale is directly proportional to the weight being measured. A distance of 4.2 in. from O on the scale would be marked 1 stone (14 lb.), a further distance of 4.2 in. would be marked 2 stones, and so on. The space between each main division would be divided evenly into 14 parts, to represent pounds.

6. Centre of gravity and stability

The position of the centre of gravity of such bodies as vehicles, walls, buildings, and ships is of importance as it determines whether or not that body is stable.

For a body to remain in stable equilibrium the line of action of its weight (which will act through its centre of gravity) must fall inside its base.

The wall (fig. 10.8) will thus be in stable equilibrium, but the loaded motor lorry (fig. 10.9), although stable on level ground, will become unstable and will overturn when tipped at such an inclination that the line of action of its weight falls outside its wheelbase.

A body is in **stable equilibrium** if, after being slightly displaced from this position, the moment of its weight acts to restore it to its original position. If this moment acts to overturn the body, it is said to be in **unstable equilibrium**. When,

however, *any* displacement from the original position does not affect the body's state of equilibrium (as, for instance, moving a ball on a flat surface), then that body is said to be in **neutral**

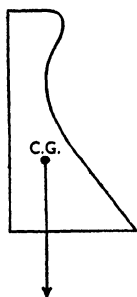


Fig. 10.8

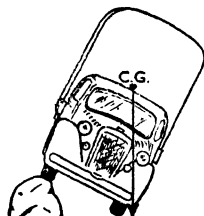


Fig. 10.9

equilibrium. For example, a cone standing on its base is in stable equilibrium; balanced on its apex (the apex being slightly flattened) it is in unstable equilibrium; lying on its side it is in neutral equilibrium (see fig. 10.10).

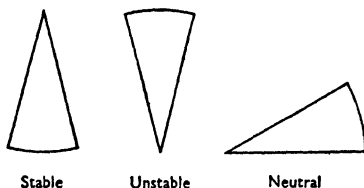


Fig. 10.10.—Equilibrium

EXERCISES ON CHAPTER X

Section A

1. A uniform beam is 12 ft. long and weighs 60 lb. It is supported at its ends A and B. Four feet from end A a load of 10 lb. is placed, and 3 ft. from the other end B a load of 24 lb. is placed. Find the reactions at A and B. (S.W.E.T.C.)

2. A beam AB is 10 ft. long and weighs 6 lb. A load of 1 lb. is suspended at B, and a load of 4 lb. at 6 ft. from B. The beam is supported at A and at another point. The reaction of the support at A is 4 lb. Where is the other support, and what is the reaction at this support? (S.W.E.T.C.)

3. A plank of weight 56 lb. and length 20 ft. is placed horizontally on two walls 15 ft. apart, one end projecting 5 ft. beyond one of the walls. A man of weight 10 stones walks along the plank towards this end. Find at what distance he will be from the end when the plank tips up. (S.W.E.T.C.)

4. A uniform isosceles triangle has its two equal sides each 5 ft. long, and its base 8 ft. long. Find its centre of gravity. If its weight be 5 lb., and a weight of 10 lb. be hung at the vertex, find the centre of gravity of the whole. (S.W.E.T.C.)

5. Define the moment of a force.

A uniform rod 8 ft. long weighs 40 lb. It is supported at its ends, and carries a single load of 120 lb. If the reactions at the supports are 100 lb. and 60 lb. respectively, calculate the distance of the load from the middle of the rod. (U.L.C.I.)

6. What is meant by "centre of gravity"?

A T-shaped piece of metal of uniform thickness has the following dimensions:

cross-piece 4 in. $\times$ 1 in.,

upright 4 in. $\times$ 1 in.

Find either graphically or by calculation the position of the centre of gravity measured from the foot of the T. (U.L.C.I.)

7. A locomotive stands on a bridge of 50 ft. span, and the wheel loads of 10, 10, 16, 14, and 14 tons are respectively distant 4, 10, 18, 25, and 32 ft. from the left end. Find the position of the centre of gravity of the loads, and the pressures on the end supports. (U.L.C.I.)

8. A load of 100 lb. is placed at one end of a uniform beam 20 ft. long. Balance is obtained at a point 6 ft. from this end. Find the weight of the beam.

9. A Roman steelyard weighs 8 lb. and a load of 20 lb. supported at 2 in. to the left of the fulcrum is balanced by a counterpoise of 5 lb. at 10 in. to the right of the fulcrum. Find the position of the centre of gravity of the steelyard.

At what distance from the fulcrum will the 5-lb. counterpoise balance a load of 30 lb? (S.W.E.T.C.)

Section B

10. The angle B of a triangle ABC is a right angle; AB is 16 in. long, and BC is 22 in. long. Masses weighing 4, 5, and 6 lb. are placed at A, B, and C respectively. Find the distance of the centre of gravity of the three masses from A. (C.P.E.)

11. A bar ABCD 36 in. long and weighing 50 lb. is supported in a horizontal position by two spring balances at B and C. The readings on the balances are 30 and 20 lb. respectively at B and C when these points are 20 in. apart and CD is 10 in. Find the distance of the centre of gravity of the bar from A.

Assuming the bar to remain sensibly horizontal, find the weight of a mass which, when suspended from D, will give equal readings on the two balances. (C.P.E.)

12. Explain the meaning of the term "centre of gravity of a body". Give examples of a body in (a) stable, (b) unstable, (c) neutral

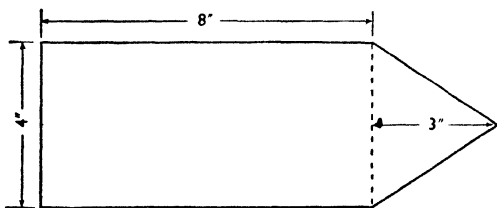


Fig. 10.11

equilibrium, and explain how the position of the centre of gravity is related to each of these cases. Calculate the position of the centre of gravity of the lamina shown in the figure. (C.P.E.)

13. A rod 24 in. long rests symmetrically on two supports 16 in. apart. The greatest masses that can be suspended from the two ends in succession without overturning the rod are 6 lb. and $7\frac{1}{2}$ lb. respectively. Find the weight of the rod and the position of its centre of gravity. (C.P.E.)

14. State the Principle of Moments. What do you understand by the centre of gravity of a body?

A telescope consists of three tubes, each 10 ft. in length, sliding within one another, and their weights are 8, 7 and 6 lb. Find the position of the centre of gravity when the tubes are drawn out to their full length. (S.W.E.T.C.)

15. Explain the meaning of the terms "weight" and "centre of gravity" of a body. How does the weight of a body depend on its

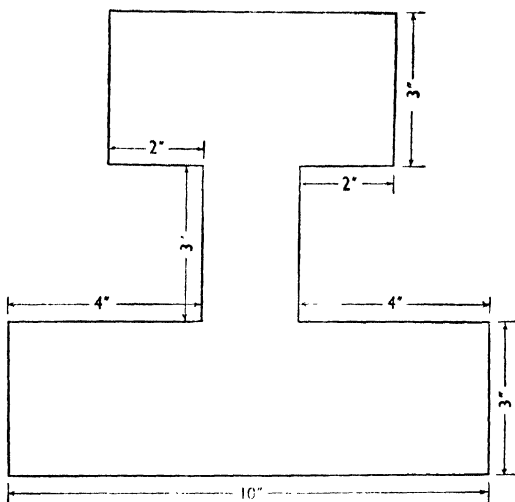


Fig. 10.12

position on the surface of the earth, and what explanation can be given? Calculate the height of the centre of gravity above the base of the piece of cardboard of the shape shown. (C.P.E.)

16. Explain the terms stable, unstable and neutral equilibrium and give examples to illustrate. A cylinder of height 9 in. and diameter 4 in. rests on a rough-surfaced inclined plane whose inclination to the horizontal can be altered. Find the angle of the plane to the horizontal when the cylinder is on the point of falling over.

(C.P.E.)

Every force, then, is really one of a pair of forces, the second being the reaction to which it gives rise. Such a pair of forces, therefore, constitute a **stress**.

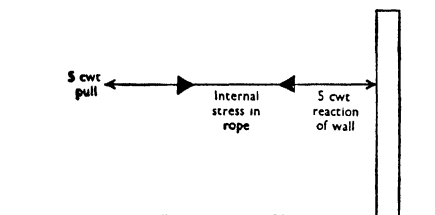


Fig. 11.6

5. Some common properties of materials

1. **Elasticity** is that property possessed by a material which enables it to return to its original dimensions (or nearly so) when the straining force is removed, provided that this force is not too great. The material's **elastic limit** is the greatest stress it can withstand whilst still retaining its elasticity.

If the straining force (load) is continued beyond the elastic limit, however, rapid alteration in size will occur, until eventually **fracture** takes place.

All materials possess elasticity to some extent, but in some it is so unnoticeable that the materials are regarded as **inelastic**, e.g. putty and wax. Rubber, however, possesses elasticity to a marked extent; steel may possess a considerable amount (e.g. spring steel); whereas cast iron has very little. (In the case of cast iron, fracture occurs in tension with practically no measurable strain, but it is strong in compression.)

2. **Ductility** is the property of being permanently extended after subjection to a tensile force. Steel, when stressed beyond its elastic limit, is said to be **ductile**, or in the **plastic** stage.

Aluminium, copper, brass and gold are ductile metals, gold and copper especially so. Ductility is usually measured in terms of the **percentage elongation** produced during a **tensile test** (see Chap. XII, § 5).

3. **Tenacity** is the property of resisting fracture whilst undergoing tension. A measure of the tenacity of a material is obtained from its **ultimate tensile strength** (U.T.S.), which can be found by a tensile test. Steels, especially alloy steels,

possess considerable tenacity. Cast iron possesses very little.

4. **Brittleness** is the tendency to fracture when subjected to a sudden blow or shock. Cast iron is very brittle.

5. **Toughness** is the ability to resist fracture when subjected to impact, bending, twisting, etc. A measure of a material's toughness or brittleness can be found by means of an **impact test** (see Chap. XXIII, § 6).

6. **Malleability** means that the material may be "worked" without fracture (i.e. its shape may be altered by hammering, etc.).

7. **Hardness** is the resistance offered to penetration, abrasion, or wear. A hard material can cut, scratch, or indent another material. By suitable heat-treatment, and by the addition of certain elements, such as tungsten or molybdenum, steel can be made very hard, and consequently is used for cutting purposes, e.g. tool steel. Diamond is the hardest known material.

8. **Softness** is the reverse of hardness—the ability to "give" to pressure without fracture. A measure of a material's relative hardness can be found by means of a **hardness test**. (See Chap. XII, § 10.)

6. Intensity of stress

The degree or intensity of stress in a material depends upon the magnitude of the force applied, and the cross-sectional area of its section. For tensile and compressive stresses this cross-sectional area is measured in a plane at right angles to the force. Numerically

$$\text{intensity of stress} = \frac{\text{load}}{\text{cross-sectional area}}.$$

This value is often expressed in either pounds or tons per square inch.

Consider a person of weight 144 lb. sitting on a chair, and having an area of contact with the chair of 36 sq. in. The total compressive stress (or force) on the chair will be 144 lb., but we could say that the intensity of stress (or pressure) is $144 \div 36 = 4$ lb./sq. in.

In general descriptions we often speak of a stress or a pressure of so many pounds per square inch, when we, strictly speaking, mean an intensity of stress or pressure.

7. Measurement of strain

As strain is the alteration caused by a load, we may express it numerically as

$$\text{strain} = \frac{\text{alteration in length}}{\text{original length}}.$$

This ratio is often referred to as “fractional strain”.

8. The elastic law. Young's modulus

If a gradually increasing series of loads are hung on a spring, and the resulting alteration in length noted for each load, a graph can be plotted of load against alteration (see fig. 11.7).

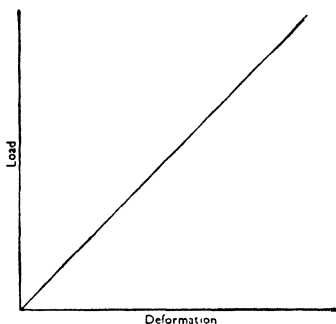


Fig. 11.7

N.B. This alteration will be an **extension** in length if the spring is being stretched, or a **contraction** if it is being compressed.

Provided that the elastic limit of the spring is not exceeded, it will be found that this graph is a straight line, i.e. **the alteration (strain) is proportional to the load (stress) producing it.**

Similar experiments may be carried out on steel wire and other engineering materials, and it will be found that a similar relationship holds good. As the matter was first investigated by Robert Hooke (1635–1703) this relationship is known as **Hooke's law.**

From this law, it will be seen that, for a given material (within its elastic limit), the ratio of stress to strain will be a constant. This constant is referred to as **Young's modulus of elasticity**, and may be denoted by the letter E , i.e.

$$\text{Young's modulus} = E = \frac{\text{stress}}{\text{strain}}.$$

As strain is a ratio (i.e. a number) then E can be expressed in the same units as stress. In engineering practice E is given in lb./sq. in. or tons/sq. in., and is sometimes defined as that stress which would produce unit strain in a material if its elastic limit were disregarded. (Unit strain is obtained when the alteration produced is equal to the original length.) In general, E should be considered as a measure of the resistance a material offers to stretching (or deformation). The higher the value of E , the greater is this resistance.

Example 1.—A wire 90 in. long and 0.125 in. in diameter extends by 0.06 in. when a load of 50 lb. is suspended from it. Find E .

Cross-sectional area of wire $= \frac{1}{4}\pi d^2 = \frac{1}{4} \times 3.142 \times 0.125^2 = 0.0123$.

$$\text{Stress in wire} = \frac{\text{load}}{\text{c.s. area}} = \frac{50}{0.0123} = 4080 \text{ lb./sq. in.}$$

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{0.06}{90} = 0.000666.$$

$$\begin{aligned} \text{Modulus of elasticity } (E) &= \frac{\text{stress}}{\text{strain}} = \frac{4080}{0.000666} = 6,120,000 \text{ lb./sq. in.} \\ &= 2730 \text{ tons/sq. in.} \end{aligned}$$

Example 2.—A steel tie-bar 2 in. $\times$ 1 in. has to withstand a load of 8 tons. If its length is 8 ft. and E is 13,000 tons/sq. in., determine the extension of the tie.

$$\text{Stress} = \frac{\text{load}}{\text{c.s. area}} = \frac{8}{2} = 4 \text{ tons/sq. in.}$$

$$E = \frac{\text{stress}}{\text{strain}}, \quad \therefore \text{strain} = \frac{\text{stress}}{E} = \frac{4}{13,000} = \frac{\text{extension}}{\text{original length}}.$$

$$\therefore \text{Extension} = \frac{4 \times 8 \times 12}{13,000} = \frac{384}{13,000} = 0.02955 \text{ in.}$$

Example 3.—A wire, 0.1 in. diameter and 15 ft. long, stretches 0.061 in. when subjected to a tensile load of 80 lb. What diameter of wire of the same kind of material should be used to give a stretch of 0.05 in. on a length of 12 ft. under a tensile load of 50 lb? (C.P.E.)

$$\text{Stress} = \frac{\text{load}}{\text{c.s. area}} = \frac{80}{\pi \times 0.05^2} = 10,180 \text{ lb./sq. in.}$$

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{0.061}{15 \times 12} = 0.0003388.$$

Modulus of Elasticity (E)

$$= \frac{\text{stress}}{\text{strain}} = \frac{10,180}{0.0003388} = 30,050,000 \text{ lb./sq. in.}$$

Let the required diameter of wire have a radius r in., then

$$\text{Stress} = \frac{\text{load}}{\text{c.s. area}} = \frac{50}{\pi r^2}.$$

$$\text{The resulting strain} = \frac{\text{extension}}{\text{original length}} = \frac{0.05}{12 \times 12}.$$

E has been found,

$$\therefore 30,050,000 = \frac{\text{stress}}{\text{strain}} = \frac{50}{\pi r^2} \times \frac{12 \times 12}{0.05}.$$

$$\therefore r^2 = \frac{50 \times 12 \times 12}{30,050,000 \times \pi \times 0.05},$$

from which

$$r = 0.03905,$$

and

$$\text{diameter} = 0.0781 \text{ in.}$$

9. Compressive stress

When materials are placed under **compressive stress**, a reduction in length may be observed. Initially this stress produces a proportional strain, i.e. Hooke's law is obeyed, and for most materials Young's modulus in compression is equal to the value in tension. Eventually, however, the limit of proportionality is passed and the material collapses.

Example 4.—A hollow, cylindrical, cast-iron column, 10 in. internal diameter and 1 in. thick, is 20 ft. high, and supports a load of 80 tons. Find the average compressive stress in the material, and

the amount it shortens under load. Take Young's modulus as 8000 tons/in.²

Internal radius = 5 in. External radius = 6 in.

Area of cross-section = $\pi(R^2 - r^2) = \pi(6^2 - 5^2) = 11\pi$ sq. in.

Average intensity of compressive stress = $\frac{80}{11\pi} = 2.315$ tons/in.²

$$\text{Strain} = \frac{\text{stress intensity}}{E} = \frac{2.315}{8000} = 0.00029.$$

But
$$\text{strain} = \frac{\text{alteration in length}}{\text{original length}},$$

$\therefore$ total shortening under load = strain $\times$ original length
 = $0.00029 \times 20 \times 12 = 0.0696$ in.

10. Shear stress

Consider the case of the riveted joint shown in fig. 11.3. The shearing force P is resisted by the material of the rivet. The intensity of **shear stress** (q) will be

$$q = \frac{\text{shear force } (P)}{\text{c.s. area of rivet}}.$$

In many cases a riveted joint (or some similar fitting) may be arranged in **double shear**, as shown in fig. 11.8. In such cases the shearing force is resisted by the material at two

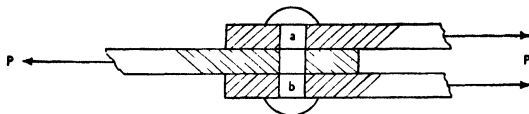


Fig. 11.8—Rivet in double shear

cross-sections (a and b). In consequence, the intensity of shear stress produced is half of that produced by single shear, and the joint is twice as strong.

N.B. In practice, to allow for errors, etc., a value of 1.75 is usually taken.

11. Shear strain

When **shear stress** is applied to a material, the form of **shear strain** occurring may be illustrated by considering a book (see fig. 11.9). The action of this stress P causes the pages to slide over one another as the top cover of the book moves to the right. The "stickiness" between pages restricts this movement.

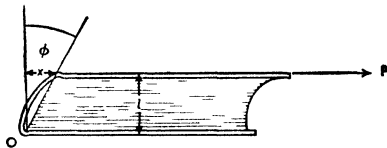


FIG. 11.9—Illustrating shear strain

The resulting shear strain (deformation) produced can be measured in terms of the angle (ϕ).

If x is regarded as part of the arc of a circle about point O , then

$$\phi \text{ (radians)} = \frac{\text{arc}}{\text{radius}} = \frac{x}{\text{thickness } (l)}.$$

This ratio constitutes shear strain.

12. Modulus of rigidity

When stress is due to shear, the ratio (shear stress) : (shear strain) is termed the **modulus of rigidity** (N), i.e.

$$N = \frac{q}{\phi}.$$

Example 5.—A riveted joint is held by two $\frac{3}{8}$ -in. diameter rivets, arranged in double shear. The shearing force is 2 tons. Calculate the theoretical shear stress in the rivets.

Cross-sectional area of rivet = $\frac{1}{4}\pi d^2 = 0.7854 \times 0.375^2 = 0.1105$ sq. in.

Area of rivet resisting shear = $2 \times 0.1105 = 0.221$ sq. in.

As 2 rivets are used, total area of material resisting shear = $2 \times 0.221 = 0.442$ sq. in.

$$\text{Shear stress in rivet} = \frac{2.0}{0.442} = 4.53 \text{ tons per sq. in.}$$

Example 6.—Four $\frac{1}{4}$ -in. diameter mild-steel rivets are used to fix two bars together. Find the shear stress and shear strain when a shearing force of 3140 lb. is applied, given that the modulus of rigidity (N) is 12,000,000 lb. per sq. in. If the thickness of the joint is 2 in., find the deformation (x) produced.

$$\begin{aligned}\text{Shear stress } (q) &= \frac{\text{shear force}}{\text{resisting area}} = \frac{3140}{4 \times \frac{1}{4} \times (\frac{1}{4})^2} \\ &= 16,000 \text{ lb. per sq. in.}\end{aligned}$$

$$\text{Shear strain } (\phi) = \frac{q}{N} = \frac{16,000}{12,000,000} = 0.00134 \text{ radian.}$$

$$\text{Deformation } (x) = \phi l = 0.00134 \times 2 = 0.00268 \text{ in.}$$

13. Punching and cutting

The workshop processes of cutting and punching holes in materials are examples of failure occurring due to shearing. The material resisting shear during such operations is the cross-sectional area of the cut, i.e. in the case of a punched hole, it is (perimeter of hole) $\times$ (thickness of material).

Example 7.—A steel strip, 4 in. wide and $\frac{1}{2}$ in. thick, has an ultimate shear stress of 26 tons/sq. in. Find the least force required to (a) shear off a length, (b) punch a 2-in. diameter hole in it.

$$(a) \text{ Area resisting shear} = 4 \times \frac{1}{2} = 2 \text{ sq. in.}$$

$$\text{Force required} = 2 \times (\text{U.S.S.}) = 2 \times 26 = 52 \text{ tons.}$$

$$(b) \text{ Area resisting shear} = \pi d \times \text{thickness} = \pi \times 2 \times \frac{1}{2} = 3.142 \text{ sq. in.}$$

$$\text{Force required} = 3.142 \times 26 = 81.69 \text{ tons.}$$

EXERCISES ON CHAPTER XI

Section A

1. A rope of 2 in. diameter is attached to a wall by means of a ring, and is pulled simultaneously by two men and a boy, exerting pulls of 112, 120, and 82 lb. respectively.

Find (a) the tension in the rope, (b) the force exerted by the ring on the rope, (c) the intensity of tensile stress in the rope.

2. A rod has to withstand a tensile load of 10 tons. Find its minimum diameter if the permissible tensile stress in the material is 5000 lb. per sq. in.

3. A hollow column 9 in. internal diameter, metal $1\frac{1}{2}$ in. thick, supports an axial load of 100 tons. Find the compressive stress in the material.

4. A steel wire 0.08 in. diameter and 50 ft. long is subjected to tension by a load of 112 lb. Determine (a) the stress in pounds per sq. in., (b) the strain, and (c) the elongation in inches. (U.L.C.I.)

5. A body which is stressed by the application of external force may be either (a) in tension, (b) in compression, or (c) in shear. Illustrate by means of simple sketches each of the three cases.

A rod 10 ft. long is subjected to a tensile stress of 5 tons per sq. in. If the modulus of elasticity for the material is 30×10^6 lb. per sq. in., calculate the extension produced in the rod. (U.L.C.I.)

6. What do you understand by the terms "intensity of stress", "strain" and "modulus of elasticity", as applied to a bar under a tensile force?

A steel wire 12 ft. long and 0.1 in. diameter is subjected to a pull of 400 lb. How much will it stretch if the modulus of direct elasticity is 13,000 tons per sq. in. (Poly.)

7. The following corresponding set of readings was taken during the test of a helical spring:

Load in lb.	..	..	4	8	12	16	20
Length of spring in inches	$4\frac{1}{8}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{3}{8}$		

Draw a graph showing the relationship between the load (horizontal) and the spring length (vertical). From the graph find the length of the unloaded spring, and the length when carrying a load of 14 lb. (U.E.I.)

8. Describe any experiment you have made to verify Hooke's law. A steel wire 0.05 in. in diameter and 15 ft. long stretches 0.061 in. when a load of 20 lb. is suspended from the free end of the wire. What extension would you expect on a steel rod of 1 in. diameter and 7 ft. long when subjected to a tensile load of 5 tons? (C.P.E.)

9. A hole 2 in. square is punched out of a metal plate $\frac{1}{8}$ in. thick. If the ultimate shear-stress intensity of the metal is 45,000 lb./sq. in., what load is required? The punching tool is attached to a ram 3 in. diameter. What is the compressive stress intensity in the ram? (S.W.E.T.C.)

10. A hole in the base of a tank is 18 in. in diameter; it is closed by a plate on the underside of the tank, and is secured by eight $\frac{3}{4}$ -in. diameter rivets. Find the stress in these rivets if the tank contains oil of specific gravity 0.9 to a depth of 20 ft. (1 cu. ft. of water weighs 62.5 lb.) (U.E.I.)

Section B

11. The following results were obtained from a tensile test on a wire 0.04 in. in diameter and 3 metres in length.

Load in lb.	..	0	7	14	21	28	35
Extension in mm.	0	1.4	2.85	3.95	5.4	6.65	

Plot the load-extension graph, using scales 1 in. = 5 lb. and 1 in. = 1 mm. extension, and obtain from it the value of Young's modulus of elasticity in lb. per sq. in. (U.E.I.)

12. A rod 12 in. long is $1\frac{1}{2}$ in. diameter over a length of 10 in., and 1 in. diameter over the remaining 2 in. of its length. The smaller portion is subjected to a tensile stress of 9 tons per sq. in. What is the stress intensity in the larger portion? How much will an 8-in. length of the larger portion stretch under the load if the modulus of direct elasticity of the material is 13,500 tons/sq. in.? (U.E.I.)

13. A vertical concrete column of 15×15 in. square section is reinforced by four steel rods 1 in. in diameter. The load on the column is 50 tons. Calculate the stress in the steel

The modulus of elasticity of steel should be taken as 12 times that of concrete. (I.E.E. Grad.)

14. Two steel bars each of $2 \times \frac{3}{8}$ in. section are to be connected in line by means of a lap joint and a single rivet. Take the shearing stress for the rivet as $\frac{3}{4}$ the tensile stress for the plate, and calculate the rivet diameter for the strongest joint. (I.E.E. Grad.)

15. A spur wheel is required to transmit 50 horse-power at 100 r.p.m. What is the torque transmitted? The wheel is keyed to a shaft 3 in. diameter by a key of width 0.875 in. and length 5 in. Find the average shear stress in the key. (U.E.I.)

16. A steel block is subjected to a shear stress of 15 tons per sq. in. Calculate the angle by which a line marked on the side of the block, and originally vertical, will differ from the vertical under the action of the stress. Take the modulus of rigidity of the material as 5200 tons per sq. in.

CHAPTER XII

Materials Testing

1. Tensile testing of materials

In order to prove the strength and to examine the behaviour of a material under tensile loading, and to check its compliance with some specification, it is customary to carry out a tensile test to destruction on a carefully prepared sample.

From the results of such a test can be found:

1. The load at fracture, and hence the ultimate tensile strength or stress (U.T.S.).
2. The elongation produced.
3. The reduction of area suffered by the sample.
4. The yield point (yield point stress).
5. The proof stress.

1, 2, 3, and 4 are employed commercially when specifying and using materials. 5 is called for in some specifications, more especially with aircraft materials.

Steels, and other materials, are often referred to by quoting their U.T.S. (e.g. 28 tons/sq. in. steel is ordinary structural steel). High-tensile alloy steels may go up to 100 tons/sq. in.

2. Tensile testing machine

Fig. 12.1 shows in outline the arrangement of a typical tensile testing machine of the single-lever type. It consists essentially of a hydraulic pump which forces oil against a piston (P), so creating the load, and a steelyard for measuring this load (Chap. X, § 5). To operate the machine the testpiece is held by shackles between the piston and steelyard, and the pump is set in motion, so applying the tensile load. The steelyard is kept in balance with the gradually increasing load by sliding the weight along it. A pointer attached to this weight records the load.

To measure the extension, two centre-punch marks are made on the testpiece, a specified distance apart. The elonga-

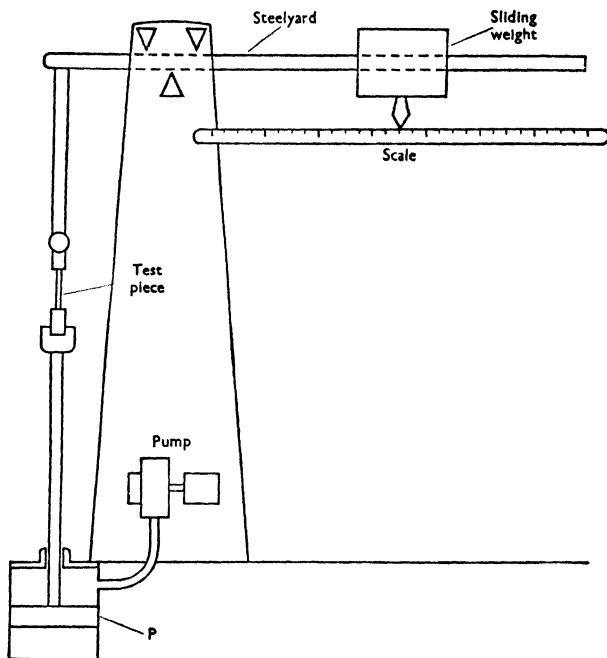


Fig 12 1.—Single-lever tensile testing machine

tion produced at fracture can be measured by putting the two pieces together, or a continuous indication of extension can be obtained by using an instrument called an **extensometer**.

3. Load-extension curve

A typical load-extension curve for mild steel is shown in fig. 12.2. As the load is applied, it will be seen that at first the extension produced is proportional to it (i.e. stress is proportional to strain) and the curve is practically straight. Beyond

A, however, the curve falls off, because stress is no longer proportional to strain, owing to the material having been taken beyond its **elastic limit**. A therefore marks the **limit of proportionality**, or the approximate elastic limit. From O to A the material closely obeys Hooke's law.

There is a sudden stretching of the material from A to B and just beyond, without any appreciable increase in loading. In an actual test, owing to this rapid extension, the load eases and the steelyard arm drops down. The point at which this occurs (B) is known as the **yield point**. (N.B. With mild steel, and also wrought iron, A and B practically coincide, but this does not necessarily follow with other materials.)

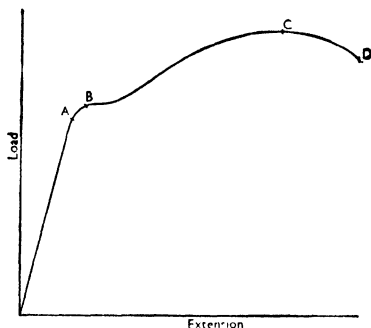


Fig. 12.2—Load-extension curve for mild steel

After this the material stretches rapidly as the load is increased, and it is said to be passing through its **plastic** (or ductile) stage. The stretch is now quite permanent.

At point C a waist begins to form in the testpiece, indicating a reduction in its cross-sectional area. This reduced cross-section will not sustain the total load without fracture, so this load must be slightly reduced if the complete test curve is to be drawn. Hence **maximum load** occurs at C.

If the stretching is carefully continued, fracture finally occurs at D. Owing to the reduction in cross-sectional area, however, **maximum intensity of stress** occurs at D, and is calculated from (load at D) $\div$ (area of waist). This is therefore the true breaking stress of the material.

In commercial practice, however, the maximum or breaking stress is always calculated from

$$\frac{\text{maximum load (i.e. load at C)}}{\text{original cross-sectional area}}$$

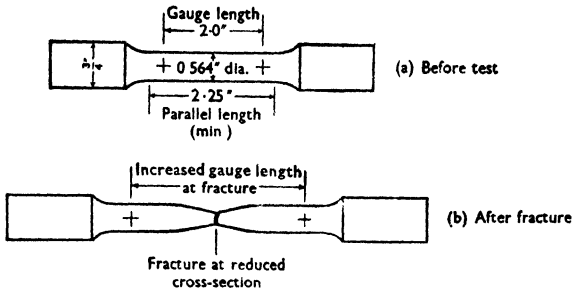


Fig. 12.3.—A British Standard tensile testpiece

This value is known as the **ultimate tensile stress** (U.T.S.) of the material.

4. Standard testpieces

In order to ensure that consistent results are obtained everywhere when testing any particular material, the specification for that material will call for the test samples to be made up in a particular manner, usually in one of the forms laid down in British Standard Specifications.

One such British Standard testpiece is shown in fig. 12.3. It will be found that the diameter of 0.564 in. is such that it will give a cross-sectional area of exactly $\frac{1}{4}$ sq. in. Other standard testpieces may have cross-sectional areas of $\frac{1}{2}$ or $\frac{3}{4}$ sq. in.

5. Percentage elongation

It will be noticed in fig. 12.3 that two centre-punch marks are shown 2 in. apart, and indicate what is known as the **gauge length**.

The increase in this gauge length at fracture is called the **elongation**, and

$$\text{Percentage elongation} = \frac{\text{actual elongation}}{\text{original gauge length}} \times 100.$$

6. Reduction of area

Knowing the original cross-sectional area (or diameter), and the cross-sectional area (or diameter) at fracture, it is easy to find the reduction of area. This is usually expressed as a percentage, thus

$$\text{Percentage reduction of area} = \frac{\text{actual reduction}}{\text{original c.s.a.}} \times 100.$$

7. Proof stress

The terms “elastic limit”, “yield point”, and “limit of proportionality” are often used indiscriminately. With some materials these points are not clearly discernible, and so, to avoid confusion in finding the stress at such points, a material specification may call for a “proof stress” check. In this, a given intensity of stress is applied to the testpiece for a given time (usually 15 sec.). Upon removal, the **permanent set** must not exceed a given amount (perhaps 1 per cent of the elongation produced under the given load, or, as in many specifications, 0.1 per cent of the original gauge length).

For instance, D.T.D. Specification 146A is for high-chromium non-corrodible aircraft steel of 30 tons, 0.1 per cent proof stress. This means that when a tensile load of 30 tons/sq. in. is applied, the permanent set must not exceed 0.1 per cent of the gauge length (0.1 per cent of 2 in., or 0.002 in.).

8. Factor of safety

In practice, a material must not be allowed to be stressed beyond its elastic limit, otherwise a permanent strain (alteration) would be set up in it. Stresses approaching the ultimate strength of the material will therefore not be encountered in machines and structures under normal conditions. In fixing the sizes of stressed parts, design is such that the maximum stress permitted does not exceed what is called the **working stress**. This working stress is obtained by dividing the ultimate stress by a number called the **factor of safety**.

$$\text{Working stress} = \frac{\text{ultimate stress}}{\text{factor of safety}},$$

and
$$\text{Factor of safety} = \frac{\text{ultimate stress}}{\text{working stress}}.$$

The value of this factor of safety, and consequently of the working stress, varies greatly, and it depends upon the material used and the conditions of loading. Its choice is largely a matter of experience. A high factor of safety would be used if the load were subject to sudden variations, or if it were moving, as in the case of a vehicle crossing a bridge, or in the case of the piston and connecting-rod of an engine. Such loads are termed **live loads**. A lower factor of safety could be used for **dead loads** (i.e. steady loads of fixed value). Sudden applications of load, frequently repeated, can cause early failure of a structure, owing to what is known as **fatigue** of the material. This can readily be imagined by considering a weight tied to a piece of elastic. The weight can be lifted safely by means of the elastic if it is done slowly and steadily. A few jerky movements, however, will set the weight dancing, and, if continued, will result in the breaking of the elastic.

Example 1.—The ultimate tensile strength of the steel used in a structure is 30 tons per sq. in. Calculate the cross-sectional area required for a member subject to a load of 18 tons, if the factor of safety allowed is 6.

$$\text{Working stress} = \frac{\text{ultimate tensile stress}}{\text{factor of safety}} = \frac{30}{6} = 5 \text{ tons per sq. in.}$$

$$\text{Cross-sectional area of member} = \frac{\text{load}}{\text{working stress}} = \frac{18}{5} = 3.6 \text{ sq. in.}$$

Example 2.—A tensile test on a British Standard mild-steel test-piece of 0.564 in. diameter and 2 in. gauge length gave the following results:

Load at yield point = 3.8 tons.

Maximum load at fracture = 7.9 tons.

Distance between gauge points after fracture = 2.48 in.

Diameter of testpiece at fracture = 0.32 in.

Find the yield-point stress, the ultimate tensile stress, the percentage elongation, and the percentage reduction of area.

$$\text{Cross-sectional area of testpiece} = 0.25 \text{ sq. in.}$$

$$\text{Yield-point stress} = \frac{3.8}{0.25} = 15.2 \text{ tons per sq. in.}$$

$$\text{Ultimate tensile stress} = \frac{7.9}{0.25} = 31.6 \text{ tons per sq. in.}$$

$$\begin{aligned}\text{Percentage elongation} &= \frac{\text{actual elongation}}{\text{original gauge length}} \times 100 = \frac{0.48}{2} \times 100 \\ &= 24 \text{ per cent.}\end{aligned}$$

$$\begin{aligned}\text{Percentage reduction of area} &= \frac{\text{actual reduction}}{\text{original c.s.a.}} \times 100 \\ &= \frac{0.25 - (\pi \times 0.16^2)}{0.25} \times 100 = \frac{0.25 - 0.0804}{0.25} \times 100 \\ &= \frac{0.1696}{0.25} \times 100 = 67.84 \text{ per cent.}\end{aligned}$$

9. Stress-strain diagrams

Instead of a load-extension curve, one of stress-strain is sometimes used. This is of exactly the same shape, as stress is calculated by dividing the load by the original cross-sectional area, and strain by dividing the extension by the gauge length. The slope of the elastic portion of a stress-strain diagram will give Young's modulus.

10. Hardness testing

An important test which is carried out commercially on most metallic materials is that for determining hardness. The degree of hardness is expressed as a number on some specified scale, and this number can be used for comparing one material, or one particular surface, with another. Hardness testing enables routine checks to be made on the consistency of manufacture, or of heat treatment, of different batches of material. In addition, as there is a close relationship between hardness number and tensile strength, a check can be kept on the latter quantity without having to test a sample to destruction.

"Hardness" is understood to mean the resistance offered by a material to abrasion or indentation. An idea of the degree of hardness of a piece of steel can be gained by rubbing it with a file, but this, of course, is not a scientific method. Hardness is measured commercially to-day by a machine which pushes an indenting tool into the surface of the material under test. The size of the impression produced, taking into account the material of the tool and the load applied, enables the hardness number to be determined.

Some investigation into the significance of hardness was carried out in the latter half of the nineteenth century, and the first hardness-testing machine was used commercially by Brinell in 1900.

11. The Brinell hardness-testing machine

The Brinell machine uses a hard steel ball (normally of 10 mm. diameter) for indenting. The load can be adjusted according to the material being tested.

For steel the load is 3000 kg.

For copper and aluminium alloys it is 1000 kg.

For copper and aluminium it is 500 kg.

For lead and tin it is 100 kg.

The construction of the machine is shown in fig. 12.4. To use it the sample is placed on the anvil (*a*), and a hand pump (*h*) is worked until the weights (*W*) rise, so forcing the ball (*b*) into the material.* The load must be sustained for 15 sec. The pressure is then released by the valve (*V*), the specimen taken from the machine, and the width of the impression (fig. 12.5) measured by

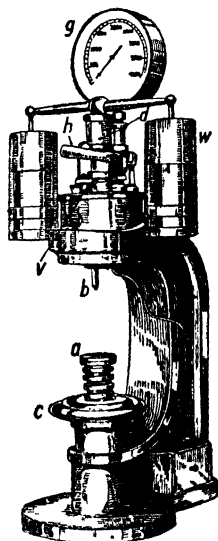


Fig. 12.4.—Brinell hardness-testing machine

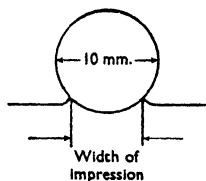


Fig. 12.5.—Brinell indenter impression

means of a graduated microscope. By referring this width to a table the **Brinell hardness number** can be determined. This table is based on the relationship:

$$\text{Brinell hardness number} = \frac{\text{load (kg.)}}{\text{spherical area of depression (sq. mm.)}}$$

It will be noticed that the hardness number increases with degree of hardness. If *D* is the diameter of the ball, and *d* the width of the impression (in mm.), then the hardness number

* An indication of the load is given by the pressure gauge (*g*).

can be calculated, if need be, from the formula

$$\text{Brinell hardness number} = \frac{\text{load (kg.)}}{\frac{1}{2}\pi D(D - \sqrt{D^2 - d^2})}$$

12. The Vickers pyramid hardness-testing machine

The Brinell machine is not suitable for testing very hard materials, because of the shallowness of the indentation produced, the consequent difficulty in measuring its vague outline accurately, and because the ball itself deforms under heavy load. In order to overcome these difficulties, the Vickers Diamond-pyramid Hardness-testing Machine was introduced



Actual size
of impression

Fig. 12.6.—Vickers diamond-pyramid indenter

(1925). This machine employs a pyramid-shaped diamond as the indenting tool, and the load can be adjusted to suit different materials over the range 1–50 kg. A load of 30 kg. is used for steel, and one of 10 kg. for non-ferrous metals. If required, the machine can be fitted with a 2-mm. diameter steel ball, loaded up to 120 kg., and used as a Brinell machine.

The impression made by the diamond is very much smaller than that made by the Brinell steel ball, which is an advantage, as it permits the testing of finished articles without disfigurement (see fig. 12.6). The method of operating the Vickers machine is designed to eliminate variations in results owing to the human element. After setting up the specimen, the machine is loaded by depressing a foot pedal, and then set in operation by pushing a lever. The width of the diamond-shaped indentation is then measured by means of a special microscope fitted with two adjusting screws. On lining-up two lines in the microscope to the width of the impression by the aid of these screws, a number appears on a counter. This value is then referred to a table which gives the hardness number.

13. Rockwell hardness-testing machine

Various other hardness-testing machines may be encountered. The Rockwell machine is an American type introduced

in 1920. It uses a diamond cone indenter for steel and a $\frac{1}{16}$ -in. diameter steel ball for other metals. Its great advantage is that it indicates the hardness value directly on a dial.

14. Hardness and tensile strength

There is a close relationship between hardness and tensile strength, as was mentioned previously

For steel this relationship can be taken as

$$\text{Brinell number} \times 0.22 = \text{tensile strength (tons/sq. in.)}$$

When using a machine other than a Brinell, the particular hardness number obtained must first be converted to a Brinell number before using the above formula. Conversion tables are supplied with the machine, or may be found in a good reference book.

It should be realized that the hardness numbers obtained with different types of machine do not correspond. Therefore, when quoting a number, the particular machine used, the load, scale, etc., should be mentioned.

Alternatively, the particular number should be converted to a Brinell number by means of a conversion table.

Example 3.—A Brinell hardness test on a sample of steel gave the following results:

Load used = 3000 kg.

Diameter of ball = 10 mm.

Measured width of impression on surface of sample = 6 mm.

Calculate the Brinell hardness number, and from it estimate the ultimate tensile strength of the material.

$$\text{Brinell hardness number} = \frac{\text{load (kg.)}}{\text{spherical area of impression (sq. mm.)}}$$

$$\begin{aligned} &= \frac{\text{load}}{\frac{1}{2}\pi D(D - \sqrt{D^2 - d^2})} = \frac{3000}{\frac{1}{2}\pi \times 10 \{10 - \sqrt{(10^2 - 6^2)}\}} \\ &= \frac{3000}{5\pi(10 - \sqrt{64})} = \frac{3000}{5\pi \times 2} = \frac{3000}{31.42} = 95.5. \end{aligned}$$

The approximate ultimate tensile strength will be

$$(\text{Brinell number} \times 0.22) = 95.5 \times 0.22 = 21 \text{ tons/sq. in.}$$

15. Compression tests

Tests of materials under compression are not so frequently made commercially as tensile tests.

With ductile materials, the test sample (fig. 12.7*a*) flattens or "barrels" out under large compressive loads, with no

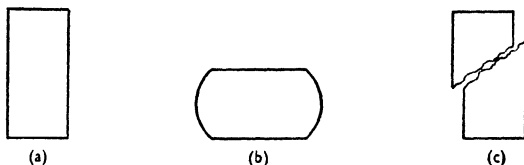


Fig. 12.7.—Materials under compression

detectable yield point (fig. 12.7*b*). With brittle materials (such as cast iron) the test sample is found to fracture on an inclined plane (fig. 12.7*c*) at a definite loading.

16. Other tests

Many other physical tests may be carried out on materials in order to study their behaviour. Shearing, bending, and torsion tests might be mentioned. An Impact Testing Machine is considered in Chapter XXIII, § 6.

EXERCISES ON CHAPTER XII

Section A

1. What do you understand by elastic limit, limit of proportionality, dead load, live load, and fatigue as applied to engineering materials?

2. How would you carry out a tensile test on a sample of mild steel?

Sketch roughly the type of load-extension curve you would expect to obtain, and explain its significance.

3. Sketch a typical load-extension curve, marking clearly the following points: (a) limit of proportionality, (b) yield point, (c) maximum load.

Calculate the diameter of a steel tie-rod which is required to transmit a pull of 6 tons, if the ultimate tensile stress of the steel is 30 tons per sq. in., and the factor of safety is to be 6. (U.L.C.I.)

4. State in tabular form the quantities you would measure in an ordinary tensile test of a ductile metal, and the deductions you would make; and give approximate values for mild steel.

(I.E.E. Grad.)

5. Define "factor of safety".

If the ultimate tensile strength of mild steel is taken as 28 tons per sq. in., what must be the c.s.a. of a mild steel bar which has to take an axial load of 9 tons with a factor of safety of 7? If the original length of the bar were 12 ft., by how much would it extend under the load if $E = 30 \times 10^6$ lb. per sq. in.? (U.L.C.I.)

6. A tie-bar has a rectangular cross-section of 3 in. $\times$ 1 in. A gauge shows it to be stretched by 0.00312 in. on a length of 8 in. when under load. If $E = 13,000$ tons per sq. in., find the intensity of stress and the total load carried by the bar. If the ultimate strength of the material is 30 tons per sq. in., what is the factor of safety? (U.L.C.I.)

Section B

7. A tie-bar in a steel structure is of rectangular cross-section 3 in. $\times$ $\frac{7}{8}$ in. A strain gauge applied to it shows a stretch of 0.0044 in. on an 8-in. length under load. Find (i) the intensity of stress, (ii) the total load carried by the tie-bar, and (iii) the factor of safety, if $E = 13,000$ tons/in.², and U.T.S. = 30 tons/in.² (S.W.E.T.C.)

8. In a Brinell test on a piece of steel the diameter of the indentation was 0.45 cm. The load was 2000 kg. and the ball diameter 1 cm. Find the Brinell hardness number. Estimate the ultimate tensile strength of the material.

9. (a) Sketch a typical stress-strain diagram for a mild-steel specimen under tension, marking the position of the elastic limit and ultimate stress.

(b) A pull of 300 lb. is applied to a signal wire 210 ft. long and $\frac{1}{8}$ in. in diameter. If the required movement at the signal is 6 in., find the movement necessary at the signal-box end of the wire if $E = 30,000,000$ lb./in.² (U.E.I.)

10. The following results were obtained during a tensile test on a British Standard testpiece of duralumin of 0.564 in. diameter and 2 in. gauge length:

Maximum load at fracture, 7.5 tons.

Distance between gauge points after fracture, 2.24 in.

Diameter of testpiece at fracture, 0.504 in.

Determine the ultimate tensile stress, the percentage elongation, and the percentage reduction of area of the material.

11. Explain what is meant by "proof stress". When and why is it used?

12. Define the following terms: "dead load", "live load", "factor of safety", "fatigue of materials".

In the basement of a mill there are 52 cast-iron pillars supporting the superstructure. Each of these pillars is 10 in. outside diameter, and the metal is $1\frac{1}{2}$ in. thick. Assuming these pillars are subject to a simple compressive stress, what total load in tons can they support, allowing for a factor of safety of 8?

N.B. The ultimate breaking load of cast iron in compression is 40 tons/in.² (U.L.C.I.)

CHAPTER XIII

Forces in Simple Structures

The Laws of Concurrent Coplanar Forces can be applied to the solution of many problems requiring the determination of the unknown forces in loaded structures, etc.

To ensure ease and uniformity of procedure, and to lessen the chance of mistakes, it is usual to letter the diagrams in accordance with what is known as "Bow's notation"

1. Bow's notation

A space diagram is drawn to scale and all the spaces between forces are marked with capital letters, this being done by going

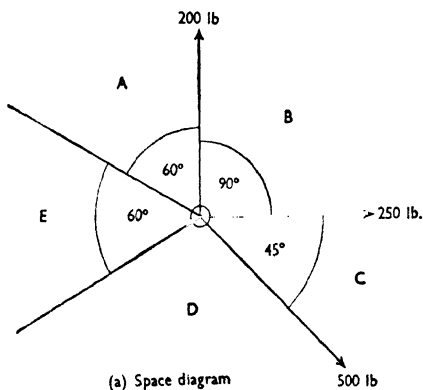


Fig 13.1a

around the diagram in a regular manner (either clockwise or anticlockwise). The forces can then be denoted by naming the capital letters which appear in the spaces on each side of them.

Their corresponding vectors (drawn on a vector diagram) are then denoted by the same, but small, letters, placed one at each end.

The method has already been used (but not mentioned) when dealing with the triangle of forces (Chap. VIII, § 4).

As a further example consider the following case:

Example 1.—Five telegraph wires are fixed to a pole and disposed at the angles shown in the plan (fig. 13.1*a*). If the pull in three of these wires is as indicated, find the magnitude of the pull in each of the other two wires.

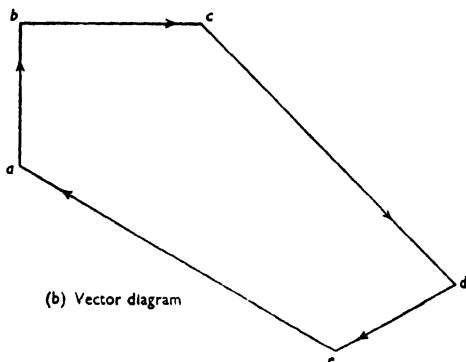


FIG. 13.1*b*

First insert capital letters A, B, C, D, E in the spaces between the wires.

Then, making use of the polygon of forces law, draw the vector diagram (fig. 13.1*b*). Make vector $ab = 200$ lb., vector $bc = 250$ lb., vector $cd = 500$ lb.; then draw de from d parallel to the force DE, and ae from a parallel to the force EA. Vectors de and ae will intersect at e . Their magnitude, found by measuring to scale, will be found to be

$$de = 190 \text{ lb.} = \text{pull in wire DE}$$

$$ae = 502 \text{ lb.} = \text{pull in wire AE.}$$

As the forces are in equilibrium, the arrowheads will all be clockwise in the closed polygon, and the sense of the two unknown forces must be outwards from the pole.

2. Framed structures

Bridges, roofs, cranes, aircraft, etc., are often built up from members arranged in triangulated form, and secured together by means of simple pinning, riveting, or some similar method. Members of the structure which are resisting tension are termed **ties**, and those which are resisting compression are termed **struts**.

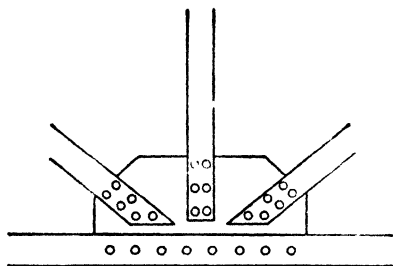


Fig. 13.2

The laws of concurrent coplanar forces can strictly be applied only to structures whose members are secured together at one *point*, i.e. pinned or hinged members, because the laws apply to forces meeting *at a point*. In practice, structures are often riveted together with the aid of a gusset plate, as shown in fig. 13.2, the forces acting through several points. When

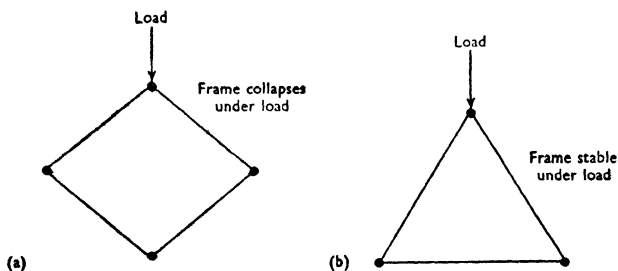


Fig. 13.3

solving practical problems, however, pin joints are assumed, as the results obtained are sufficiently accurate.

Triangulated frames are used because of their inherent stability. A four-sided frame, loaded as in fig. 13.3a, would

collapse concertina fashion, but the triangulated frame (fig. 13.3*b*) would remain stable.

As an example of a simple framed structure, we will proceed to find the forces in a simple jib frame.

Example 2.—A simple warehouse crane consists of a jib of length 10.5 ft., held at a radius of $7\frac{1}{2}$ ft., measured horizontally from the jib-head to the wall. Find the forces in these members when a load of 2.5 cwt. is suspended from the jib-head.

The method of tackling this type of problem is illustrated in fig. 13.4, and is outlined below.

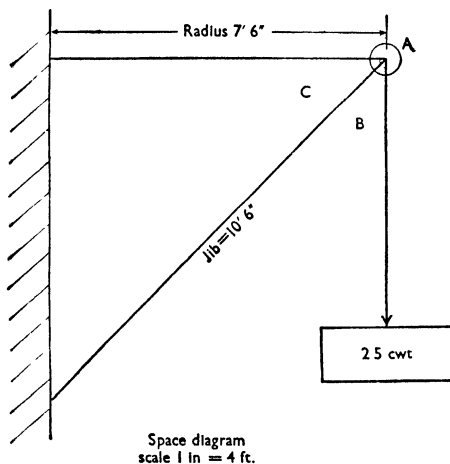


Fig. 13.4*a*

(1) Draw the **space diagram**, showing the actual physical layout of the problem, using some convenient scale of feet.

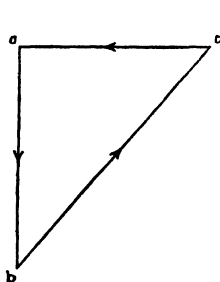
(2) Making use of **Bow's notation**, letter all the spaces in order.

(3) Now draw the **vector or force diagram** of the forces acting at the jib-head, using some convenient scale. The vector *ab*, representing the only known force AB to a scale of 2.5 cwt., should be drawn first, parallel to AB.

From *b* draw a vector *bc* of indefinite length (representing the unknown force BC) parallel to BC. From *a* draw a vector *ac* of indefinite length (representing the unknown force AC) parallel to AC. These last two vectors will be found to intersect, and this point of intersection will be *c*.

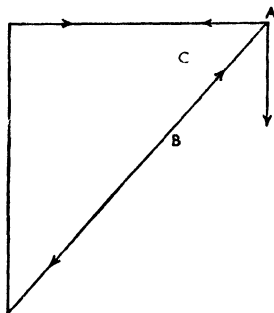
(4) Now measure the length of the vectors bc and ac on the vector diagram to the scale employed. This gives the forces in those members.

(5) Ascertain the direction of the forces in the members. In the vector diagram we have a closed triangle of forces representing equilibrium, and as the vector ab acts downwards (i.e. from a to b), the vectors bc and ca must follow round in that order. (N.B. The order of writing represents the direction: i.e. ca means from c to a .)



Vector diagram
scale 1 in. = 2 cwt.

Fig. 13.4b



Stress diagram

Fig. 13.5

Arrowheads may therefore be drawn on the vector diagram, and transferred to the **stress diagram** at the *jib head* (see fig. 13.5). From this it will be seen that the member BC thrusts at the jib-head, and the member AC pulls at the jib-head. To secure equilibrium the member BC will also thrust at its joint with the wall, and the member AC will pull on the wall. Arrowheads to indicate this should be inserted on the stress diagram. It will be seen therefore that the member BC is a **strut** (being in compression) and the member AC a **tie** (being in tension).

(6) Tabulate results in the following manner:

Member	Force, cwt.	Nature
AB	2.5	(given)
BC	3.3	strut (compression)
CA	2.15	tie (tension)

N.B. In order to save space, the stress diagram with its arrowheads is often incorporated into the space diagram.

3. Roof trusses

A collection of triangulated frames may be built up to form a **truss**. The joints are often termed **nodal points** or **nodes**. The design is arranged so that the load is transmitted to the frame only at the nodes (see fig. 13.6).

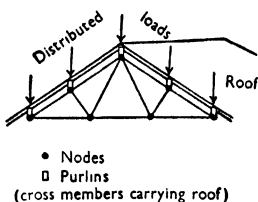


Fig. 13.6.—Roof truss

It should be noted that when a simple plank is used as a bridge it will bend considerably under load (fig. 13.7a). If the plank is arranged edgewise, however, its resistance to bending is increased considerably (fig. 13.7b). A girder bridge consisting of two trusses, or a roof supported by a number of trusses, makes use of this principle. The effect is the same as if the

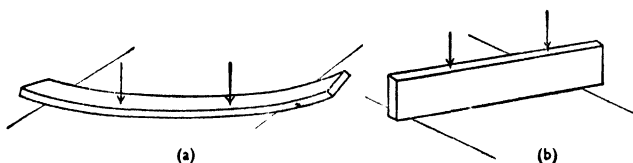


Fig. 13.7

trusses were solid planks arranged edgewise, but the weight of each truss will be much less, owing to the open triangulated construction.

The weight of the roof is carried by cross-members, or **purlins**, and is transferred by them to the nodes of each roof truss (see fig. 13.6). In the case of a railway girder bridge, the load is transferred to the nodal points by some such method

as fixing the rails to cross-members fixed between corresponding nodes on each girder (see fig. 13.8).

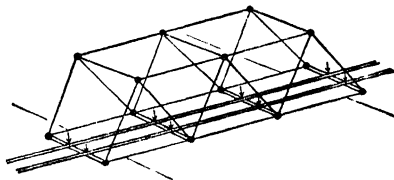


Fig. 13.8.—Railway bridge showing transference of load to nodes

Example 3.—Find the forces in the simple roof truss shown in fig. 13.9 (p. 152). State their nature.

Method of procedure:

(1) Find reactions at supports. (In this case, as the truss is symmetrically loaded, they will be 2 tons each.)

(2) Draw space diagram and letter it using Bow's notation. (Letter each space in order, going round externally first.)

(3) Draw vector diagram for the three forces at apex (vectors ab , bd and da , inserting arrowheads).

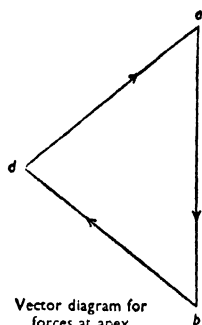
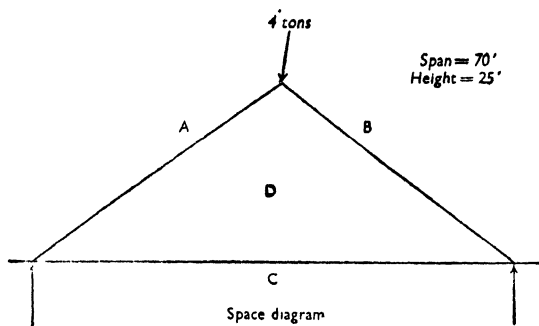
(4) Draw vector diagrams for the forces meeting at the left-hand support, and then the right-hand support (inserting arrowheads).

(5) As several vectors appear more than once in each vector diagram, it is convenient to draw a combined diagram known as a *reciprocal figure*. (N.B. This figure does not indicate sense of direction, so arrowheads must not be inserted.)

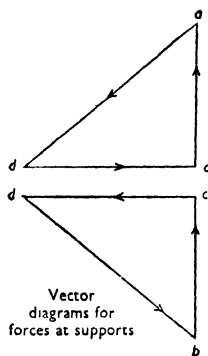
(6) Show the sense of direction of the forces at the apex and supports on the stress diagram (fig. 13.10) (or transfer to space diagram).

(7) Draw up a table to show the members, the forces in them, and their nature. (For the loading and dimensions given, the results are as shown in fig. 13.10.)

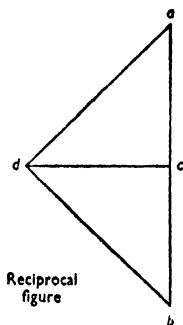
N.B. In this example the reactions at the supports could have been found from the vector diagrams by measuring the vectors ca and cb (vectors ad and bd being of known value, obtained from the vector diagram of the forces at the apex). In more complex examples, however, the method of finding the reactions at the supports, by taking moments about them, is often of value as a starting point in solving the problem graphically.



Vector diagram for
forces at apex

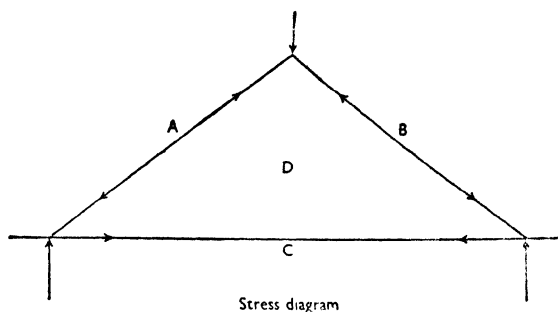


Vector
diagrams for
forces at supports



Reciprocal
figure

Fig. 13.9



Member	Stress	Nature
AD	3.4 tons	Strut
BD	3.4 "	Strut
CD	2.8 "	Tie
AC (reaction)	2.0 "	Support
BC (reaction)	2.0 "	Support

Fig. 13.10

Example 4.—Find the forces acting in the crane shown in fig. 13.11a. Indicate their nature.

This is a slightly more difficult problem. The load is suspended by a rope passing over a pulley at the jib-head. The pull in this rope

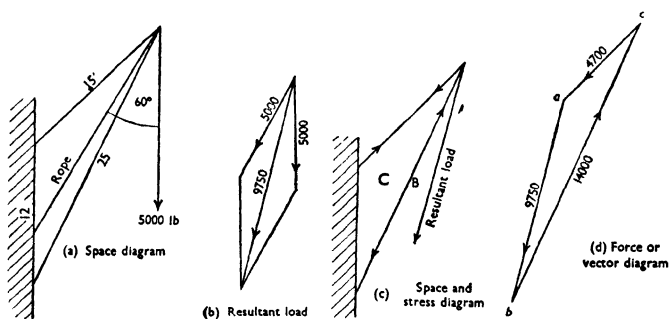


Fig. 13.11

will act downwards from the jib-head on both sides of the pulley, the force in both cases being equal to the load (action and reaction being equal and opposite). One method of procedure is as follows:

(1) First find the resultant load due to the pull in the two portions of the rope (fig. 13.11b).

(2) Then proceed to find the forces in the members by the procedure indicated for the simple jib crane in Example 2, using the space diagram incorporating the resultant load (fig. 13.11c).

For the load and dimensions given the vector diagram will be as shown in fig. 13.11d.

The results are tabulated below:

Member	Force, lb.	Nature
AB	9,750	Resultant load
BC	14,000	Strut
CA	4,700	Tie

4. Graphical representation of the forces acting on a body on a smooth inclined plane

Consider a body placed on a frictionless inclined plane, and supported by means of a string (fig. 13.12a). As it is assumed that there is no friction present, then the body would slide down the plane under its own weight unless so supported.

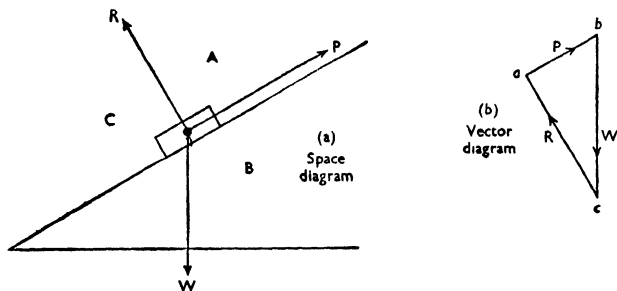


Fig. 13.12

The forces acting on the body and maintaining equilibrium are:

1. The weight (W) of the body acting vertically downwards.
2. The pull or effort (P) in the string.
3. The equilibrant or reaction (R) of the plane on the body, acting at right angles to the plane.

These forces can therefore be represented by the three sides of a triangle, and a vector diagram may be drawn (fig. 13.12b). (N.B. The reaction of the plane is the opposite to the force the body exerts on the plane, as action and reaction are equal

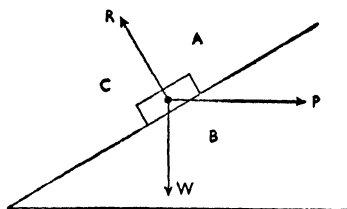


Fig. 13.13



and opposite. This reaction must be present, as otherwise the body would penetrate the plane.)

As it has been assumed that no force of friction is present (which force would tend to prevent sliding), the body will slide freely in any position. Its reaction must therefore be at right angles to the plane, so as not to oppose this motion. *This*

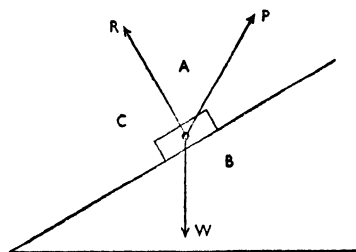


Fig. 13.14



is the direction of the reaction at any smooth surface. In practice, when friction is present, this reaction will not be at right angles to the plane, but will differ from it by a small angle, known as the **angle of friction** (see Chap. XIV, § 4).

The pull or effort (P) may be applied:

- (i) parallel to the plane (as in fig. 13.12), or
- (ii) horizontally (as in fig. 13.13), or
- (iii) at any angle with the plane (as in fig. 13.14).

The corresponding vector diagrams for these cases are shown.

Example 5.—A body weighing 50 lb. is supported on a smooth inclined plane by a rope acting parallel to the plane. If the angle of inclination is 30° , find (i) the force in the rope, (ii) the pressure on the plane.

This problem may be solved graphically by drawing a vector diagram as in fig. 13.12*b*, making $bc = 50$ lb. and angle $bca = 30^\circ$.

By measurement:

$$\text{Force in rope } (P) = ab = 25 \text{ lb.}$$

$$\text{Pressure on plane } (R) = ac = 43\frac{1}{3} \text{ lb.}$$

By trigonometry:

In the triangle abc (fig. 13.12*b*),

$$\sin 30^\circ = P \div W,$$

$$\therefore P = W \sin 30^\circ = 50 \times 0.5 = 25 \text{ lb.}$$

$$\cos 30^\circ = R \div W,$$

$$\therefore R = W \cos 30^\circ = 50 \times 0.866 = 43.3 \text{ lb.}$$

Where the effort (P) is applied horizontally as in fig. 13.13, it will be seen from the vector diagram that

$$\tan acb = P \div W,$$

$$\therefore P = W \tan acb;$$

and, as $(ac)^2 = (ab)^2 + (bc)^2$,

$$R^2 = P^2 + W^2,$$

$$\therefore R = \sqrt{P^2 + W^2}.$$

5. Determination of forces in structures by moments

An alternative method of solving some structural problems is by means of moments.

Consider again the example regarding the inclined plane (Example 5), and refer to fig. 13.15.

In the triangle ABC , as $\angle BAC = 30^\circ$, then the sides $AB:BC:CA = 1:0.5:0.866$ respectively. (This result could be obtained by drawing to scale and then measuring.)

Imagine rotation is possible about A . Then take moments about A .

$$\text{Clockwise moments} = 0.866 \times W = 0.866 \times 50.$$

$$\text{Anticlockwise moments} = 1 \times R.$$

(Note that the perpendicular distances from the line of action of the forces to the fulcrum have been taken, and that the moment of P about A in this case will be zero.)

When in equilibrium the moments will balance,

$$\therefore R = 0.866 \times 50 = 43.3 \text{ lb. (as obtained before).}$$

To find P.—Project R down to the baseline at D , and imagine rotation to be possible about D . Then taking moments about D :

$$\text{Clockwise moments} = P \times BD.$$

$$\text{Anticlockwise moments} = W \times CD.$$

$$\therefore P = W \times \frac{CD}{BD}.$$

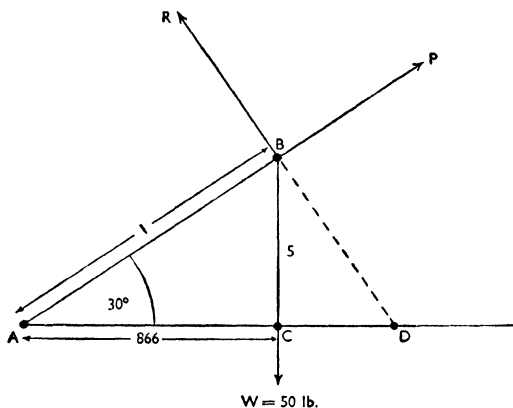


Fig. 13.15

(Note in this case that the moment of R about D is zero.)

The values of CD and BD can be found by drawing to scale and then measuring, or they can be found by using trigonometry.

In the triangle ABD , $\angle BDC = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$.

$$\cos \angle BDC = \cos 60^\circ = \frac{CD}{BD} = 0.5,$$

$$\therefore P = W \times \frac{CD}{BD} = W \cos 60^\circ = 50 \times 0.5 = 25 \text{ lb.}$$

(as obtained before).

Example 6.—Find the force in the tie-bar CB in the structure shown in fig. 13.16. A is embedded in the wall AC.

Take moments about A.

$$\begin{aligned}\text{Clockwise moments} &= (4 \times 10) + (2 \times 5) = 40 + 10 \\ &= 50 \text{ cwt.-ft.}\end{aligned}$$

Anticlockwise moments = tension in BC $\times$ distance AD.

$$\therefore \text{ Tension in BC} = \frac{50}{AD}.$$

The distance AD can be found by measurement; or by trigonometry,

$$AD = AB \sin 30^\circ = 10 \times 0.5 = 5 \text{ ft.}$$

$$\therefore \text{ Tension in tie-bar BC} = \frac{50}{5} = 10 \text{ cwt.}$$

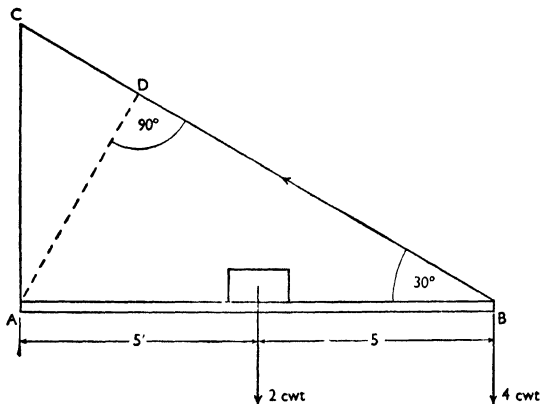


Fig. 13.16

The forces in the simple roof truss dealt with in Example 3 can also be found by means of moments (see fig. 13.17).

The reactions at A and B will be 2 tons each as the truss is symmetrical. Now imagine that rotation is possible about A as fulcrum (as would occur if support B were removed). Consider the forces acting in the members due to the load and which tend to cause rotation. Only that in CB will exert any turning moment about A.

Turning moment = force in CB $\times$ perpendicular distance of its line of action from fulcrum
 = force in CB $\times$ distance AD.

In order to prevent rotation this must be balanced by the anticlockwise moment of the reaction at the support B about A.

$$\begin{aligned}\text{Anticlockwise moment} &= \text{reaction at B} \times \text{span} \\ &= 2 \times 70 = 140 \text{ ton-ft.}\end{aligned}$$

By measurement, $AD = 40.5 \text{ ft.}$

$$\therefore \text{Thrust in CB} = \frac{140}{40.5} = 3.46 \text{ tons.}$$

(CA, being symmetrical, is also 3.46 tons.)

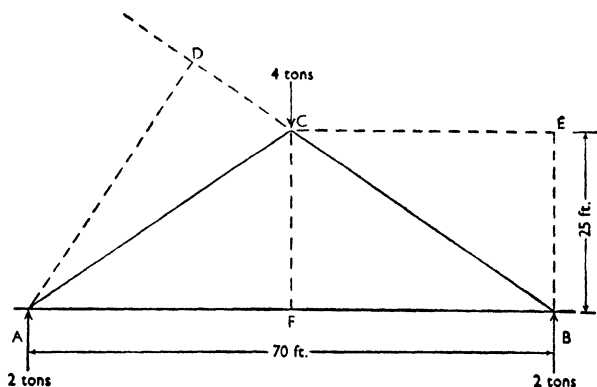


Fig. 13.17

Now imagine rotation possible about the apex (C) as would occur if support A were removed, and consider the forces in members tending to rotate CB. Only that in AB will exert any turning moment about C.

$$\text{Turning moment} = \text{force in AB} \times \text{distance CF} = \text{force in AB} \times 25.$$

To prevent rotation this must be balanced by the anticlockwise moment of the reaction at support B about C.

$$\begin{aligned}\text{Anticlockwise moment} &= \text{reaction at B} \times \text{distance CE} \\ &= 2 \times 35 = 70 \text{ ton-ft.}\end{aligned}$$

$$\therefore \text{Pull in AB} = \frac{70}{25} = 2.8 \text{ tons.}$$

If the distance AD is found by trigonometry, we can check the thrust in CB. First find $\angle CBF$.

$$\tan \angle CBF = \frac{CF}{BF} = \frac{25}{35} = 0.7143.$$

From tables, $\angle CBF = 35\frac{1}{2}^\circ$ (approx.).

Then $AD = 70 \sin 35\frac{1}{2}^\circ = 70 \times 0.5807 = 40.65 \text{ ft.}$

from which $\text{Thrust in CB} = \frac{140}{40.65} = 3.444 \text{ tons.}$

EXERCISES ON CHAPTER XIII

1. State the principle of the triangle of forces. Three members of a pin-jointed frame meet at a point. One of the members is vertical and connected to the other two at its lower end. The second and third members make angles of 120° and 210° respectively with the vertical member, the angles being measured in a clockwise direction from it. If the force in the vertical member is 10 tons compressive, find the magnitude and nature of the forces in the other two members. (U.L.C.I.)

✓ 2. The jib of a derrick makes an angle of 60° to the horizontal while the tie makes an angle of 30° to the horizontal. Find, *either* graphically, *or* by calculation, the forces in the jib and tie when the derrick supports a load of 2 tons, and state whether they are tensile or compressive.

What fundamental principle of applied mechanics was made use of when solving this problem? (U.L.C.I.)

3. A piece of cord which will break under a tensile load of 7 lb. is used to hang a picture weighing 4 lb. from a nail. The two sides of the cord include an angle of 150° at the nail. Assuming the picture hangs vertically, will the cord break? Your answer must be supported by clear reasoning to obtain marks. (U.L.C.I.)

✓ 4. The jib and tie of a crane make angles of 45° and 15° respectively with the horizontal. Find the stress in each when a load of 12 tons is suspended from the hook.

5. Explain what is meant by the "equilibrant" of a number of forces. A shaft weighs 1 ton. A crane lifts the shaft by means of a rope sling, and each half of the sling makes an angle of 50° with the shaft. By calculation, or by measurement, find the force in each half of the sling. (U.E.I.)

6. A weight of 40 lb. is held in equilibrium on a smooth inclined plane by a force X parallel to the plane. The angle the plane makes with the horizontal is 25° .

Find, by measurement or calculation, the force X , and the reaction of the plane. (U.F.I.)

7. Find the force in each bar of the roof truss, stating whether it is compressive or tensile. Find also the forces at the supports (fig. 13.18). (S.W.E.T.C.)

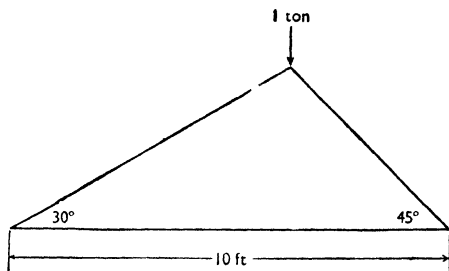


Fig. 13.18

8. A diagram is given below of a small roof truss. What are the forces in the members BD, DA and DC? (S.W.E.T.C.)

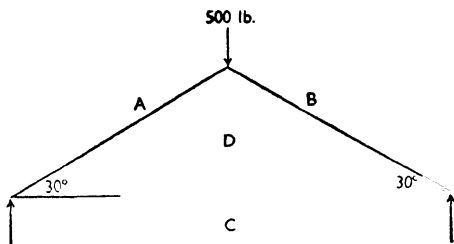


Fig. 13.19

9. A uniform rod is 10 ft. long and weighs 55 lb. Two strings each 12 ft. long are attached, one at each end of the rod. The other ends of the strings are attached to two points on the ceiling 20 ft. apart. Find the tension in each of the strings. (S.W.E.T.C.)

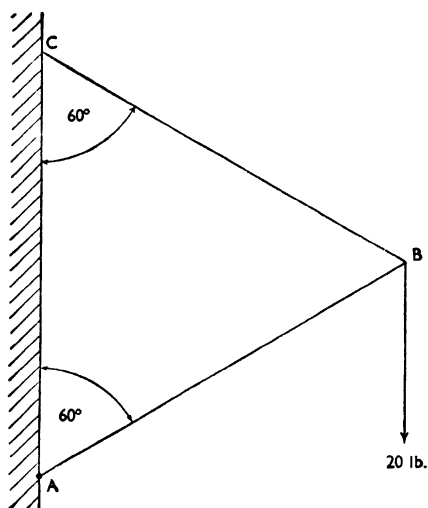


Fig. 13.20

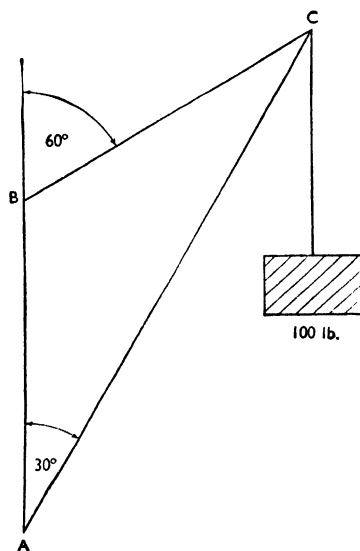


Fig. 13.21

10. AB is a rod 5 ft. long, weighing 32 lb., and is pivoted at A to the wall. The end B carries a load of 20 lb., and the rod is held in position by a wire BC. Find the tension in the wire (fig. 13.20).

(S.W.E.T.C.)

11. In fig. 13.21 ABC represents a simple crane. The jib AC is 18 ft. long and makes an angle of 30° with the vertical, the tie BC making an angle of 60° with the vertical. Find, by a graphical method, the forces and the nature of the forces in the jib and tie.

(S.W.E.T.C.)

12. A vertical flagstaff is secured by two wire ropes which make an angle of 60° with the horizontal ground. The pull in each rope is 100 lb., and the flagstaff weighs 80 lb. What is the resultant thrust of the flagstaff in the ground?

(S.W.E.T.C.)

CHAPTER XIV

Friction

1. Friction

When two bodies in contact move over each other a *resistance to movement* is set up. The amount of this resistance depends upon the nature of the surfaces in contact, i.e. the material used, whether it is smooth or rough-surfaced, whether wet or dry, etc.

This resistance to movement is termed **friction**, and the force necessary to overcome it may be called the **force or effect of friction** (see fig. 14.1).

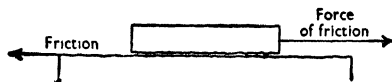


Fig. 14.1

Everyday experience indicates that a greater force is required to start a body sliding than is needed to keep it moving once it has been started. Consequently, the force of friction at starting will be greater than when movement is actually taking place; i.e. **static friction** will be greater than **sliding friction**.

2. Kinds of friction

Friction may be considered under the following headings:

1. *Solid (dry) friction*.—This occurs when solid surfaces are in contact, and are perfectly clean and dry. The amount of friction set up is high. An example of this type of friction, and the use to which it can be put, occurs in the friction clutch.

2. *Liquid (or fluid) friction*.—This occurs when particles of a fluid (liquid or gas) are in contact with each other or with a moving surface. In some cases this fluid may form a *complete* sandwich between two solid surfaces.

The amount of friction set up is small.

The resistance set up to the passage of a ship through water is an example of this kind of friction. Fluid friction may also

occur in systems of "forced lubrication" designed to make a surface "float" on oil.

3. *Boundary friction*.—This occurs when the surfaces in contact are smeared, for instance, with a thin layer of lubricant which does not entirely fill up all the interstices between them. This often happens in practice, as the pressure between the surfaces tends to squeeze the lubricant out; in consequence, partial solid contact may occur. Therefore this type of friction is a boundary case between 1 and 2.

Most ordinary methods of greasing, oiling, etc., are examples of boundary friction, the solid friction of the dry surfaces having been reduced by the action of the lubricant. For example, just prior to launching a ship, the ways are greased, so reducing friction and facilitating movement.

4. *Rolling friction*.—This occurs when the surfaces in contact roll over each other, as in the case of a wheel on a road. As it is obviously much easier to push a given load in a truck than to slide it along the ground, the amount of resistance set up with this kind of friction is small. It depends upon the amount of "give" in the surfaces in contact. A wheel on a soft road would sink into it, and would set up more frictional resistance than a wheel on a steel rail.

3. The coefficient of friction

Consider a body of weight W resting on a horizontal surface (fig. 14.2). A horizontal force P is applied to it, and is of such

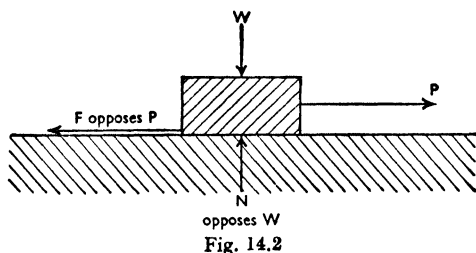


Fig. 14.2

a value as to just make the body slide over the surface at a steady rate. The ratio

$$\frac{\text{force of friction } (P)}{\text{pressure between surfaces } (W)}$$

is termed the **coefficient of sliding friction**, and is denoted by the Greek letter mu (μ) (pronounced "mew"). If P were the force required to *start* movement, then μ would represent the **coefficient of static friction**.

As friction always opposes motion, the force necessary to overcome friction ($\bar{P}$) will be opposed by the equal and opposite frictional resistance (F). Similarly, the pressure between the two surfaces (in this case the weight of the body W) will be opposed by the reaction (N) at right angles to ("normal" to) these surfaces. Consequently we may write

$$\mu = \frac{P}{W} = \frac{F}{N}.$$

4. Angle of friction

Now consider the body at rest under the action of W and the reaction N (see fig. 14.3). Let a small force p , insufficient to cause motion, be applied horizontally to it. This force will immediately be opposed by an equal and opposite force f .

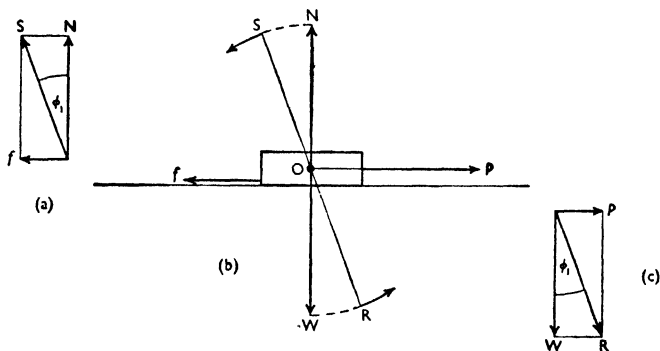


Fig. 14.3

The resultant of p and W can be found by using the parallelogram of forces (fig. 14.3c), and it will be seen that this resultant (R) is inclined at an angle ϕ_1 to the vertical. Similarly, the resultant of f and N is inclined at this same angle (fig. 14.3a). We may therefore draw a line SOR , inclined at angle ϕ_1 to the vertical, through the centre of gravity of the body.

The body is now in equilibrium under the action of the applied forces p and W , and the equilibrant OS .

If this applied force p is now gradually increased, we can imagine the line SOR gradually swinging out anticlockwise about O (fig. 14.3b).

This action will continue until the force p reaches the value P sufficient to cause motion (the frictional resistance f then reaching its maximum value F), when the angle between SOR and NOW will have swung out to its maximum limiting value.

This maximum value of ϕ , which is reached just prior to motion taking place, is called the **angle of friction**.

Referring to fig. 14.4, it will be seen that when movement is about to take place, the tangent of the angle of friction

$$\tan \phi = \frac{F}{N} \text{ or } \frac{P}{W}.$$

$$\text{But} \quad \mu = \frac{P}{W} = \frac{F}{N},$$

$$\therefore \tan \phi = \text{coefficient of friction} = \mu.$$

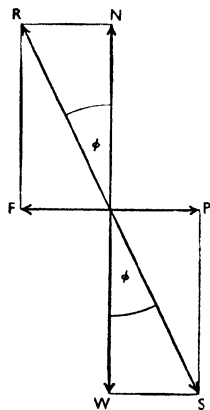


Fig. 14.4

5. Angle of repose

Consider this same body on the same flat plane, one end of which is now gradually raised. An angle of slope will eventually be reached beyond which the body would commence to slide. We will investigate this limiting angle (see fig. 14.5a).

The weight (W) of the body, which acts vertically downwards, can be considered as being divided into two components:

P , parallel to the plane, tending to cause movement down it;
 Q , perpendicular to the plane, representing the pressure exerted on it.

Under the stated conditions the friction set up will reach its maximum limiting value F , this exactly opposing P .

Consequently, Q may be regarded as being balanced by N , the "normal" reaction to the pressure between the surfaces, and W by an equal and opposite reaction R .

(Note $F = \mu N$).

The equilibrium of forces can be represented by a vector diagram (fig. 14.5b).

$$\tan \angle nfw = \frac{F}{N} = \frac{\mu N}{N} = \mu;$$

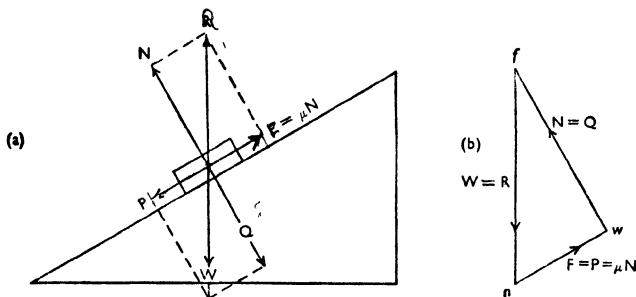


Fig. 14.5

but μ is the tangent of the angle of friction (ϕ)

$$\therefore \text{angle } nfw = \phi$$

(i.e. in fig. 14.5a the angle between N and R is ϕ).

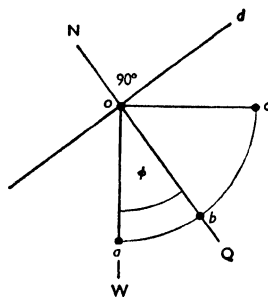


Fig. 14.6

In fig. 14.6, $\angle aob = \phi$. $\angle aoc = 90^\circ$. $\angle bod = 90^\circ$.

Then

$$\phi + \angle bod = \angle aoc + \angle cod,$$

i.e.

$$\phi + 90^\circ = 90^\circ + \angle cod;$$

$$\therefore \text{angle of slope of plane } (\angle cod) = \phi.$$

Consequently the steepest angle of slope to the horizontal at which a particular body can remain unsupported on an inclined plane without sliding is the same as the angle of friction. This angle may therefore be called the **angle of repose**.

The coefficient of friction between a material and a surface may readily be found experimentally, by tilting the surface until the material just begins to slide. Then the angle of slope is ϕ and

$$\tan \phi = \text{coefficient of friction} = \mu.$$

This is strictly known as the **coefficient of limiting or static friction**, because the frictional resistance reaches an upper *limiting* value just prior to movement occurring.

6. Elementary laws of dry friction

By pulling various materials along various surfaces in the laboratory, the following general laws may be established:

1. Friction always opposes motion. (The frictional force may be sufficient in some cases to prevent motion altogether.)
2. The force of friction depends upon the materials and quality of the surfaces in contact.
3. The force of friction is directly proportional to the total pressure between the two surfaces, i.e.

force of friction = (coefficient of friction)

× (normal, i.e. perpendicular, pressure between surfaces),

$$\therefore P = \mu W.$$

4. The force of friction is independent of the areas of the surfaces in contact, i.e. if an ordinary brick was slid, first on its face and then on its side, the force of friction required would be the same.

N.B. These laws are approximately true for normal practice. At high speeds and pressures they require to be modified. Again, if two surfaces are extremely dry and flat, and are kept under pressure, the friction may increase up to such a point that adhesion occurs. Workshop slip gauges readily show this tendency when "wrung" together to build up a dimension; consequently they should not be left in this state (under pressure) for any length of time, as they tend to adhere even more firmly the longer they are left. The force of friction

needed to slide them apart might rise to 70 lb. per sq. in. of contact area.

Example 1.—The slide valve of a steam engine is 15 in. long and 8 in. wide. The pressure of steam on this surface is 100 lb. per sq. in. If the coefficient of friction between the slide valve and the valve face is 0.12, calculate the force required to move the valve.

The force required = $P = \mu W$.

$$\begin{aligned} W &= \text{total pressure on valve} = \text{area} \times 100 \\ &= (15 \times 8) \times 100 = 120 \times 100 = 12,000 \text{ lb.} \end{aligned}$$

$$\therefore \text{Force required} = P = 0.12 \times 12,000 = 1440 \text{ lb.}$$

Example 2.—It is found that an iron block of weight 1 cwt. can be pulled along a slightly wet horizontal surface of oak by a horizontal force of 30 lb., the motion being steady. Calculate (a) the coefficient of sliding friction for these two surfaces under these conditions, (b) the angle of slope of an oak slipway down which an iron hull would just slide.

$$(a) \quad \mu = \frac{\text{force of friction } (P)}{\text{pressure between surfaces } (W)} = \frac{30}{112} = 0.268.$$

$$(b) \quad \text{Angle of slope} = \text{angle of repose} = \text{angle of friction} = \mu = \tan \phi = 0.268.$$

Therefore ϕ is the required angle.

From trigonometrical tables, the angle whose tangent is 0.268 is 15° .

7. Inclined plane with friction

It was seen in Chap. XIII, § 4, that, when a body is placed on a frictionless inclined plane and is supported by means of a string, the reaction at the surface of the plane is always at right angles to the plane.

When friction is present, the direction of this reaction will differ from the normal to the plane (the right angle) by the *angle of friction* (see § 4). The reaction will always act on that side of the normal where it will *oppose* the motion of sliding.

Problems involving friction on an inclined plane can be solved graphically if this is remembered.

Example 3.—If the coefficient of friction between the 50-lb. body and the plane inclined at 30° to the horizontal (see Chap. XIII,

Example 5) is 0.3, find the force now required in the rope to just pull up the body when applied parallel to the plane.

$$\mu = 0.3 = \tan \phi,$$

$\therefore$ angle of friction (ϕ) = 17° approx. (from tables).

The reaction (R) can then be drawn 17° from the normal to the plane. As friction always opposes motion, R will be drawn on the left of the normal. The vector diagram (fig. 14.7) is then drawn, from which, by measurement, $P = 37.5$ lb. approx.

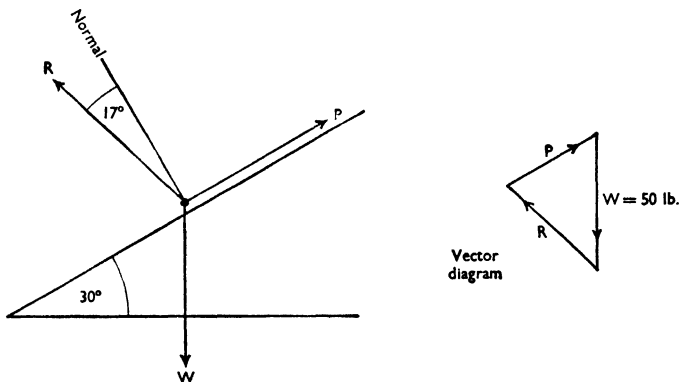


Fig. 14.7

If the body was allowed to slide down the plane [which it would do if the rope was cut, as the angle of inclination (30°) is steeper than the angle of repose (17°)], then the reaction R would take up a position 17° to the right of the normal, so as to tend to oppose this motion of sliding.

8. Wanted and unwanted friction

Friction is often very necessary, but is only required in its right place. In a frictionless world, for instance, walking would be impossible (note what happens if one walks carelessly on the smooth surface of ice); motors and trains could not run (their wheels would merely slip round uselessly); and even if vehicles could move, we would not be able to stop them by means of the friction brake.

On the other hand, as friction opposes motion, some energy is lost whenever motion occurs, but owing to the law of con-

servation of energy, it reappears as unwanted heat. Thus, as friction wastes energy, it will decrease the efficiency of machines. Therefore all machinery should be kept lubricated, to reduce this loss of energy as far as possible.

Friction and generation of heat will occur in such places as bearings, the driving wheels of locomotives, and between the plates of a clutch. The bearings would be lubricated, because here friction must be reduced as far as possible. The driving wheels (or rather the surface between wheels and rails) would not be lubricated, as here some friction is required in order to move the train along the rails. The rims of the wheels would become warm, however, but if we attempted to reduce this loss the locomotive would slip badly. Therefore here we have to compromise. The pressure between surfaces (adhesion weight of locomotive) (W), and the coefficient of friction between the rolling wheels and rails (μ), decides the amount of friction present ($F = \mu W$). When this is not sufficient to prevent slipping, μ may be increased temporarily (as at starting) by introducing sand. On a wet day the lubricating effect of the water on the rails may also cause slipping, and here again the sandblower may be used to increase the friction when starting a heavy train.

In the case of the clutch, the object is to make the friction between plates so great that adhesion occurs and the plates turn together as if they were one body. This is achieved by using clutch lining material with a high coefficient of friction, and by making the pressure between plates very high.

9. Bearing friction

The friction, with consequent generation of heat and therefore waste of energy, which occurs due to the rotation of a shaft in a bearing, is an important practical case. The frictional resistance between the circumference of the shaft and the bearing sets up a retarding torque on the shaft. This frictional resistance (F) = μW , where W = pressure exerted on the bearing by the shaft (i.e. the load on the bearing).

Therefore frictional torque

= moment of frictional resistance about centre of shaft

$$= \mu W r,$$

where r = radius of shaft.

Example 4.—An engine shaft of $2\frac{1}{2}$ in. diameter revolves at 240 r.p.m. The load on the shaft is 300 lb., and the coefficient of friction between shaft and bearing is 0.03.

Find the frictional torque and the energy wasted in friction per minute.

$$\text{Frictional torque} = \mu Wr = 0.03 \times 300 \times 1.25 = 11.25 \text{ lb.-in.}$$

The energy wasted per minute can be found in foot-pounds (see Chap. XV), and is equal to (torque $\times$ angle turned through in radians).

$$\begin{aligned} \text{Work done against friction per minute} &= 11.25 \times 2\pi \times 240 \\ &= 16,960 \text{ ft.-lb.} \end{aligned}$$

Knowing the mechanical equivalent of heat (Chap. XXVII, § 4), we can determine the amount of heat generated by bearing friction per minute.

As 778 ft.-lb. of work done against friction will generate 1 B.Th.U., then

$$\text{Heat generated by friction per minute} = \frac{16,960}{778} = 21.8 \text{ B.Th.U.}$$

10. Lubrication

By interposing a layer of oil or grease between surfaces, the sliding “solid friction” between bodies is considerably reduced. The laws of solid friction do not completely apply, and those appertaining to “fluid friction” (not considered in detail here) are approached as the layer of lubricant between the surfaces approaches a complete sandwich. Whilst the sandwich is not complete we have the conditions of “boundary friction”.

The lubrication of a machine may be carried out:

1. *By gravity or contact.*—In this case the oil may drip down from a container, be wiped on by a pad, etc., or be splashed on to the surfaces in an oil-bath.

2. *By forced lubrication.*—In this case a pump is used to force the oil around the various surfaces in contact at such a pressure as to ensure the maintenance of a film of oil between them. This method is used especially with high-speed machines, and in cases where the pressure between surfaces would normally squeeze out the lubricant.

11. Ball and roller bearings

The frictional resistance offered to rolling a ball or cylinder along a surface is much less than that offered to sliding an object of similar weight. This is obvious if we consider moving a box of, say, 1 cwt. Sliding would entail considerable effort, but if wheels or rollers were placed underneath it would be an easy task.

Advantage of this fact is taken by using ball and roller bearings which have very free-running properties, as will be evident if a bicycle wheel is set spinning and the time taken to come to rest noted. Such bearings need only a small coating of lubricant, mainly for the purpose of preventing rust.

12. Bearing linings

Most plain bearings are fitted with a lining of some soft "anti-friction" material, such as white metal. This has a low coefficient of friction, and also a comparatively low melting-point. If the bearing becomes seriously overheated through lack of lubrication, this metal will melt and, by so giving warning, will enable steps to be taken to prevent the machine "seizing up". Such linings can be readily replaced.

Lining materials have been recently developed which contain a lubricant actually embodied in their structure. Such materials can be incorporated in bearings which need no lubricating arrangements, and which will run for years with the minimum of attention.

13. Thrust bearings

An interesting case occurs with a ship. Here the engines turn the propeller which *thrusts* the water backwards and so makes the ship travel forward. This thrust, in accordance with the law regarding action and reaction being equal and opposite, has a reaction which is transmitted along the propeller shaft, and has to be taken up by a suitable "thrust bearing".

To cope with the terrific thrust set up in the shaft of, say, an Atlantic liner, very large multiple thrust-blocks were needed, containing collars bearing on to the frame. Lubrication of

these was a problem, and playing a hose-pipe on to a troublesome bearing in mid-Atlantic was not unknown.

The *Michell Thrust Block*, however, gets over the problem in an interesting manner, and without occupying a great deal of space (see fig. 14.8). Pads or blocks are so pivoted that they are able to tilt slightly out of parallelism, but can still be held

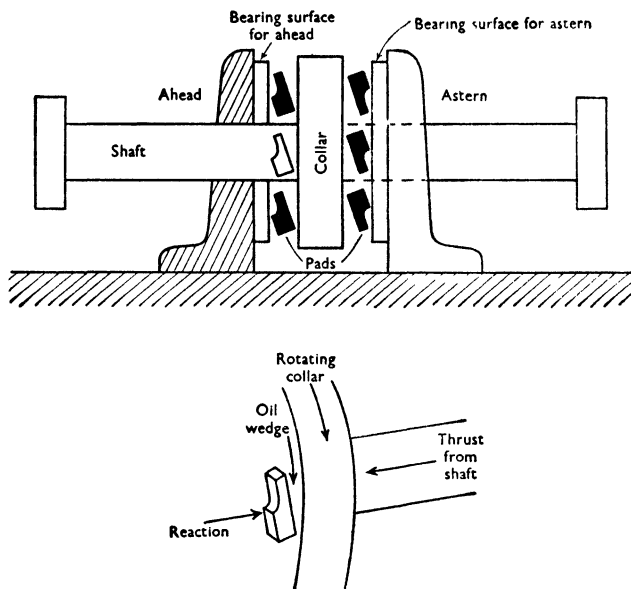


Fig. 14.8.—Michell thrust bearing for marine use
The lower sketch shows a line-pivoted pad, 8 such pads (gunmetal with white metal facings) being arranged in a circle

by the fixed casing. Oil is forced between each pad and the rotating collar by the motion, and forms a wedge-shaped film which will carry a very considerable thrust without breaking down. This thrust is transmitted via the ring of pads to the fixed casing, and so to the ship's framework. A separate set of pads is needed on either side of the collar, one for ahead and the other for astern working. Similar thrust blocks may be used in turbines, geared drives, etc.

14. Tractive resistance

All vehicular traffic makes use of the fact that rolling friction is less than solid friction. We have already seen that some frictional resistance is necessary, however, to ensure movement, but that for efficiency it must be reduced to the minimum compatible with requirements. This frictional resistance is termed **tractive resistance**, and, for a train on rails, may be about 12 lb. per ton of the train's weight. The locomotive must produce a draw-bar pull or **tractive effort** equal to this in order to maintain movement on a level track. To provide acceleration, to climb a gradient, or to start a heavy train, however, the locomotive must be capable of producing a considerably greater pull.

For example, the tractive effort required to maintain a steady speed on level track with a train of 500 tons weight, tractive resistance being 12 lb. per ton, would be

$$12 \times 500 = 6000 \text{ lb. wt.}$$

The nominal maximum value* of tractive effort developed by a typical medium-powered general-purpose locomotive, such as the L.M.S. (British Railways, London Midland Region) Class 5, 4-6-0 mixed-traffic type, is 25,455 lb. The difference between the maximum tractive effort achieved, and that required to overcome the tractive resistance of the train, will be the pull available for climbing and accelerating purposes. Should this not be sufficient to ensure adequate climbing ability, acceleration, and speed with a heavy train, then a more powerful express locomotive, such as the Class 7P "Duchess" 4-6-2 of the same railway system, developing a nominal tractive effort of 40,000 lb., would be used.

It should be noticed that it is no use increasing tractive effort unless the adhesion weight (the weight on the coupled driving wheels) is sufficient to prevent slipping. The limiting value of useful tractive effort is equal to $\mu \times$ (adhesion weight), where μ is taken as the coefficient of friction between the rolling

* This nominal value is used mainly for purposes of comparison. It is rarely, if ever, achieved, as it is calculated on the assumption that the pressure in the cylinders is maintained at 85 per cent of the boiler pressure throughout each stroke, whereas in practice the steam is used "expansively", being "cut off" at from about 10 to 70 per cent of the stroke, according to the immediate needs.

wheels and rails. Consequently, locomotives are of heavy construction, with a large proportion of their weight supported on their driving wheels. Goods locomotives, requiring to make use of the greatest possible pulling power, are often designed so that all of their weight is available for adhesion. An express locomotive sacrifices some adhesion weight in order to ensure safe running around curves at high speed, being fitted with a leading bogey, and sometimes trailing wheels, for this purpose.

Experiments have shown that with good modern railway vehicles the tractive resistance on straight level track is about 11 lb. per ton of the train's weight at 60 m.p.h. This value varies slightly with speed, and with the type of vehicle, bearings, etc., and, of course, it increases considerably on curves.

An electric street tramcar, using as it does a type of rail of such a section that it presents more friction to movement than ordinary railway lines, has a tractive resistance of about 25 lb. per ton.

With road transport, the tractive resistance between wheels and road is higher than that of railway transport. This accounts for the great advantage the railways have over road transport, as it enables them to pull a much greater load for a given expenditure of effort. The actual tractive resistance will, of course, depend upon the type of wheel and its condition, and the type of road surface. A cart on a rough gravel road may present a tractive resistance of 150 lb. per ton, and on a macadam road a value of 45-65 lb. per ton. A pneumatic-tyred vehicle may present 35-45 lb. per ton on a secondary road, and this may fall to about 25-35 lb. per ton on a smooth road.* If the tyres are not fully inflated, the value will increase considerably.

The pull required to maintain movement of a heavily laden barge is even less than with rail transport. Consequently canal transport of heavy loads can be very economical, provided that speed is not important.

15. Streamlining

With high-speed transport, air friction or resistance has to be considered, and may approach values comparable to trac-

* Many first-class roads are purposely made slightly rough to prevent skidding.

tive resistance. To reduce this air resistance we may make use of **streamlining**.

The amount of air resistance increases very rapidly as the speed goes up.

The saving achieved by streamlining is not generally worth while unless speeds of at least 80–90 m.p.h. are required. It is therefore of great importance in aircraft design, but did not come into practical use on railways until such speeds became necessary. The first streamlined trains were introduced into Great Britain by the London and North Eastern and the London Midland and Scottish Railway Companies during the period 1935–37. One such train, the L.N.E.R. “Silver Jubilee”, covered the distance from London to Newcastle in 4 hours, necessitating an average speed throughout of 67 m.p.h. The schedule demanded sustained speeds of 85–90 m.p.h. on certain parts of the route. It was found that streamlining made a saving of approximately 10 per cent in the horse-power required at 90 m.p.h. During a test run in 1938 with a train of this type, the L.N.E.R. recorded a maximum speed of 126 m.p.h., a world’s record for steam traction.

Of more practical importance in increasing the load on a locomotive is the effect of a cross wind blowing on the train. This can set up a very considerable amount of frictional resistance, by causing the wheel flanges on one side to press hard against the rails. This effect cannot be reduced by streamlining.

The frictional resistance between the hull of a ship and the water through which it passes also increases rapidly at high speeds. Considerable saving in fuel consumed, etc., can be achieved by ensuring a suitable design of hull form, so as to reduce this resistance to a minimum. Experiments are usually carried out by towing models in a tank of water, and noting the force required to do so, in order to find the most suitable hull form for a new design of ship.

Example 5.—The adhesion weight of the L.N.E.R. (British Railways, Eastern Region) locomotive “Mallard” is 66 tons. If the coefficient of friction between driving wheels and rails under starting conditions be taken as 0.21, find the maximum tractive effort which it can exert under these conditions before slipping occurs.

If the tractive resistance is 11 lb. per ton, and the total weight of the locomotive and train is 500 tons, determine the steepest gradient

up which it can climb at uniform speed, assuming the above conditions.

$$\mu = \frac{P}{W} = \frac{\text{permissible tractive effort}}{\text{adhesion weight}} = \frac{\text{p.t.e.}}{66} = 0.21.$$

$\therefore$ Maximum permissible tractive effort = $0.21 \times 66 = 13.86$ tons
= 31,000 lb.

Force required to overcome tractive resistance (friction) = $11 \times 500 = 5500$ lb. If there is no acceleration, then

Force available for climbing = $31,000 - 5500 = 25,500$ lb.

With this force available for moving up the plane the angle of slope (ϕ) can be increased until there is no acceleration.

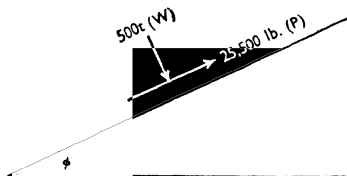


Fig. 14.9

Under these conditions (see fig. 14.9)

$$\frac{P}{W} = \frac{25,500}{500 \times 2240} = \tan \phi = \text{gradient.}$$

Expressing this gradient in the usual form, this becomes

$$\begin{aligned} & \frac{25,500}{25,500} \div \frac{500 \times 2240}{25,500} \\ &= 1 \text{ in } 44 \text{ (approx.).} \end{aligned}$$

N.B. (1) In fig. 14.9 the train's weight 500 tons (W) would act vertically downwards, but the difference between this and the normal to the hypotenuse is small for all ordinary railway gradients (ϕ being small), and can be ignored.

(2) The true gradient is $\tan \phi$, but railway gradients, etc., are measured in terms of $\sin \phi$ (opposite $\div$ hypotenuse), and expressed in the form of a rise of 1 (opposite) in x units (hypotenuse), e.g. 1 in 200. As ϕ is never very large in such cases, the difference between $\tan \phi$ and $\sin \phi$ can usually be neglected.

EXERCISES ON CHAPTER XIV

Section A

1. Define "coefficient of friction".

Mention one instance where it is desirable to reduce friction, and the means by which this may be done; also one instance where use is made of the friction between two surfaces.

A horizontal force of 60 lb. is required to slide a casting along a workshop floor at a uniform speed. If the coefficient of friction between casting and floor is 0.4, calculate the weight of the casting. (U.L.C.I.)

2. The area of the working face of a slide valve is 140 sq. in., and the steam pressure on the back of the valve is 100 lb. per sq. in. If the coefficient of friction for the surfaces in contact is 0.09, find the thrust in the slide-valve rod necessary to overcome friction. (S.W.E.T.C.)

3. (a) Find the least force required to slide an electric iron weighing $5\frac{1}{2}$ lb. over a surface, if the coefficient of friction between the iron and the surface is 0.3.

(b) To what angle may this surface be tilted before the iron commences to slide of its own accord?

4. Find the greatest weight a man can pull along a floor if the coefficients of friction between his boot soles and the floor, and between the weight and the floor, are 0.5 and 0.3 respectively.

5. What force would be necessary just to maintain motion in a body weighing 18 cwt. (a) along a horizontal plane, (b) up an incline of 1 in 10? The coefficient of friction in each case is 0.08. (U.L.C.I.)

6. What is the "coefficient of friction"?

A variously loaded slider was moved slowly, but uniformly, over a horizontal surface, and the different total weights (W) of the cradle and the corresponding horizontal forces (F) necessary to move it were as follows:

W lb.	10	15	20	25	30	35
F lb.	1.9	3.1	4.1	4.9	6	7.1

Plot a graph connecting F and W ; and determine the coefficient of friction for the surfaces. What force would be necessary to move the cradle when loaded with 27 lb? (U.L.C.I.)

7. The adhesion weight of a locomotive is 60 tons, and the coefficient of friction between its wheels and the rails is 0.15. Determine the maximum tractive effort which it can develop when pulling a train without its wheels slipping.

If the tractive resistance is 12 lb. per ton, and the weight of locomotive and train is 500 tons, find the value of the steepest gradient it can climb without losing speed.

Section B

(A knowledge of Work, Power, and Heat is required)

8. Define the term "coefficient of friction".

A body weighing 72 lb. rests on a horizontal surface. The coefficient of friction between the surfaces in contact is .072. If the body is pushed a horizontal distance of 15 ft., how much work is done?

(S.W.E.T.C.)

9. The piston of a horizontal engine weighs 55 lb. The engine runs at 120 r.p.m. The coefficient of friction between the piston and the cylinder is 0.085. The length of stroke of the piston is 18 in. What power is lost in overcoming friction?

(S.W.E.T.C.)

10. A table of a planing machine weighs 2350 lb., and slides backwards and forwards on a bed. The coefficient of friction between the table and the bed is 0.082. The table moves a horizontal distance of 9 feet in 5 seconds. Find

(a) the horizontal force required to move the table;

(b) the horse-power required to move the table. (S.W.E.T.C.)

↓11. Define the term "coefficient of friction".

A piston in an engine weighs 5 lb. and slides in a cylinder. If the coefficient of friction is 0.1, what horizontal force on the piston is necessary to just move it? If the piston moves 2 ft. in 0.75 sec., what horse-power is lost in overcoming friction? (S.W.E.T.C.)

12. A body weighing 15 lb. is placed on an inclined plane, and is found to slide down the plane at a uniform velocity when the angle of inclination of the plane is 30° . (a) What is the coefficient of friction? (b) If the body is pulled up the plane a distance of 12 ft., measured along the plane, what work is done? (S.W.E.T.C.)

13. The total pressure on the bearings of a horizontal shaft is 5 tons. The shaft is 7 in. diameter and runs at a speed of 200 r.p.m. If the coefficient of friction is 0.02, find the number of ft.-lb. of work absorbed by friction per minute, and the heat equivalent of this work. (U.L.C.I.)

14. Explain what you understand by "angle of friction", and establish the relationship between the "angle of friction" and the "coefficient of friction". A rotor weighing 5 tons is mounted in bearings of 4 in. diameter, and the torque necessary to make the engine revolve is 20 lb.-ft. Determine the "coefficient of friction" at the bearings and the horse-power absorbed by friction when the speed is 250 r.p.m. (U.L.C.I.)

15. A mass weighing 30 lb. is on the point of moving down a rough inclined plane when supported by a force of 12 lb. weight acting up the plane and parallel to it, and is on the point of moving up the plane when a force of 18 lb. weight acts up the plane and parallel to it. Find the value of the coefficient of friction. (C.P.E.)

CHAPTER XV

Work and Power

1. Work

We have seen that energy is that agent capable of doing work. This energy is utilized in overcoming some form of resistance, and so producing movement of a body, or of some particle of matter. To overcome resistance, force is needed. Hence work is done whenever force acts on a body and causes it to move against a resistance. If there is no movement, or no resistance, then no work is done, however great the force. A man pushing hard against a wall does no work from the scientific viewpoint, in spite of the sweat running down his brow, unless, of course, the wall collapses owing to his efforts. If we could imagine a frictionless surface, then a man skating on to it would move without the expenditure of any energy, his speed remaining that with which he approached the surface.

To perform work in moving a ship through a distance, a force must be exerted to overcome the frictional resistance of the water. Similarly, with a vehicle of any kind, some form or forms of resistance must be overcome in order to produce movement. When lifting a weight, the resistance to be overcome is that due to the earth's pull, or gravity.

2. Units of work

In the gravitational system of units work is usually measured in **foot-pounds**.

A "foot-pound" is the amount of work done when a resistance of one pound is overcome through a distance of one foot, *measured in the direction in which the force acts*, i.e.

$$\begin{aligned} \text{work done} &= \frac{\text{resistance overcome}}{(\text{or force exerted})} \times \frac{\text{distance moved in the}}{\text{direction of the force,}} \\ (\text{foot-pounds}) &= (\text{pounds}) \times (\text{feet}). \end{aligned}$$

For example:

1. The work done in lifting a 2-lb. weight vertically through 6 ft. is $2 \times 6 = 12$ foot-pounds (ft.-lb.).

2. The work done by a locomotive of 80 tons weight in moving through 10 ft. on the level when the tractive resistance is 14 lb. per ton is

$$(80 \times 14) \text{ lb.} \times 10 \text{ ft.} = 11,200 \text{ ft.-lb.}$$

Other units of work employed when more convenient include foot-tons and inch-pounds.

Units employed in the metric (gravitational) system include gram-centimetres and kilogram-metres

In the absolute (C.G.S.) system of units used in the scientific field, the unit of work is called the **erg**. This is the work done when a force of one dyne acts through a distance of 1 centimetre (i.e. a dyne-cm.). It is a rather small unit, so a larger unit, the **joule**, consisting of 10 million ergs, is employed where necessary.

The joule is the unit of work or energy in the M.K.S. system of units, and is equal to a force of 1 newton acting through a distance of 1 metre.* (1 joule = 0.7372 ft.-lb.)

3. Work done in climbing a slope

In cases where work is done against gravity, whether it be in lifting (such as raising coal from a mine), or in climbing an incline, work done = weight raised $\times$ *vertical* distance lifted.

In fig. 15.1, the work done in both cases in raising the car of

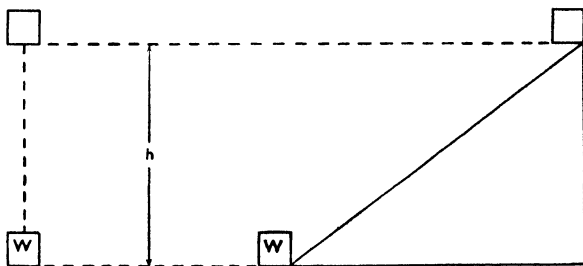


Fig. 15.1

* Units of force are explained more fully in Chap. XXII.

tion of motion of the barge is 30° , and the average pull in the rope is 60 lb. Find the work done per hour.

$$\begin{aligned}\text{Work done per hour} &= \text{force} \times \cos \theta \times \text{distance moved per hour} \\ &= 60 \times 0.866 \times 5280 \times 3 = 823,046 \text{ ft.-lb.}\end{aligned}$$

6. Work diagrams

The amount of work done by a force acting through a distance may be represented diagrammatically.

Fig. 15.4a shows a graph of force against distance for a force of constant magnitude; in fig. b, the force is increasing uniformly; in fig. c, it is varying irregularly. In each case the

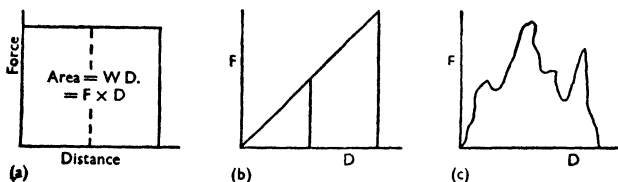


Fig. 15.4

work done by the force in travelling through the distance D is given by the area under the curve. This area can be found arithmetically, or by means of a planimeter if necessary.

N.B. If the force is expressed in pounds and the distance in feet, then the work done will of course be in foot-pounds.

Use of a "work diagram" where the force varies in a regular manner

Example 4.—The rope of a hoist weighs $1\frac{1}{4}$ lb. per ft., and the weight of the hook is 16 lb. Find the total work done in lifting a load of 2 cwt. through 200 ft.

First find the weight to be lifted (or force required) at various heights.

- (1) At commencement of the lift (0 ft.), the force required
 $= \text{wt. of load} + \text{hook} + \text{wt. of full length of rope}$
 $= 224 + 16 + 200 \times 1\frac{1}{4} = 490 \text{ lb.}$
- (2) Half-way up (100 ft.) the force required
 $= 240 + 100 \times 1\frac{1}{4} = 365 \text{ lb.}$
- (3) At finish of the lift (200 ft.) the force required
 $= 240 + 0 = 240 \text{ lb.}$

These weights when plotted on a work diagram (fig. 15.5) will be found to lie on a straight line. (Any other values calculated will also lie on the same straight line.)

$$\begin{aligned}
 \text{Work done} &= \text{area of work diagram} \\
 &= \text{area of rectangle OABC} + \text{area of triangle BCD} \\
 &= 200 \text{ ft.} \times 240 \text{ lb.} + \frac{1}{2} \times 200 \text{ ft.} \times 250 \text{ lb.} \\
 &= 48,000 + 25,000 = 73,000 \text{ ft.-lb.}
 \end{aligned}$$

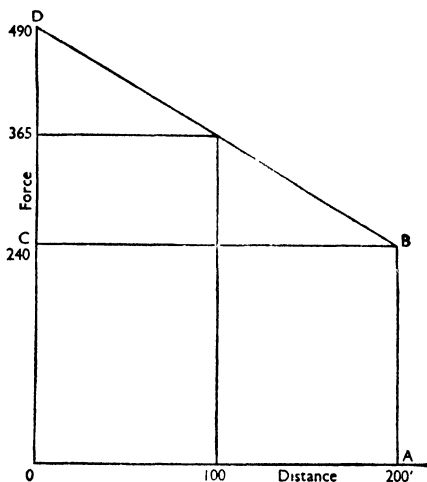


FIG 15.5

Alternatively:

As the curve is a straight line, its mean height (or the average force) can be found as follows:

$$\text{Average force} = \frac{1}{2}(\text{OD} + \text{AB}) = \frac{1}{2}(490 + 240) = \frac{1}{2} \times 730 = 365 \text{ lb.}$$

$$\therefore \text{work done} = \text{average } F \times D = 365 \times 200 = 73,000 \text{ ft.-lb.}$$

In a work diagram where the force varies irregularly (as in fig. 15.4c), the average value of the force can be found by making use of the "mean-ordinate rule" (see Chap. II, § 7).

The area under or enclosed by the curve is divided into a number of strips of equal width. The height along the middle

of each strip is measured, and the average height is thus found.

$$\text{Average height} = \frac{\text{sum of heights of ordinates}}{\text{number of ordinates}}.$$

This height, expressed to the appropriate scale, will then give the average force.

Alternatively, the area can be found by any desired method, e.g. by means of a planimeter (see Chap. V, § 16). Then

$$\text{average force} = \frac{\text{area of work diagram}}{\text{length of diagram } (D)}$$

(expressed in the appropriate units).

7. The indicator diagram

The indicator diagram shown in fig. 15.6 is an example of a work diagram showing the variation of steam pressure inside an engine cylinder during the outward and return movements

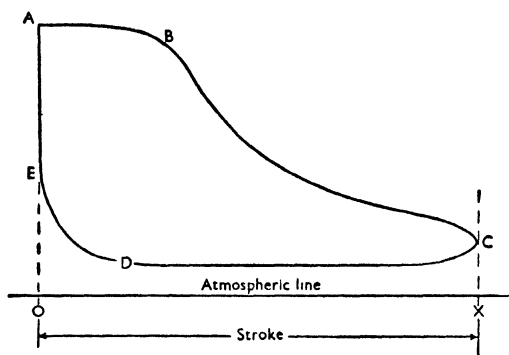


Fig. 15.6.—Steam-engine indicator diagram

of the piston. It forms a closed curve, and consists really of two curves, (1) the curve ABC representing the variation of steam pressure above the datum, or **atmospheric-pressure line**, during the outward stroke, and (2) the curve CDE representing the variation (due to back pressure) during the return stroke.

The area ABCXO represents the work done *by* the steam *on* the piston during the outward stroke, and the area EDCXO

represents the work done *on* the steam in the cylinder *by* the piston during the return or exhaust stroke. The energy of this exhaust stroke is used to force the steam out of the exhaust (from point C to D) and then to compress the residue trapped in the cylinder (from D to E).

The area enclosed by these two curves (the area ABCDEA) will therefore represent the net work done in one revolution by the engine, and the average height of the enclosed curve will represent the **mean effective pressure** (m.e.p.) of the steam during that period.

$$\text{Thus m.e.p.} = \frac{\text{area enclosed by indicator diagram}}{\text{length of diagram (stroke)}}$$

(expressed in the appropriate units)

An **indicator diagram** is obtained by using an instrument known as an "engine indicator", which is essential when

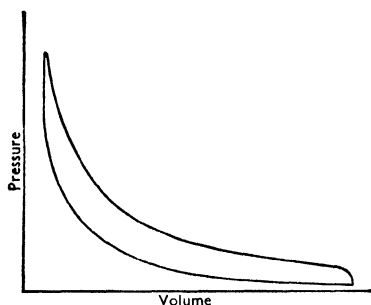


Fig 15.7.—Gas-engine indicator diagram

carrying out engine tests. From the indicator diagram we can obtain the m.e.p., as shown above, which can then be used to determine the **indicated horse-power** of the engine (see § 14). Specially designed planimeters are often used for finding the area of an indicator diagram.

From a study of an indicator diagram it is also possible to assess the behaviour of the engine, and to track down faults due to valve adjustment, etc. An indicator diagram taken from a gas engine is shown in fig. 15.7.

In most practical cases the m.e.p. of an engine is determined by finding the mean height of the indicator diagram (in inches), and multiplying this by the spring strength (the number of pounds required to compress the indicator spring one inch). This spring strength is called the **spring number**, and is stamped on the indicator spring.

8. Space average of a force

In all cases where the average value of a quantity is determined with respect to distance, this average is known as the **space average**.

9. Power

The time taken to do a certain amount of work is of great importance. For instance, 200 tons of bricks have to be moved 10 miles. A man with a wheel-barrow holding 1 cwt. could make 200×20 journeys at, say, 4 hours per journey, and would complete the work in 16,000 hours net. The same work could be done by a motor lorry of 10 tons capacity, making 20 journeys at, say, $\frac{1}{2}$ hour per journey, and taking 10 hours net. A railway train, however, could do the work in one journey of, say, 20 min.

We say that the motor lorry is more powerful than the man, and the railway locomotive is more powerful than the lorry. **Power**, then, means the **rate of doing work**, and is measured by the amount of work done in one unit of time.

$$\text{Power} = \frac{\text{work done}}{\text{time}}.$$

Since work done = force $\times$ distance, and speed = distance $\div$ time, then

$$\text{Power} = \frac{\text{work done}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}} = \text{force} \times \text{speed}.$$

10. Units of power

1. In the *British gravitational (F.P.S.) system*, the unit of power is one foot-pound per second, i.e. the power when 1 ft.-lb. of work is done per second.

2. In the *metric (C.G.S.) system*, the unit of power is one erg per second. But, as the erg is small, the joule per second may

be used (1 joule = 10 million ergs). A power of one joule per second is termed one **watt**.

3. The watt is the unit of power in the M.K.S. system.

In electrical engineering, the units are so arranged that in a direct-current circuit (or in any purely resistive circuit) the power in watts is given by the product of potential difference (volts) and current (amperes), i.e.

$$\text{volts} \times \text{amperes} = \text{watts.}$$

$$E \times I = W.$$

According to Ohm's law, the resistance (R) of the above circuit is given by $E \div I$,

$$\therefore E = IR,$$

and

$$W = I^2 R.$$

Larger units of power are in common use:

1. *The horse-power* (h.p.) is the power when 33,000 ft.-lb. of work are done per minute (or 550 ft.-lb. per sec.).

2. *The kilowatt* (kW.) consists of 1000 watts.

The **kilowatt** and the **megawatt** (one million watts) are used when dealing with large electrical machines and very large powers, whilst the **milliwatt** ($\frac{1}{1000}$ watt) and the **micro-watt** ($\frac{1}{1,000,000}$ watt) are used when dealing with very small powers, such as occur in radio circuits.

One horse-power is equivalent to 746 watts, and one kilowatt is equivalent to $(1000 \div 746) = 1.34$ h.p.

It is usual to rate mechanical machines in terms of horse-power, and electrical machinery in terms of kilowatts.

11. Origin of the term horse-power

The use of this unit is due to James Watt, of steam-engine fame. He needed some standard of comparison by which he could express the power of his engines. As, in many cases, these engines were displacing horse-driven machinery, it was natural that he should consider that animal.

Experiments were made something on the lines indicated in fig. 15.8. A strong, well-fed brewer's-dray horse was used, and the amount of work it could do in one minute was determined by measuring the distance various weights were lifted

up a mine shaft. It was found on the average that the horse could lift in one minute loads such as the following:

	220 lb.	through a distance of 100 ft.,
or	100 lb.	„ „ „ 220 ft.,
or	440 lb.	„ „ „ 50 ft.

The rate of doing work in each case thus approximated to 22,000 ft.-lb./min.

To guard against the possibility of someone having specially strong horses, and using them for adverse comparison with

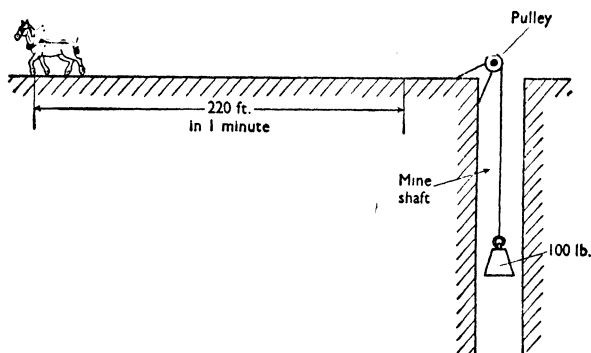


Fig. 15.8.—To explain horse-power

his engines, Watt decided to make his horse-power $1\frac{1}{2}$ times as great, i.e. 33,000 ft.-lb. per minute. Although this power is greater than that which any horse can maintain for any length of time, it has remained the standard ever since, and is in universal use.

Example 5.—A load of 1 ton is raised 30 ft. by a crane. If this takes $\frac{3}{4}$ min., what is the effective h.p. of the crane motor?

$$\text{Work done} = 2240 \times 30 = 67,200 \text{ ft.-lb.}$$

$$\text{Work done per minute} = 67,200 \div \frac{3}{4} = 89,600 \text{ ft.-lb.}$$

$$\text{H.P. of motor} = 89,600 \div 33,000 = 2.714.$$

(N.B. This answer neglects the effects of friction in the crane, and also the efficiency of the motor. The actual h.p. of the motor used would have to be somewhat higher.)

Example 6.—A 4·7-in. anti-aircraft shell weighs 50 lb., and is fired with a muzzle velocity of 3000 ft. per sec. If it is fired vertically upwards, find the h.p. developed by the gun.

Work done against gravity per second = 50×3000 ft.-lb.,

$$\therefore \text{h.p.} = \frac{50 \times 3000}{550} = 272\cdot7.$$

Example 7.—An electric motor develops a torque of 70 lb.-ft., and works at a speed of 3000 r.p.m. Find its horse-power output, and express this also in kilowatts.

Work done per minute = torque $\times$ angle turned through (radians) per minute = $70 \times 3000 \times 2\pi = 1,319,600$ ft.-lb.,

$$\therefore \text{h.p.} = \frac{1,319,600}{33,000} = 39\cdot99.$$

As 1 h.p. = 0·746 kW., then $39\cdot99$ h.p. = $0\cdot746 \times 39\cdot99 = 29\cdot8$ kW

Example 8.—Find the horse-power developed by the engine whilst climbing the gradient in Example 1 if it is travelling at 45 m.p.h.

$$45 \text{ m.p.h.} = 66 \text{ ft./sec.}$$

The train will climb the 500 ft. in $500 \div 66 = 7\cdot575$ sec., and it will do 3,640,000 ft.-lb. of work in this time.

Its rate of doing work will be $3,640,000 \div 7\cdot575$ ft.-lb./sec., or

$$\frac{3,640,000}{7\cdot575 \times 550} = 874 \text{ h.p.}$$

12. Larger units of energy

The foot-pound and the joule are rather small units for dealing with large amounts of work or energy.

Such amounts can conveniently be expressed in the following units:

(1) **The horse-power-hour.**—This is the amount of work done by one horse-power maintained for one hour, and is equal to

$$33,000 \times 60 = 1,980,000 \text{ ft.-lb.}$$

(2) **The kilowatt-hour (kWh).**—This is the amount of work done by a power of one kilowatt maintained for one hour. It is the commercial unit used in the sale of electrical energy. It is also known as the **Board of Trade unit** (familiarily as a unit).

As 1 joule per second = 1 watt, then the work done by a power of one watt maintained for one second (1 watt-second) is equal to one joule.

$$\begin{aligned}\therefore 1 \text{ kWh} &= 1000 \times 60 \times 60 = 3,600,000 \\ &= 3.6 \times 10^6 \text{ watt-seconds or joules.}\end{aligned}$$

Example 9.—Define the commercial unit of electrical energy.

The electrical load of a building is made up as follows: twenty 100-watt lamps used 6 hours daily, thirty 40-watt lamps used 5 hours daily, two electric motors of 1.0 b.h.p. each, and 81.4 per cent efficiency when fully loaded, used for 3 hours daily. Calculate the total energy consumption over a period of 65 days, and the cost, if electrical energy costs three farthings per unit. (U.L.C.I.)

Daily energy consumption:	watt-hours
20 lamps at 100 W. for 6 hours	= 12,000
30 lamps at 40 W. for 5 hours	= 6,000
2 electric motors at $\frac{1 \times 746}{0.814} = 916\frac{2}{3}$ W. for 3 hours	= 5,500
	<u>23,500</u>

Energy consumption for 65 days = 23.5 kWh (units) $\times$ 65
= 1527.5 units.

Cost = 1527.5 $\times$ $\frac{3}{4}$ d. = 1145.6d. = £4. 15s. 5.6d.

13. Horse-power of engines

Two methods of expressing the horse-power of an engine are commonly used:

1. *The Indicated Horse-power* (i.h.p.) is the horse-power actually produced in the engine by the action of steam or gas pressure on a piston or pistons. It is thus only applicable to piston-type engines.

2. *The Brake Horse-power* (b.h.p.) is the horse-power available for doing useful work after the losses (such as those due to friction) in the engine have been accounted for.

Indicated horse-power derives its name from the fact that it is determined by using an instrument known as an "engine indicator", whilst **brake horse-power** is so named because it is measured by some form of brake mechanism arranged to absorb or transmit the actual power output of the engine. (See Chap. XVI, § 19.)

14. Determination of i.h.p.

An outline of a simple piston-type engine is shown in fig. 15.9.

In order to determine its i.h.p., we need to know:

1. The average or mean effective pressure on the piston during its stroke (denoted by P lb. per sq. in.).
2. The area of the piston (A sq. in.).
3. The length of stroke (denoted by L measured in feet).
4. The number of *working* strokes per minute (denoted by N).

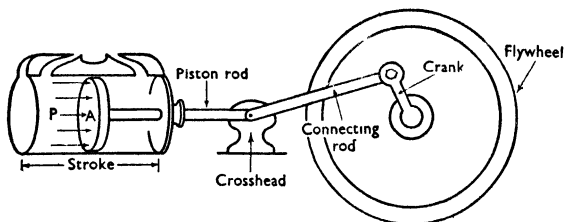


Fig. 15.9—Reciprocating engine

Then total force on piston = $P \times A$ lb.,

work done per stroke = $P \times A \times L$ ft.-lb.,

and work done per minute = $P \times A \times L \times N$ ft.-lb.,

$$\therefore \text{i.h.p.} = \frac{P \times A \times L \times N}{33,000} = \frac{PLAN}{33,000}.$$

15. Number of working strokes per minute

The significance of the number of working strokes per minute should be realized.

An engine may be either **single-acting** or **double-acting**.

With a single-acting steam engine, steam is admitted at one end of the cylinder only, where it presses on the face of the piston, so forcing it to the other end of the cylinder, thus completing one working stroke. The energy stored in the flywheel is drawn upon to bring the piston back on its return stroke to its original position, so completing one revolution. This return stroke is *not* a working stroke. Therefore with a single-acting steam engine there is one working stroke per revolution.

Most reciprocating steam engines are double-acting. Steam is admitted first at one end of the cylinder, so causing the piston to make one working stroke, and then at the other end of the cylinder, where it acts on the opposite side of the piston, causing it to make the return stroke, which is also a working stroke. Therefore with a double-acting steam engine there are two working strokes per revolution.

High-speed internal-combustion engines of the type used in cars and aircraft are invariably single-acting, but heavy internal-combustion engines, working at a slower speed, such as those used for ship propulsion, are sometimes double-acting.

Internal-combustion (i.c.) engines work either on a **four-stroke cycle** or a **two-stroke cycle** (see § 17).

With a four-stroke cycle, every fourth stroke is a working stroke, and with a two-stroke cycle every second stroke is a working stroke.

Summarizing, therefore, in the formula $\frac{PLAN}{33,000}$, the value of N to be used is as follows:

- For a single-acting steam engine, N = number of r.p.m.
 „ double-acting steam engine, $N = 2 \times (\text{no. of r.p.m.})$.
 „ single-acting i.c. 4-stroke engine, $N = \frac{1}{2} \times (\text{no. of r.p.m.})$.
 „ single-acting i.c. 2-stroke engine, N = no. of r.p.m.
 „ double-acting i.c. 4-stroke engine, N = no. of r.p.m.
 „ double-acting i.c. 2-stroke engine, $N = 2 \times (\text{no. of r.p.m.})$.

N.B. In some i.c. engines the value of N to be used may be less than indicated above, owing to the method of governing adopted. In such cases the number of explosions per minute must be taken.

16. Piston area

When determining the area of the piston (A) in the case of double-acting engines, it must be realized that the effective area on the piston-rod side is reduced by an amount equal to the cross-sectional area of the piston rod. Calculations of indicated horse-power (i.h.p.) can therefore be made separately for each side of the piston, or an average value of A can be used. In the latter case the mean effective pressure (m.e.p.) used should be the average for both sides of the piston.

17. Internal-combustion engine cycles

In this type of engine combustion of the fuel takes place in the cylinder, no boiler being required. The fuel used may be petrol, gas, heavy oil, etc. The four-stroke cycle consists of the following events continuously repeated:

1. A suction stroke during which the piston moves outward, drawing air, or a fuel/air mixture, through an **inlet** valve.

2. A compression stroke. All valves are kept closed and the piston moves inward, compressing the charge.

3. A working stroke. The charge is fired and forces the piston outward, the valves remaining closed.

4. An exhaust stroke. The **exhaust** valve opens, and the piston moves inward, expelling the products of combustion.

With petrol and gas engines the fuel/air mixture is fired by a spark from an electrical ignition system. Modern heavy-oil engines employ the compression-ignition system, and are often called diesel engines. In these, air only is drawn in during the suction stroke, and is then highly compressed, its temperature rising to a high value in consequence (see Chap. XXV). At or around top dead centre a small quantity of fuel oil is injected into the cylinder and ignites immediately upon contact with this high-temperature air.

In the two-stroke cycle the inlet and exhaust valves are normally replaced by ports, which are covered and uncovered by the moving piston. Every outward stroke is a working stroke.

As the piston starts to move inward the charge is forced through the inlet port, which, however, soon becomes covered and the charge is then compressed. Firing takes place around top dead centre and the piston now moves outward on its power stroke. Near the end of this stroke it uncovers the exhaust port, so permitting the products of combustion to escape. The inlet port is now uncovered, and a fresh charge rushes in, this action facilitating the exit of the exhaust gases.

Example 10.—A single-cylinder double-acting steam engine has a cylinder of 10 in. diameter and 18 in. stroke. The diameter of the piston rod is 2 in. The average m.e.p. as determined by indicator diagrams is 40 lb./sq. in. Find its indicated horse-power at 240 r.p.m.

$$P = 40 \text{ lb./sq. in.} \quad L = 1\frac{1}{2} \text{ ft.} \quad N = 2 \times \text{r.p.m.} = 480.$$

$$\text{Area of piston, back end} = 3.142 \times 5^2 = 78.54 \text{ sq. in.}$$

$$\begin{aligned} \text{" " " front end} &= 78.54 - (\text{area of piston rod}) \\ &= 78.54 - 3.142 \times 1^2 = 75.4 \text{ sq. in.} \end{aligned}$$

$$\text{Average piston area (A)} = \frac{1}{2}(78.54 + 75.4) = 76.97 \text{ sq. in.}$$

$$\text{I.H.P.} = \frac{PLAN}{33,000} = \frac{40 \times 1\frac{1}{2} \times 76.97 \times 480}{33,000} = 67.2.$$

Example 11.—The following are details concerning a single-cylinder 4-stroke motor-cycle engine:

$$\text{m.e.p.} = 110 \text{ lb./sq. in.} \quad \text{Stroke} = 4 \text{ in.} \quad \text{Bore} = 3 \text{ in.}$$

Calculate the i.h.p. developed when it is turning at 2800 r.p.m. (Motor-cycle engines are invariably single-acting.)

$$N = \frac{1}{2} \times \text{r.p.m.} = 1400.$$

$$\begin{aligned} \text{I.H.P.} &= \frac{PLAN}{33,000} = \frac{110 \times 4 \times \pi \times 3 \times 3 \times 1400}{12 \times 4 \times 33,000} = \frac{7\pi}{2} \\ &= 11 \text{ h.p. (approx).} \end{aligned}$$

Example 12.—An express passenger locomotive of the "Duchess" class (British Railways, London Midland Region) has the following dimensions: 4 cylinders, $16\frac{1}{2}$ in. diameter $\times$ 28 in. stroke. Piston-rod diameter 3 in. Driving wheels 6 ft. 9 in. diameter.

If on a test run at 60 m.p.h. it is found that the m.e.p. on the piston-rod side of the cylinder is $45\frac{1}{2}$ lb./sq. in., and on the other side it is $42\frac{1}{2}$ lb./sq. in., calculate the i.h.p. being developed.

$$\text{Distance travelled in one rev. of driving wheels} = 6\frac{3}{4}\pi = 21.21 \text{ ft.}$$

$$\text{Speed of locomotive} = 60 \text{ m.p.h.} = 5280 \text{ ft./min.}$$

$$\therefore \text{No. of r.p.m. of driving wheels} = \frac{5280}{21.21} = 249.$$

$$\text{No. of working strokes per minute (N)} = 4 \times 2 \times \text{r.p.m.} = 1992.$$

$$\text{Stroke (L)} = 2\frac{1}{2} \text{ ft.}$$

$$\text{Average m.e.p.} = \frac{1}{2}(45\frac{1}{2} + 42\frac{1}{2}) = 44 \text{ lb./sq. in.}$$

$$\text{Area of piston} = \pi \times (8\frac{1}{4})^2 = 213.9 \text{ sq. in.}$$

$$\text{" " " (piston-rod side)} = 213.9 - \pi \times (1\frac{1}{2})^2 = 206.83 \text{ sq. in.}$$

$$\text{Effective piston area (A)} = \frac{1}{2}(213.9 + 206.83) = 210.36 \text{ sq. in.}$$

$$\text{I.H.P.} = \frac{PLAN}{33,000} = \frac{44 \times 2\frac{1}{2} \times 210.36 \times 1992}{33,000} = 1304.$$

EXERCISES ON CHAPTER XV

Section A

1. A lift cage weighs 560 lb. If the electric motor raising the cage is developing $10\frac{1}{2}$ h.p., at what speed in ft./sec. is the cage moving? (S.W.E.T.C.)

2. Water is piped from a reservoir and falls a distance of 50 ft. The pipe is large enough to allow 1000 gal. to flow in 5 min. What is the work done by the water in 1 min., and what is the h.p. available? (S.W.E.T.C.)

3. An electric motor developing 250 b.h.p. is driving a hoisting drum at 300 r.p.m. If the diameter of the drum is 5 ft., calculate (a) the pull on the rope, (b) the torque. (S.W.E.T.C.)

4. Define "work" and "power".

Calculate the horse-power a man must exert to pump 20 gal. of water per minute through a height of 15 ft., if the efficiency of the pump is 60 per cent. (U.L.C.I.)

5. (a) Find the work done and the tractive effort required if a train weighing 400 tons moves 500 ft. up a slope of 1 in 100 when the tractive resistance is 14 lb. per ton of its weight. (b) If it takes 9 sec. to do this, find the h.p. exerted by the engine.

6. Find the work done in each of the following cases:

(a) Lifting $4\frac{1}{2}$ tons of coal to a height of 50 ft.

(b) By a force of 9 newtons exerted through 760 cm.

(c) In hoisting 3 cwt. up a ropeway inclined at 45° , the base of the ropeway being 120 ft.

(d) By a jet exerting a reactive force of $\frac{3}{4}$ lb. at a radius of 3 in. and causing rotation through 500 revolutions.

7. Define "work" and "power". In climbing a slight incline 300 yd. long, the average force exerted by an engine on a trailer is 240 lb. Calculate the total work done. If the speed is 15 m.p.h., calculate the average horse-power exerted during the climb. (U.E.I.)

8. 500 lb. of material contained in a cage are lifted from a shaft 600 ft. deep by means of a rope weighing 1.2 lb. per ft. length. Show by means of a carefully drawn diagram the work done in lifting the cage to the surface of the shaft, stating the total amount of work done. If the cage is lifted to the surface in 5 min. at uniform speed, what horse-power has been exerted? (U.E.I.)

9. A spring balance was inserted between a motor and a truck in order to measure the pull exerted by the motor. The readings P lb.

of the balance were taken at distances S yd. from the starting point, and were as follows:

P lb.	180	130	104	88	86	96	108	195.5	125	125	115
S yd.	0	100	200	300	350	400	450	500	550	600	700

Plot a curve connecting P and S , plotting P vertically. Using this curve, find the average pull exerted. What is the work done in ft.-tons in hauling the truck over the 700 yd? (U.E.I.)

Section B

10. (a) Define work, energy, power, horse-power.

(b) A round water tank 10 ft. diameter, 50 ft. high, is filled from a pond at its base by a motor-driven pump of 64 per cent combined efficiency, running at 1500 r.p.m. and taking current at 200 volts. Calculate:

- The work required to fill the tank.
- The horse-power required to do this in 1 hr. 20 min.
- The torque on the pump shaft.
- The input current in amperes.

1 h.p. = 746 watts. 1 cubic foot of water weighs 62.3 lb.

(S.W.E.T.C.)

11. A four-stroke oil engine of 24-in. stroke is found to develop 25.5 i.h.p. when running at a speed of 160 r.p.m. The indicator data are as follows: Heights of mid-ordinates 0.2; 0.26; 0.35; 0.40; 0.45; 0.54; 0.64; 0.75; 0.88 and 1.06 in. Spring strength 100 lb. per in. Find: (i) The mean effective pressure in the cylinder; (ii) the bore of the cylinder. (S.W.E.T.C.)

12. Find the effective horse-power required to drive a vehicle weighing $1\frac{1}{2}$ tons at a speed of 40 m.p.h. along a level track against resistances of 45 lb. per ton.

What additional horse-power is required to climb a gradient of 1 in 80 at the same speed? (C.P.E.)

13. A cyclist travels along a level road at 12 m.p.h. and is working at the rate of 0.1 h.p. What is the value of the resistance to motion?

If the total weight of the cyclist and his machine is 170 lb., find his speed when climbing a gradient of 1 in 60, assuming that he increases his rate of working to 0.15 h.p. (C.P.E.)

14. A loaded cage weighs 2000 lb. and is raised from the bottom of a mine 280 ft. deep by means of a wire rope attached to the cage. The rope weighs 2 lb. per ft. Find the work done in raising the cage to the surface, and calculate the average horse-power of the engine if the cage is raised in 50 sec. What is the work done during the last 100 ft. of lift? (C.P.E.)

15. Given an indicator diagram for a gas engine, state what data you would require in order to calculate the i.h.p. of the engine, and describe how the calculation would be made. (S.W.E.T.C.)

16. An electric tramcar weighs 15 tons, and is travelling up an incline of 1 in 50 at a uniform speed of 10 m.p.h. Taking the track resistance at 20 lb. per ton weight of car, find the horse-power of the driving motor if the mechanical efficiency of the transmission gearing is 85 per cent. (U.E.I.)

17. If in the preceding question the tram motor makes use of 90 per cent of the electrical energy supplied to it, and the supply voltage is 500 volts, find (a) the current taken, and (b) the cost of running the tram at this rate for one hour if electricity is supplied at 0.9d. per unit.

18. The drawbar pull exerted by an engine on a train weighing 450 tons, when it is travelling at 71 m.p.h., is 2.35 tons. Find the resistance to the motion of the train in lb. per ton weight, and the rate at which work is being done measured as drawbar horse-power. (U.L.C.I.)

CHAPTER XVI

Introduction to Machines

1. Simple machines

A “machine” is a device for doing work by the transmission of force and motion. It may overcome a large resistance (or move a large “load”) by the use of a small “effort”, to the advantage of the operator. It may apply a force, and so overcome a resistance, at some point remote from where the effort is applied, to the convenience of the operator. The direction and magnitude of the applied force may be different from the direction and magnitude of the resisting force.

Examples of simple machines include the lever, pulley, wheel and axle, screw, gearing, and wedge. All complicated machines are combinations of a number of simple machines.

The **load** is that part of the resistance which the machine has to overcome and which is of use to the operator.

The **effort** is that force required to operate the machine, and so overcome the load and any other resistance to movement.

The **velocity ratio** (V.R.) is the ratio of the speeds of the load and effort.

$$\begin{aligned}\text{Velocity ratio} &= \frac{\text{speed of effort}}{\text{speed of load}} \\ &= \frac{\text{distance moved by effort}}{\text{distance moved by load}} \quad \text{in the same time.}\end{aligned}$$

2. The principle of work

In any machine the work done by the effort is equal to the work done in overcoming resistance. This follows from the principle of conservation of energy. The work done (or energy expended) in overcoming resistance is made up of the useful

work done in moving the load, plus the work done in overcoming the effects of friction in the machine itself, i.e.

$$\begin{aligned} \text{work done by effort} &= \text{useful work done on load} \\ &+ \text{work done in overcoming friction.} \end{aligned}$$

Owing to friction the useful work done by a machine (i.e. its output of energy) is always less than the work done by the effort (i.e. its actual input of energy). The work done in overcoming friction represents the mechanical losses in the machine. Therefore

$$\begin{aligned} \text{losses due to friction} \\ &= \text{work done by effort} - \text{work done on load.} \end{aligned}$$

(N.B. These losses are here expressed in terms of energy, e.g. in ft.-lb.)

3. Mechanical advantage

The greater the load that can be moved by a given effort, the greater becomes the machine's **mechanical advantage**. This is expressed as the ratio of load to effort, i.e.

$$\text{mechanical advantage (M.A.)} = \frac{\text{load } (L)}{\text{effort } (E)}$$

It should be realized that a large mechanical advantage is not necessarily a desirable quality in a machine, as advantages of convenience in the application of the effort may sometimes be of more importance.

Also, owing to the principle of work, a large mechanical advantage implies that the effort must move through a large distance in order to raise the load through a small distance (i.e. the velocity ratio must be large).

4. Mechanical efficiency

The efficiency of any device may be expressed by the ratio of output to input, which may be rewritten, as convenient, in any of the following forms:

$$\text{efficiency } (\eta) = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

The **mechanical efficiency** of a machine is always expressed as the ratio of the useful work done to the actual work put into it. In consequence of the effects of friction the mechanical efficiency of a practical machine is always less than 100 per cent, and can be found as follows:

$$\begin{aligned}\text{mechanical efficiency} &= \frac{\text{work output}}{\text{work input}} \\ &= \frac{\text{load} \times \text{distance moved by load}}{\text{effort} \times \text{distance moved by effort}}\end{aligned}$$

But $\frac{\text{load}}{\text{effort}} = \text{mechanical advantage,}$

and $\frac{\text{distance moved by effort}}{\text{distance moved by load}} = \text{velocity ratio.}$

$$\therefore \text{mechanical efficiency} = \text{M.A.} \times \frac{1}{\text{V.R.}} = \frac{\text{M.A.}}{\text{V.R.}}$$

As efficiencies are often expressed as a percentage, then

$$\text{percentage mechanical efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100.$$

From the above it will be seen that, in an ideal machine (i.e. one without friction, and therefore having no losses, its mechanical efficiency thus being 100 per cent),

$$\text{mechanical advantage (M.A.)} = \text{velocity ratio (V.R.).}$$

5. Ideal effort

In an ideal machine, the mechanical efficiency being unity (i.e. 100 per cent), output would be equal to input.

In such a case, then

$$\frac{\text{load} \times \text{distance moved by load}}{\text{effort} \times \text{distance moved by effort}} = 1 \text{ (or 100 per cent),}$$

and the effort required to raise a given load would be

$$\text{load} \times \frac{\text{distance moved by load}}{\text{distance moved by effort}} = \text{load} \div (\text{velocity ratio}).$$

This effort is known as the "ideal effort".

An alternative expression for mechanical efficiency is sometimes useful, and can be deduced as follows:

$$\begin{aligned} \text{As mechanical efficiency} &= \frac{\text{M.A.}}{\text{V.R.}}, \text{ and } \text{V.R.} = \frac{L}{\text{ideal effort}}, \\ \text{then} \quad \text{mechanical efficiency} &= \frac{\text{M.A.}}{\text{V.R.}} \\ &= \frac{L}{\text{actual effort}} \div \frac{L}{\text{ideal effort}} = \frac{\text{ideal effort}}{\text{actual effort}}. \end{aligned}$$

6. Effect of friction

The actual effort required to raise the load is always greater than the ideal effort because of friction, and the difference between actual and ideal efforts is called the "friction effort" (or sometimes the "effort" effect of friction).

$$\text{Friction effort} = \text{actual effort } (E) - \text{ideal effort } (L \div \text{V.R.}).$$

The effect of friction can also be expressed in terms of that additional load, due to friction, which has to be overcome by the effort before any useful work can be done.

Friction load (or "load" effect of friction)

$$= \text{friction effort} \times \text{V.R.}$$

$$= \left(E - \frac{L}{\text{V.R.}} \right) \times (\text{V.R.}) = (E \times \text{V.R.}) - L.$$

The value $(E \times \text{V.R.})$ represents that load which could be raised by the given effort in an ideal machine, and is termed the "ideal load".

7. Classification of simple machines

A classification of simple machines may be made as follows:

(1) **The lever.** This includes:

- (a) The lever proper in all its forms (see Chap. IX).
- (b) The wheel and axle.

In this class of machine, the body of the machine can move about a fixed point, and in so doing lifts a load.

(2) **The pulley,** ranging from a simple pulley to a complex arrangement of pulley blocks and tackle. In this class of machine the shortening of a rope is made to lift a load.

(3) **The inclined plane.** This includes:

- (a) The inclined plane proper.
- (b) The wedge.
- (c) The screw.

In this class of machine, force is used to slide a load up a plane rigid surface.

8. The lever

The lever has been used since ancient times to raise a large load by means of a small effort, to the advantage of the operator.

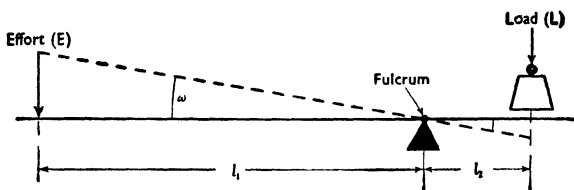


Fig. 16.1

Referring to fig. 16.1, by the principle of moments (see Chap. IX) we obtain

$$El_1 = Ll_2.$$

$$\therefore \text{mechanical advantage} = \frac{L}{E} = \frac{l_1}{l_2}.$$

(This neglects the effects of any friction present at the fulcrum.)

Consider the lever to turn through an angle of omega (ω) radians, then

$$\text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{\omega l_1}{\omega l_2} = \frac{l_1}{l_2}.$$

$\therefore$ mechanical advantage = velocity ratio (if friction be neglected).

(N.B. The effect of friction would be to reduce the mechanical advantage, but it would not change the velocity ratio.)

9. The wheel and axle

This consists of a wheel of diameter D (and radius R), rigidly fastened to an axle of diameter d (and radius r) (fig. 16.2). Examples of its use are afforded by the windlass, capstan, water wheel, etc.

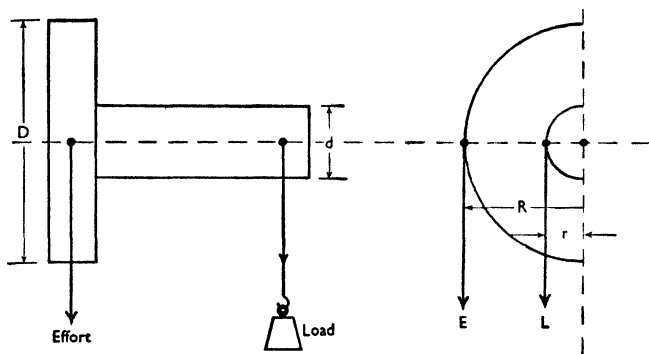


Fig. 16.2.—Wheel and axle

Examination of the figure shows that it resolves itself into a lever with the centre as fulcrum.

Taking moments about the centre we obtain

$$ER = Lr.$$

$$\therefore \text{mechanical advantage} = \frac{L}{E} = \frac{R}{r}.$$

Velocity ratio (assuming 1 revolution of wheel)

$$= \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{2\pi R}{2\pi r} = \frac{R}{r}.$$

$$\therefore \text{M.A.} = \text{V.R. (assuming no friction is present).}$$

The values of load and effort obtained, neglecting the effects of friction, are known respectively as the **ideal load** and the **ideal effort**.

$$\therefore \text{ideal load} = \text{ideal effort} \times \text{velocity ratio.}$$

The mechanical advantage of this machine can be made as large as desired by making $(D - d)$ large. If this is taken to

excess, however, the machine becomes unwieldy, or the axle becomes too weak to sustain the load.

By using a wheel and differential axle this disadvantage is overcome. This consists of two cylinders of different radii rigidly attached to a common axle (see fig. 16.3). When the

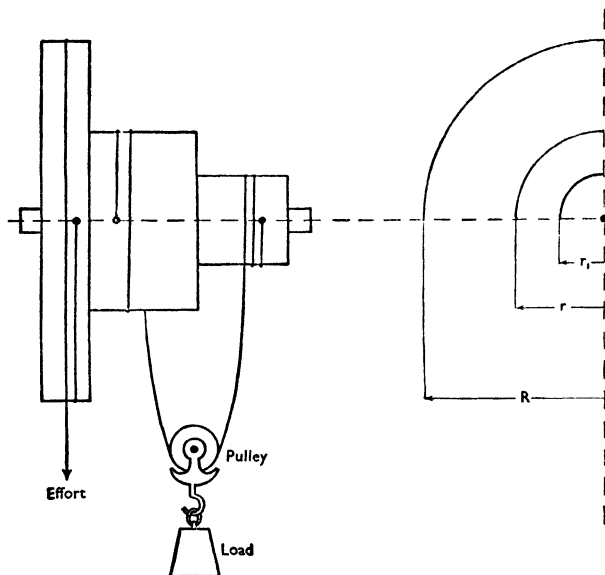


Fig. 16.3.—Wheel and differential axle

wheel is turned, the rope winds on to one cylinder and unwinds from the other. The bight of the rope is passed under a movable pulley, to which the load is attached. Note that each revolution of the effort wheel shortens the rope by $2\pi(r - r_1)$, and that, as half this shortening occurs on each side of the bight, then the distance moved by the load is $\frac{1}{2} \times 2\pi(r - r_1)$. Neglecting friction,

$$\text{mechanical advantage} = \frac{L}{E} = \text{velocity ratio}$$

$$= \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{2\pi R}{\frac{1}{2} \times 2\pi(r - r_1)} = \frac{2R}{r - r_1}.$$

10. Pulley systems

An arrangement of blocks containing several pulleys with a rope passing around them forms a pulley system. It is much used for hoisting work, especially on cranes, and in marine work, as it enables heavy loads to be lifted, with the expenditure of a comparatively small effort, by hauling in a comparatively long length of rope.

This rope may be either

- (a) continuous,
- (b) non-continuous.

Continuous-rope systems are commonest, and these only will be considered here.

The load is attached to the lower block (fig. 16.4 shows a typical arrangement), and is supported by one, two or more, ropes according to the complexity of the system. For a continuous-rope system (neglecting friction), the load will be supported by, and divided between, the number of ropes,

$$\therefore \text{effort required} = \frac{\text{load}}{\text{number of ropes } (N)}$$

$$\therefore \text{M.A.} = \frac{L}{E} = L \div \left(\frac{L}{N}\right) = L \times \frac{N}{L} = N.$$

If the load is lifted through one foot, each rope will have been shortened by one foot, i.e. if there are N ropes, then N feet of rope will have been hauled in.

$$\therefore \text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{N}{1} = N.$$

Thus again M.A. (neglecting friction) = V.R. (N.B. The weight of the lower pulley block is not part of the useful load.)

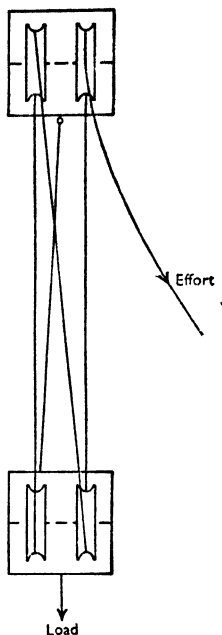


Fig. 16.4.—Pulley tackle

11. Weston differential pulley block

This particular form of lifting tackle will be described because it is much used in factories and workshops, and because it resembles in principle the wheel and differential axle, although essentially a pulley tackle.

It consists of two blocks, the top one containing two pulleys of slightly differing diameters fastened rigidly to the same axle, and the bottom one being a simple one, arranged to carry the load (see fig. 16.5). An endless chain is used as a rope, and the pulleys are designed with suitable teeth, etc., to accommodate it. A loop of this chain reaches down to the ground, and the effort is applied by hauling on it. The pull is transmitted over pulley A, through C, over B, and back to the operator.

If R is the radius of the larger pulley (A) and R_1 that of the smaller (B), then hauling in the rope sufficiently to cause one revolution of pulley A will shorten the rope supporting the load by $2\pi R - 2\pi R_1$. The load will be raised by half this amount, owing to pulley C,

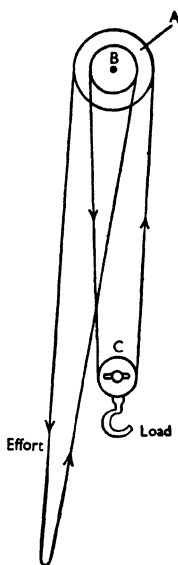


Fig. 16.5.—Weston differential pulley system

$$\begin{aligned} \therefore \text{velocity ratio} &= \frac{\text{distance moved by effort}}{\text{distance moved by load}} \\ &= \frac{2\pi R}{\frac{1}{2}(2\pi R - 2\pi R_1)} = \frac{2\pi R}{\frac{1}{2} \times 2\pi(R - R_1)} \\ &= \frac{2R}{R - R_1} \end{aligned}$$

Notice that the dimensions of pulley C do not affect the velocity ratio.

With this type of hoist it is possible to obtain a higher velocity ratio than that obtainable with ordinary pulley tackles.

Example 1.—In a Weston differential pulley block, if the numbers of teeth in the two pulleys are respectively 59 and 60, find the effort needed to raise a weight of 1000 lb., assuming the efficiency to be 50 per cent.

As the teeth in the two pulleys will be of equal pitch, then the number of teeth in each will be proportional to their circumferences.

$$\therefore \text{V.R.} = \frac{2\pi R}{\frac{1}{2} \times 2\pi(R - R_1)} \text{ becomes } \frac{60}{\frac{1}{2}(60 - 59)} = \frac{60}{\frac{1}{2}} = 120.$$

As efficiency = $\frac{\text{M.A.}}{\text{V.R.}}$ = 50 per cent, then

$$L/E = 50 \text{ per cent of } 120 = 60,$$

and
$$E = \frac{L}{60} = \frac{1000}{60} = 16\frac{2}{3} \text{ lb.}$$

Example 2.—A lifting tackle has an upper block with three sheaves and a lower block with two sheaves, one end of the rope being fastened to the lower block. What is the velocity ratio?

During an experiment with this tackle it was found that an effort of 64 lb. applied to the loose end of the rope was just sufficient to raise a load of 160 lb. with uniform velocity. Calculate the efficiency of the tackle. You may neglect the weight of the lower block.

(U.L.C.I.)

Velocity ratio = number of ropes (N) supporting load.

By using a 3 : 2 pulley tackle, and fixing rope to the lower block, it will be seen that 5 ropes are required. (Draw a diagram to check this.)

$$\therefore \text{V.R.} = 5.$$

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}} = \frac{160}{64} = 2.5.$$

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{2.5}{5} = 0.5 \text{ or } 50 \text{ per cent.}$$

12. The inclined plane as a machine

A man can push, or pull, a load up an inclined plane when he would be unable to lift it by his own unaided efforts. This method of raising heavy weights has been used since ancient times, and it is said that the ancient Egyptians made use of it when erecting the pyramids and other massive structures.

The forces acting on an inclined plane have been considered in Chap. XIII, § 4.

Consider the inclined plane as a machine, and neglect the effect of friction.

(a) With the effort acting along the plane (fig. 16.6),

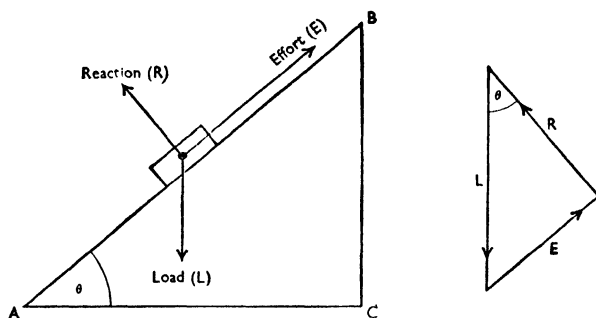


Fig. 16.6—Inclined plane

$$\text{M.A.} = \frac{L}{E} = \frac{1}{\sin \theta} = \operatorname{cosec} \theta.$$

$$\text{V.R.} = \frac{\text{distance moved by } E}{\text{distance moved by } L} = \frac{AB}{BC}.$$

But $\sin \theta = \frac{BC}{AB}$; $\therefore \frac{AB}{BC} = \frac{1}{\sin \theta} = \operatorname{cosec} \theta.$

$$\therefore \text{M.A.} = \text{V.R.} = \frac{1}{\sin \theta} = \operatorname{cosec} \theta \text{ (if friction absent).}$$

(b) With the effort acting horizontally (fig. 16.7),

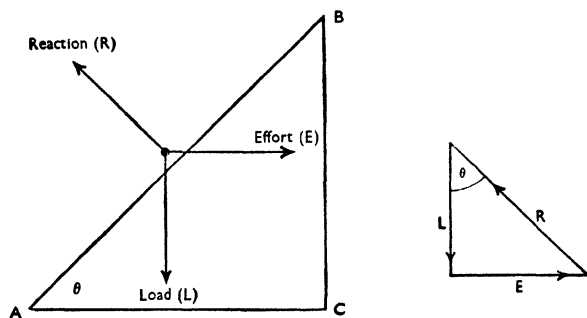


Fig. 16.7

$$\text{M.A.} = \frac{L}{E} = \frac{1}{\tan \theta}.$$

$$\text{V.R.} = \frac{\text{distance moved by } E}{\text{distance moved by } L} = \frac{AC}{BC} = \frac{1}{\tan \theta}.$$

In any practical inclined plane, some friction will be present and will cause the resultant reaction R to deviate from a right angle to the plane to an angle of $(90 + \phi)^\circ$, where ϕ is the angle of friction of the materials in contact (see Chap. XIV). This will cause the effort required to lift a given load to be greater than the ideal effort, and will consequently reduce the mechanical advantage. The velocity ratio will, however, remain unchanged.

13. The wedge

The wedge is a special case of the inclined plane. It has long been used for splitting open materials by forcing it into a small opening, and for raising heavy loads through a small

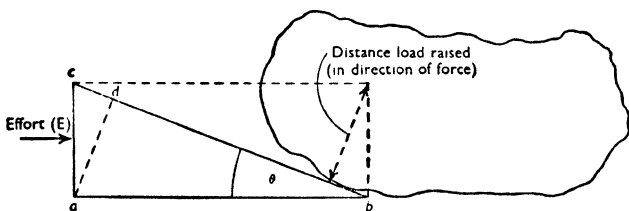


Fig. 16.8.—Wedge raising a load

distance. It is used also to-day as a fastening device, as for example a taper key, and a cotter pin* on a bicycle.

If friction be neglected, the application of an effort, such as a hammer blow, or the force developed by tightening a nut, will cause a strain to be set up at right angles to the face of

* The cotter pin is a wedge of conical form with a screwed stud attached axially to its narrow end. By inserting the stud through a clearing hole passing through two bodies and tightening a nut on to it the conical pin is drawn into the hole, thus locking the bodies together.

the wedge. This strain will either tend to raise a load or to create a bursting force, according to the use to which the wedge is put.

In the right-angled triangle-shaped wedge shown in fig. 16.8, neglecting friction,

$$\text{M.A.} = \text{V.R.} = \frac{\text{distance moved by } E}{\text{distance moved by } L} = \frac{ab}{ad} = \frac{1}{\sin \theta}.$$

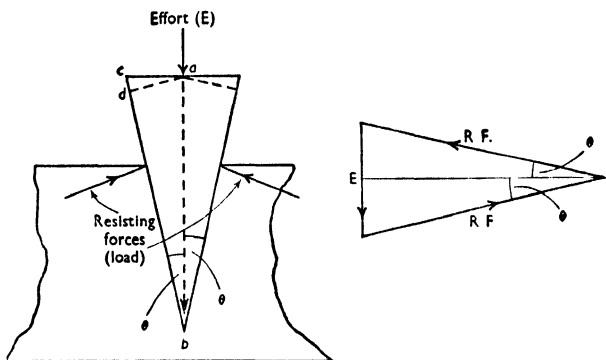


Fig. 16.9.—Wedge creating a bursting force

The wedge of isosceles form shown in fig. 16.9 really consists of two of the previous pattern, thus:

$$\text{V.R.} = \frac{\text{distance moved by effort}}{\text{distance moved by load (in direction of force)}} = \frac{ab}{2ad} = \frac{1}{2 \sin \theta}.$$

$$\text{M.A.} = \frac{\text{resisting forces (R.F.)}}{\text{effort (E)}}$$

From vector diagram,

$$\frac{1}{2}E = (\text{R.F.}) \sin \theta \quad \text{and} \quad E = 2(\text{R.F.}) \sin \theta,$$

$$\therefore \text{M.A.} = \frac{(\text{R.F.})}{2(\text{R.F.}) \sin \theta} = \frac{1}{2 \sin \theta},$$

which is the same as V.R. (neglecting friction).

14. The screw

The screw has long been used as a machine for transmitting force and motion. Its invention is credited to Archimedes, who made use of it in the times of ancient Greece.

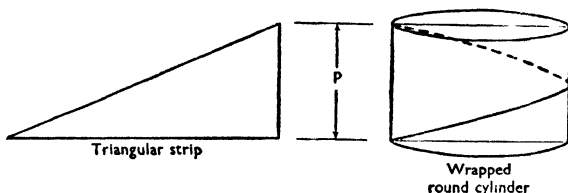


Fig. 16.10

It consists essentially of an inclined plane in spiral form, wound around a cylinder (or sometimes a cone in the case of "taper" threads). This may be shown by winding a triangular strip of paper around a cylinder (see fig. 16.10). The vertical distance p between successive turns of the thread of the screw is called the **pitch** of the screw.

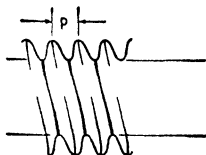


Fig. 16.11.—Single-start screw

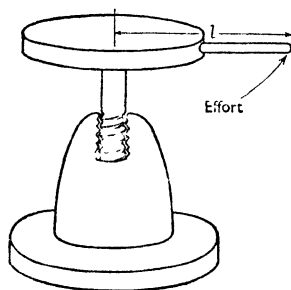


Fig. 16.12.—Screw jack

The British Standard definition of the pitch of a screw thread is the distance, measured parallel to its axis, between corresponding points on adjacent thread forms in the same axial plane.

The **lead** of a screw thread is the distance it advances axially in one revolution.

Thus a nut working on the screw thread shown in fig. 16.11

would advance through a distance equal to the lead, and this distance will also be equal to the pitch in this case (as the thread is a "single-start" thread).

Besides its present common use as a fastening device, the screw is a machine which can be used for:

1. Raising a load by means of a small effort.
2. Changing rotary into straight-line motion.

In fig. 16.12 an effort (E) is applied at the end of a tommy bar at a distance l from the centre. If this bar is turned through a complete revolution, then

$$\text{distance moved by effort} = 2\pi l.$$

The load will then be raised through a distance equal to the lead of the screw,

$$\therefore \text{velocity ratio} = \frac{\text{distance moved by } E}{\text{distance moved by } L} = \frac{2\pi l}{\text{lead}}.$$

Neglecting friction (which will be high in the case of a screw thread),

$$\text{M.A.} = \frac{L}{E} = \frac{2\pi l}{\text{lead}}.$$

Example 3.—Calculate the velocity ratio when a spanner with an effective length of 15 in. is used to tighten a nut with 8 threads per inch. When the force applied to the spanner is 50 lb., assume a mechanical efficiency of 30 per cent, and find the pressure exerted by the nut. (I.E.E.)

The arrangement of spanner and nut will be similar to that in fig. 16.12.

$$\begin{aligned} \text{Velocity ratio} &= \frac{\text{distance moved by } E}{\text{distance moved by } L} = \frac{2\pi l}{\text{lead}} = \frac{2\pi \times 15}{\frac{1}{8}} \\ &= 2\pi \times 15 \times 8 = 754. \end{aligned}$$

$$\text{Mechanical efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{L/E}{\text{V.R.}}$$

$$\therefore \frac{30}{100} = \frac{L/50}{754}.$$

The pressure exerted by the nut is represented by L ,

$$\therefore L = \frac{754 \times 30 \times 50}{100} = 11,310 \text{ lb.}$$

15. Screw threads

The following definitions should be studied in conjunction with fig. 16.13.

Major diameter (sometimes called full diameter).—The diameter of an imaginary coaxial cylinder which just touches the crest of an external thread or the root of an internal thread.

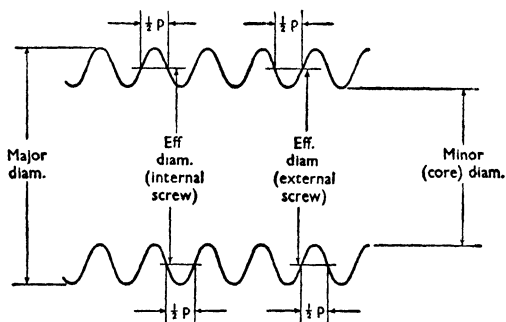


Fig. 16.13.—Screw-thread terms

Minor diameter (sometimes called core diameter).—The diameter of an imaginary coaxial cylinder which just touches the root of an external thread or the crest of an internal thread.

Effective diameter (called pitch diameter in America).—The diameter of an imaginary coaxial cylinder which cuts the flanks of the thread at a point where they are of a width equal to half the pitch.

16. Types of screw threads

A screw thread may be

- (a) right-handed (the normal type),
- (b) left-handed.

If, when it is assembled with a fixed nut and turned clockwise, the screw thread moves away from the operator it is right-handed; if it approaches the operator it is left-handed.

A **single-start thread** is the normal type and consists of one continuous helical groove (see fig. 16.11).

A **multiple-start thread** consists of two or more separate and continuous helical grooves equally spaced in the axial direction. This type of thread gives a quick traverse, as one complete turn advances the screw axially through a distance $N \times p$, where N = number of starts. Thus for any thread

$$N \times p = \text{lead.}$$

17. Thread forms

In general, a screw thread may be of

- (a) square form,
- (b) V form.

(a) Square threads, because of their strength, are used mainly for transmitting force and motion (fig. 16.14).

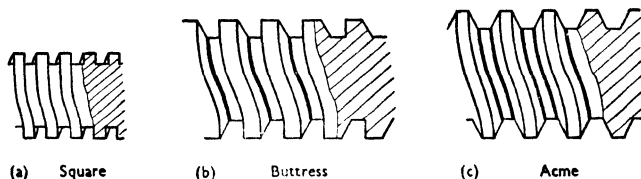


Fig. 16.14.—Types of square thread

If force has to be transmitted, mainly in one direction, as in a vice, a **buttress thread** (fig. 16.14b) may be used.

A variation of the square thread is the **acme thread** (fig. 16.14c).

(b) In GREAT BRITAIN, two kinds of V-thread forms are commonly used.

- (1) **British Standard Whitworth (B.S.W.) form.**
- (2) **British Association (B.A.) form.**

(1) The profile of the B.S.W. form is shown in fig. 16.15.

Basic values of nominal diameter and number of threads per inch are laid down for B.S.W. threads. If a finer-pitched thread is required for a given diameter of thread than is catered for

in the B.S.W. sizes, then Whitworth form threads of the following two designations may be used:

(i) **British Standard Fine (B.S.F.)**

(ii) **British Standard Pipe (B.S.P.)**

B.S.F. threads, because of their greater effective diameter, are stronger for a given size than the corresponding B.S.W. threads. Consequently they are much used in aircraft work.

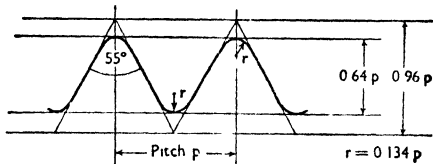
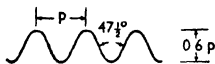


Fig. 16.15.—B.S. Whitworth thread

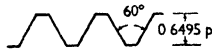
B.S.P. threads are used for steam, water, and gas pipes, as well as for general engineering purposes. The size of a B.S.P. thread is given in terms of the bore of the corresponding pipe.

(2) In the B.A. thread form, the angle between the threads is 47.5° (fig. 16.16). This type of thread is much used for instrument and electrical work.



B A

Fig 16.16



Sellers

Fig. 16.17



Sharp V

Fig. 16.18

In AMERICA, the V-thread form in common use is the **Sellers thread**, or **United States Standard thread** (fig. 16.17). The angle between threads is 60° .

The **Sharp V-thread** (fig. 16.18) is used to some extent for small screws.

The desirability of some measure of standardization of the screw threads used in Great Britain and America has long been recognized, in view of the increasing amount of equip-

ment made in one country which has to be serviced in the other.

In the **unified screw-thread system**, the countries concerned have agreed upon the thread form, and a series of diameters and pitches for bolts, nuts, etc., which will be standard.* The form of thread selected is one having a 60° angle. The bolt thread has a rounded root, with the alternative of a rounded crest for use in the U.S.A. The pitch/diameter series comprises fine, coarse, and special design patterns (designated UNF, UNC and UNS respectively). It should not be thought, however, that the unified system will immediately replace all others. It will be adopted by Service departments where practicable and by industry where convenient.

Example 4.—Calculate the mechanical advantage and the efficiency of a screw-jack in which the screw has four turns to the inch, a mean diameter of 2 in., and a coefficient of friction 0.1, the lever being 12 in. long. (I.E.E.)

$$\text{V.R.} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{2\pi \times 12}{\frac{1}{4}} = 96\pi = 302.$$

Let load (L) = 10 lb., then as $\text{V.R.} = L \div \text{ideal effort}$,

$$\text{ideal effort} = \frac{L}{\text{V.R.}} = \frac{10}{302} = 0.033 \text{ lb.}$$

$$\text{As coefficient of friction} = \mu = \frac{F}{W(\text{load})} = 0.1,$$

$$\text{friction effort} (F) = \mu W = 0.1 \times 10 = 1 \text{ lb.}$$

Work output in 1 rev. = work done on load in 1 rev. = $10 \times \frac{1}{4}$ = 2.5 in.-lb.

Work input in 1 rev. = work done by ideal effort per rev. + work done by friction effort per rev.

Work done by ideal effort per rev. = $0.033 \times 2\pi \times 12 = 0.8\pi$ in.-lb.

Work done by friction effort per rev. = $F \times \text{mean circumference}$ = $1 \times \pi \times 2 = 2\pi$ in.-lb.

Total work input = $2.8\pi = 8.8$ in.-lb.

$$\text{Mechanical efficiency} (\eta) = \frac{\text{work output}}{\text{work input}} = \frac{2.5}{8.8} = 0.284 \text{ or } 28.4 \text{ per cent.}$$

$$\text{As } \eta = \frac{\text{M.A.}}{\text{V.R.}} = 0.284,$$

then mechanical advantage = $0.284 \times 302 = 85.6$.

* At present (1951) agreement is restricted to sizes of $\frac{1}{4}$ in. and above.

18. The reciprocating engine as a machine

The elements of a reciprocating-engine mechanism were explained in Chap. XV, together with the method used for determining its indicated horse-power.

It will be useful to consider this engine as a machine for converting reciprocating straight-line motion into rotary motion.

An effort is applied by pressure on a piston, causing this piston to move in a straight line for the length of a stroke. This motion is changed into rotary motion by the action of the piston-rod, connecting-rod, and crank, and applied to the load through a flywheel or some other suitable power-transmission machine. The effort will be

$$\text{pressure (lb./sq. in.)} \times \text{piston area (sq. in.)} = PA \text{ lb.}$$

The load will be that force which is overcome at the periphery of the flywheel (see fig. 15.9).

Calling this load W lb., then

$$\text{mechanical advantage} = \frac{L}{E} = \frac{W}{PA}$$

For a single-acting steam engine, the effort will be effective for the length of one stroke (L) per revolution, and the load will be moved $2\pi \times r$ during this revolution, where r is the distance of the point of application of the load from the wheel centre (i.e. the flywheel radius).

$$\therefore \text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{\text{stroke } (L)}{2\pi r}$$

Consequently the mechanical efficiency of this engine will be

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{W}{PA} \div \frac{L}{2\pi r} = \frac{2\pi Wr}{PAL} = \frac{\text{work output per rev.}}{\text{work input per rev.}}$$

As $2\pi Wr$ = torque (Wr) $\times$ angle turned through (radians),
then power output at N r.p.m. = $2\pi WrN$

or $\frac{2\pi WrN}{33,000}$ b.h.p. (if appropriate units are used).

Power input to engine at N working strokes per min. = $PLAN$

or $\frac{PLAN}{33,000}$ i.h.p. (if appropriate units are used).

For a single-acting steam engine, number of working strokes per min. = number of r.p.m. Therefore its mechanical efficiency may be written as

$$\eta = \frac{2\pi WrN}{PLAN}.$$

This is also true for other cases if the appropriate value of N in the expression $PLAN$ is used (see Chap. XV, § 15).

Therefore mechanical efficiency (η) of any reciprocating engine may be written as $\frac{\text{b.h.p.}}{\text{i.h.p.}}$

Example 5.—An oil engine develops an i.h.p. of 22 at 250 r.p.m., and is found to overcome an external torque of 5200 lb.-in. Determine its mechanical efficiency.

$$\text{Mechanical efficiency} = \frac{\text{b.h.p.}}{\text{i.h.p.}},$$

$$\therefore \text{b.h.p.} = \frac{2\pi WrN}{33,000} = \frac{2\pi \times 5200 \times 250}{12 \times 33,000} = 20.6.$$

$$\therefore \text{mechanical efficiency} = \frac{20.6}{22} = 0.936 \text{ or } 93.6 \text{ per cent.}$$

Example 6.—A single-acting steam engine has a mean effective pressure of 41 lb./sq. in. Its stroke is 1 ft. 9 in., piston area 110 sq. in., and speed 80 r.p.m. If its mechanical efficiency is 83 per cent, determine its b.h.p.

$$\begin{aligned} \text{B.H.P.} &= 83 \text{ per cent} \times \text{i.h.p.} = 0.83 \times \frac{PLAN}{33,000} \\ &= 0.83 \times \frac{41 \times 1\frac{3}{4} \times 110 \times 80}{33,000} = 15.88 \text{ h.p.} \end{aligned}$$

Example 7.—During the test on the “Duchess” class locomotive mentioned in Example 12, Chap. XV, the train is climbing a gradient of 1 in 530. If the total weight of the train (including locomotive) is 400 tons, and the tractive resistance is 11 lb. per ton, determine the efficiency of the locomotive, in terms of the actual power output compared to input.

In the example quoted the speed was 60 m.p.h., and i.h.p. 1304.

Work done by locomotive in overcoming tractive resistance over a distance of 1 mile

$$= 400 \times 11 \times 5280 = 23,232,000 \text{ ft.-lb.}$$

Work done by locomotive in climbing gradient over a distance of 1 mile

$$= 400 \times 2240 \times \frac{5280}{530} = 8,944,000 \text{ ft.-lb.}$$

Total work done by locomotive in 1 mile

$$= 32,176,000 \text{ ft.-lb.}$$

H.P. output of locomotive during this period

$$= \frac{32,176,000}{33,000} = 975.$$

$$\text{Efficiency} = \frac{\text{h.p. output}}{\text{h.p. input (i.h.p.)}} = \frac{975}{1304} = 74.7 \text{ per cent.}$$

N.B. This answer assumes that the speed remains constant throughout. Any acceleration would mean an increase of h.p. output.

19. Determination of brake horse-power and efficiency of an engine

As the brake horse-power of an engine is the useful power obtainable from it, it follows that

$$\text{i.h.p.} - \text{b.h.p.} = \text{h.p. lost in friction, and}$$

$$\text{mechanical efficiency} = \frac{\text{i.h.p.} - \text{friction h.p.}}{\text{i.h.p.}} = \frac{\text{b.h.p.}}{\text{i.h.p.}}$$

To measure b.h.p., a machine called a **dynamometer** is used. Two forms of mechanical dynamometer are in common use:

(1) The **absorption dynamometer**, in which all the output is absorbed by some form of friction brake, and thereby converted into heat.

(2) The **transmission dynamometer**, in which the output is not wasted, but is measured, and then transmitted to where it can be utilized.

Various forms of electrical dynamometer may be used, the simplest being a generator of known efficiency. In this case

$$\text{b.h.p. of driving engine} = \frac{\text{generator output (kW.)}}{0.746 \times \text{generator efficiency}}.$$

This output may be utilized if convenient.

Two absorption-type dynamometers will now be described to illustrate the principle involved.

In the **Prony brake** (fig. 16.19) two semicircular blocks are adjusted to exert a braking torque on the engine pulley; the tendency of these blocks to turn with it is counter-balanced by adding weights (W_1) to the lever arm (r_1), until equilibrium is obtained. Under these conditions, the clockwise moment

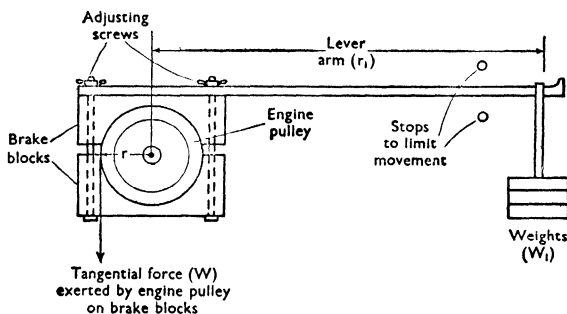


Fig. 16.19.—Prony brake

exerted by the weighted lever is balanced by the anticlockwise moment of the turning force (W) at the circumference of the pulley, acting at a distance r from the pulley centre, i.e.

$$\text{brake torque } (W_1 r_1) = \text{turning moment } (Wr).$$

As work done in rotation = torque $\times$ angle turned through (radians), then

$$\text{work done per revolution} = 2\pi Wr,$$

$$\text{and work done per minute} = 2\pi WrN,$$

where N = r.p.m.

$$\therefore \text{b.h.p.} = \frac{2\pi WrN}{33,000}.$$

In rope and band brakes (fig. 16.20), the engine flywheel, or a special pulley, is made to turn against the friction of a rope or band wound around it and held by the action of weights or spring balances. During rotation, one end of the rope will be pulled tight whilst the other will become slack.

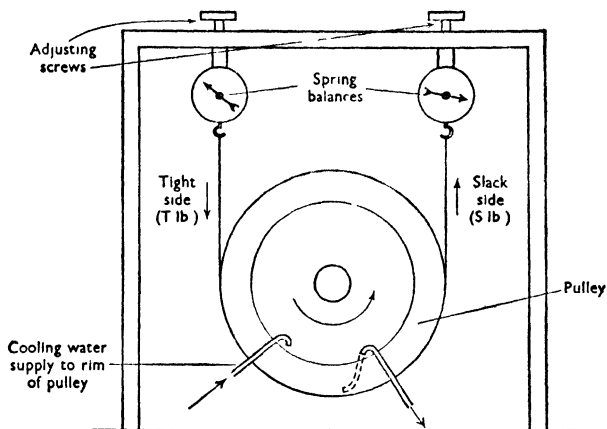


Fig. 16.20.—Principle of rope and band brakes

The effective load on the brake will be the difference between the loads on the tight and slack sides of the rope, i.e.

$$\text{effective pull } (W) = T - S.$$

This load, which can be adjusted, acts at an effective radius (r) from the centre of the pulley. Therefore, whilst the speed remains constant,

$$\text{b.h.p.} = \frac{2\pi W r N}{33,000}.$$

Example 8.—From the following data taken in a test on a gas engine, calculate the indicated horse-power, the brake horse-power, and the mechanical efficiency.

Cross-sectional area of piston, 78.5 sq. in.; stroke, 18 in.; average number of revolutions per minute, 202; average number of explosions

per minute, 88; mean effective pressure, 82 lb. per sq. in.; effective diameter of brake wheel, 5 ft. 6 in.; net average brake load, 200 lb. (Poly.)

$$\text{I.H.P.} = \frac{PLAN}{33,000} = \frac{82 \times 18 \times 78.5 \times 88}{12 \times 33,000} = 25.75.$$

$$\text{B.H.P.} = \frac{2\pi WrN}{33,000} = \frac{2\pi \times 200 \times 2\frac{3}{4} \times 202}{33,000} = 21.2.$$

$$\text{Mechanical efficiency} = \frac{\text{b.h.p.}}{\text{i.h.p.}} = \frac{21.2}{25.75} = 0.822 \text{ or } 82.2 \text{ per cent.}$$

20. Cams

The cam can be considered as a machine for converting rotary motion into straight-line reciprocating motion (the reverse of the reciprocating engine).

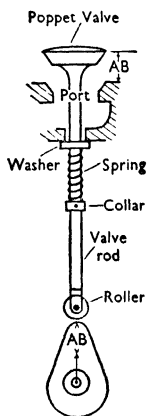


Fig. 16.21

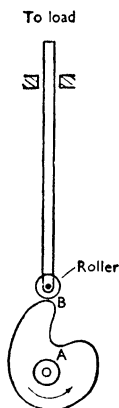


Fig. 16.22

It is used for such purposes as operating the valves of internal-combustion engines (and sometimes steam engines), and for operating the controls in various kinds of automatic machinery.

Fig. 16.21 shows a cam designed to operate a valve on an

internal-combustion engine. The camshaft is driven by means of suitable gearing from the main crankshaft.

$$\text{The velocity ratio} = \frac{\text{circumference of cam}}{\text{valve movement (AB)}}$$

By carefully designing the contour of the cam, almost any timing of the straight-line movement of the load (or the valve) can be achieved. Thus the cam in fig. 16.22, in turning anti-clockwise from point A to point B, imparts a slow opening movement to the valve, followed by a sudden rapid closure.

EXERCISES ON CHAPTER XVI

Section A

1. What do you understand by the principle of work as applied to a machine? Describe with the aid of diagrams some pulley-block system by which a load of $\frac{1}{2}$ ton could be raised; assuming 25 per cent of the effort is wasted, find the efficiency. (C.P.E.)

2. A screw has 4 threads per in. Find the magnitude of the couple which would have to be exerted on it to produce an axial thrust of 2000 lb. if the mechanical efficiency is 20 per cent. (C.P.E.)

3. Sketch a simple screw jack.

If the screw has a pitch of $\frac{3}{8}$ in. and the length of lever 10 in., calculate the ideal load which can be raised when the effort is 42 lb. (S.W.E.T.C.)

4. Explain the terms "velocity ratio" and "mechanical advantage". In a simple windlass, the load is raised 16.5 in. for one complete turn of the handle, which describes a circle of 18 in. radius. The efficiency of the windlass is 90 per cent. What is (a) the velocity ratio, (b) the mechanical advantage of the windlass. (c) What force applied to the handle is required to raise a load of 120 lb.?

5. Define (a) the mechanical advantages and (b) the velocity ratio of a machine.

A lifting tackle consists of two pulley blocks, the upper having three sheaves and the lower one two. Find (i) the velocity ratio; (ii) the work done against friction in raising a load of 300 lb. through a distance of 6 in. if the effort required is 80 lb.; (iii) the efficiency of the tackle. Neglect the weight of the lower block. (U.L.C.I.)

6. Define (a) the mechanical advantage, and (b) the velocity ratio, of a machine.

The pitch of the screw of a screw-jack is $\frac{1}{2}$ in., and the distance

from the centre of the screw to the end of the arm is 18 in. Calculate its velocity ratio. If its efficiency is 40 per cent, calculate the effort required to raise a load of 120 lb. (U.L.C.I.)

7. Find the mechanical advantage and velocity ratio of a perfectly smooth inclined plane when a force of 28 lb. parallel to the plane is sufficient to push a weight of 2 cwt. up the plane with uniform velocity. If the surface of the plane became roughened so that the applied force had to be increased to 40 lb., what would then be the mechanical advantage, the velocity ratio, and the efficiency of the plane? (U.L.C.I.)

8. A screw-jack is operated by a bar pushed through a hole in the spindle. The pitch of the screw is $\frac{1}{2}$ in. A force of 60 lb. applied at right angles to the bar, at a radius of 14 in. from the axis of the screw, is found to lift a casting weighing 2 tons which rests upon the top of the jack. Find the velocity ratio, the mechanical advantage, and the mechanical efficiency of the machine under these conditions. (U.E.I.)

9. What is the meaning of the term "machine"? Give diagrams of two machines which will enable a load of 160 lb. wt. to be lifted by a force of 40 lb. wt. assuming no friction. Give an adequate explanation of the diagrams. Explain the term "efficiency" in connection with machines, and work out its relation to the mechanical advantage and velocity ratio, illustrating by reference to a suitable machine. (C.P.E.)

Section B

10. In a lifting machine a load W is raised through a height of 3 ft. by an effort of 50 lb. The velocity ratio of the machine is 40, and the work done in overcoming frictional resistance is 3900 ft.-lb. Determine the value of W and the efficiency of the machine at this load. (C.P.E.)

11. Explain what is meant by the efficiency of a simple lifting machine.

In a hoist block, velocity ratio 24, an effort of 118 lb. is required to lift a load of 1100 lb. Find the efficiency of the block and determine how much work is expended in overcoming friction in lifting the 1100-lb. load through a height of 10 ft. (C.P.E.)

12. Give a well-proportioned sketch of a differential wheel and axle.

If the diameters of the two parts of the axle are 5 and 8 in. respectively and the diameter of the wheel is 36 in., find what effort is required to raise a load of 400 lb. when the efficiency is 80 per cent. How much of the effort is required to overcome frictional resistance? (C.P.E.)

13. A single-cylinder oil engine has a cylinder 7 in. diameter; the stroke is 13 in.; the mean effective pressure 41 lb./sq. in.; and the number of explosions per minute 90. The mechanical efficiency of the engine is 87.5 per cent. What is the b.h.p.? (S.W.E.T.C.)

14. A crane is driven by an electric motor, and has to lift a load of 10 tons at a uniform speed of 2 ft./sec. The efficiency of the crane is 65 per cent. The motor pulley is 10 in. diameter and runs at 250 r.p.m. What force must be supplied at the circumference of the motor pulley? (S.W.E.T.C.)

15. A body weighing 1 cwt. is suspended at a height of 20 ft. from the ground. What is (i) its potential energy, (ii) its kinetic energy when it hits the ground? Assume it to fall freely.

A 6-cylinder 4-stroke cycle petrol engine has cylinders 2.4 in. diameter, 4 in. stroke. If the mean effective pressure is 135 lb./in.² and the mechanical efficiency is 90 per cent, what brake horse-power will the engine develop at 1800 r.p.m.?

16. The following particulars relate to a rope brake dynamometer used in an engine test: Effective brake radius, 32 in.; spring balance readings: W_1 , 89 lb.; W_2 , 27 lb.; engine speed, 276 r.p.m. (a) Calculate the b.h.p. (b) If the mechanical efficiency is 92 per cent, what is the i.h.p.? (S.W.E.T.C.)

17. The mean effective pressure found from the indicator diagrams taken during a test on a four-stroke cycle internal-combustion engine was 88 lb. per sq. in. The diameter of the cylinder was 6 in. and the stroke 10 in. The speed of the engine was 385 r.p.m. and the number of explosions per min. 176.

Find the indicated horse-power of the engine. Assuming a mechanical efficiency of 75 per cent, find the average torque exerted by the engine. (U.L.C.I.)

18. A 4-stroke 4-cylinder petrol engine of 4 in. bore and 5 in. stroke, developing 30 brake horse-power at 1450 r.p.m., drives a generator whose output is 20 kilowatts. Find (a) the efficiency of the generator, (b) the brake torque in lb.-ft., (c) the mean effective pressure of the engine if it has mechanical efficiency of 75 per cent. (U.E.I.)

19. A smooth wedge of isosceles-triangular shape, having sides of 10, 13, and 13 in. long, is used to split a stone. The effort applied to the back of the wedge is 200 lb. Find the force exerted by each surface of the wedge on the stone.

CHAPTER XVII

Testing of Machines

1. Laboratory testing of machines

In any practical course of mechanics it is customary for the student to carry out laboratory tests on a variety of simple machines. The objects of these tests (besides the important one of gaining familiarity with the equipment) are:

- (a) To determine the effort required to raise various loads (i.e. to determine the mechanical advantage of the machine).
- (b) To study the effect of friction at various loads (i.e. to examine the losses).
- (c) To determine the mechanical efficiency at various loads.

In order to attain these objectives, it is necessary to determine the velocity ratio of the machine. This can be calculated from its dimensions (as in the examples given in Chap. XVI), or may be determined by direct experimental measurement, e.g. by measuring the distance moved by the effort to secure a one-foot drop in the load. It should be realized that the velocity ratio of a machine is a constant, and does not depend upon load.

From the results of such tests, it will be seen that, owing to the variation in the effects of friction at various loads, neither mechanical advantage nor efficiency is constant, but both vary with the load. The mechanical efficiency will generally be at a maximum at around the maximum load for which the machine is designed. At heavy overloads, or when only lightly loaded, this efficiency will fall off considerably.

The procedure to be followed when carrying out a laboratory test of a machine will be similar, irrespective of the type of machine. As an example of such procedure, details of a typical experiment carried out by students on a wheel and differential axle are given below.

2. Experimental test on a wheel and differential axle

Object.—(a) To determine the effort required, the amount of friction set up, the mechanical advantage, and the mechanical efficiency of the machine.

(b) To study these results.

Apparatus.—Wheel and differential axle fitted with rope and snatch block. Suitable weights.

Method of procedure.—The velocity ratio is first calculated from the actual physical dimensions of the machine, using the formula given in Chap. XVI § 9. Care must be taken to measure the diameters from the centre of each rope. The calculated velocity ratio is then checked by measuring the movement of the effort rope required to lift the load a distance of 6 in.

The effort required to raise various loads at constant speed is then determined, and the results recorded, care being taken that these loads are within the capabilities of the machine.

From these observed results, values of mechanical advantage, the effect of friction, and of mechanical efficiency are then derived for each value of load.

Mechanical advantage is obtained by dividing the load by the effort.

The effect of friction can be evaluated as the “load” effect (friction load), or alternatively as the friction effort (see Chap. XVI, § 6).

Mechanical efficiency is obtained by dividing the mechanical advantage by the velocity ratio.

The output of the machine is compared with the input by comparing the work done in raising the load through one foot with the corresponding work done by the effort ($E \times \text{V.R. (ft.-lb.)}$). The difference between input and output represents the friction losses (in foot-pounds) set up when the load is raised through one foot.

The mechanical efficiency can be found by dividing output by input, and this will coincide with the values obtained from $(\text{M.A.}) \div (\text{V.R.})$.

Results.

Diameter of effort wheel	..	=	10 in.
„ „ major axle	..	=	5 in.
„ „ minor axle	..	=	2 in.
„ „ effort rope	..	=	$\frac{1}{8}$ in.
„ „ load rope	..	=	$\frac{3}{16}$ in.
„ between rope centres		=	$10\frac{1}{8}$ in. (effort wheel)
„ „ „ „		=	$5\frac{3}{16}$ in. (major axle)
„ „ „ „		=	$2\frac{3}{16}$ in. (minor axle)

$$\text{Calculated velocity ratio} = \frac{2R}{r - r_1} = \frac{2 \times 5\frac{1}{16}}{2\frac{1}{32} - 1\frac{3}{32}} = \frac{10\frac{1}{8}}{1\frac{1}{2}} = 6.75.$$

Measured velocity ratio

$$= \frac{\text{distance moved by effort}}{\text{corresponding distance moved by load}} = \frac{40\frac{1}{2}}{6} = 6\frac{3}{4}.$$

Load, lb.	Effort, lb.	Mechanical advantage, L/E	Output, ft.-lb. $L \times l$	Input, ft.-lb. $E \times V.R.$	Effect of friction, $(E \times V.R. - L)$ lb., or friction loss, ft.-lb.	Mech. efficiency (%) $\left(\frac{M.A.}{V.R.} \text{ or } \frac{\text{output}}{\text{input}}\right) \times 100$
2	0.55	3.64	2	3.712	1.712	54
6	1.19	5.04	6	8.03	2.03	74.5
10	1.85	5.40	10	12.48	2.48	80
14	2.49	5.62	14	16.8	2.8	83.3
18	3.15	5.71	18	21.26	3.26	84.6
22	3.8	5.79	22	25.65	3.65	85.7
26	4.44	5.86	26	30	4.0	86.9
30	5.1	5.89	30	34.42	4.42	87.3
34	5.75	5.91	34	38.81	4.81	87.6

To enable the variation of effort, friction, and efficiency

with the load to be studied in more detail than the table of results permits, graphs are then drawn of

- (a) load v. effort,
- (b) load v. effect of friction,
- (c) load v. efficiency.

These graphs are shown in fig. 17.1.

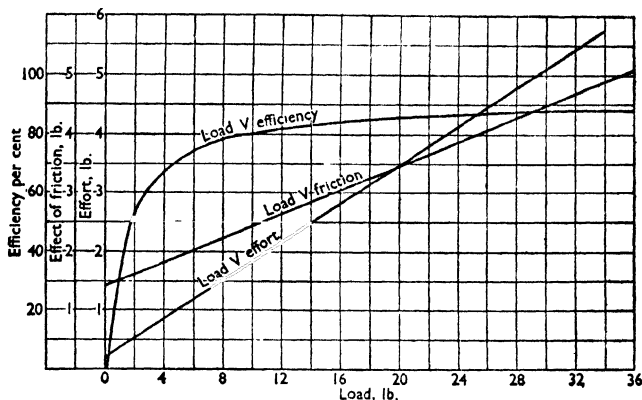


Fig. 17.1

Sometimes a graph of load against mechanical advantage may be added. This will be of similar form to that of load against efficiency.

N.B. As friction effort $\times$ V.R. = friction load, then the load v. friction graph will be of the same shape irrespective of whether friction load or friction effort values are plotted. The vertical scale will, of course, be different in each case.

3. Law of a machine

The relationship between the load (L) and the effort (E) can be expressed in mathematical form, and is then known as the "law of the machine".

It will be seen from the graph of load against effort that this relationship can be represented by a straight line. The mathematical equation for a straight-line graph is of the form

$$y = mx + c,$$

where the y axis is vertical and the x axis is horizontal, and m and c are constants. m is represented by the slope of the graph, and c by the intercept of the graph with the y axis. In the graphs, the x axis is used to indicate the load (L), and the y axis the effort (E). Thus the equation for the machine will be of the form

$$E = mL + c.$$

To find the law of the wheel and differential axle.—For convenience the load v. effort graph has been redrawn in fig. 17.2.

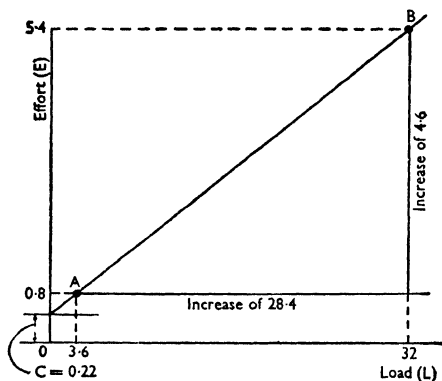


Fig. 17.2

1. *To find the slope m .*—Take two points A and B on the graph as far apart as convenient.

$$\text{At A: } L = 3.6 \quad E = 0.8$$

$$\text{At B: } L = 32.0 \quad E = 5.4$$

$$\text{Increase} \quad \underline{28.4} \quad \underline{4.6}$$

$$\therefore \text{Slope } (m) = \frac{\text{increase of } E}{\text{increase of } L} = \frac{4.6}{28.4} = 0.162.$$

2. *To find the intercept c .*— c is the value of E where the graph cuts the y axis. (It is therefore a constant representing the minimum amount of friction present in the machine, and this friction must be overcome before any load can be raised.)

By measurement from graph, $c = 0.22$. Therefore the law of the machine is

$$E = 0.162L + 0.22.$$

This result could also have been obtained algebraically by taking the values of E and L at any two points (such as at A and B) and substituting them in the equation $E = mL + c$.

Thus at A, $L = 3.6$ and $E = 0.8$,

$$\text{so equation becomes } 0.8 = 3.6m + c$$

and at B, $L = 32$ and $E = 5.4$,

$$\text{so equation becomes } 5.4 = 32m + c$$

$$\text{Subtract to eliminate } c: \quad -4.6 = -28.4m$$

$$\therefore m = \frac{-4.6}{-28.4} = 0.162, \text{ as before.}$$

Then

$$c = E - mL.$$

Substitute the values of E and L at either A or B.

$$c = 5.4 - 0.162 \times 32 = 5.4 - 5.184 = 0.216.$$

Therefore the law of the machine is

$$E = 0.162L + 0.216.$$

Knowing the law of a machine, it is then easy to calculate the effort necessary to raise any load with the machine.

4. General conclusions from the test

From a study of the results, graphs, and the law of the wheel and differential axle, or of any other machine, it is possible to reach the following general conclusions:

1. The effort required increases proportionally with the load raised. This will be readily apparent because the load v. effort graph is a straight line.

2. The friction losses increase proportionally with the load raised. This is again apparent as the load v. effect of friction graph is a straight line.

3. Some friction is present at no load, hence some effort is required to operate the machine at no load. This is indicated by the fact that the load v. effort, and load v. effect of friction graphs do not pass through the origin, but intercept the y axis at a distance from the origin.

4. The work put into the machine is always greater than the work output, the difference being equal to the friction losses.

5. The mechanical efficiency (and also the mechanical advantage) increases with load, but is not proportional to it, as is indicated by the load v. mechanical efficiency graph being a curve. A limiting value of efficiency is approached at around the maximum designed load. (If this load should be exceeded the efficiency would fall off.)

The efficiency of a wheel-and-axle machine is relatively high. Simple pulley tackles also have a high efficiency, but it falls off as the number of pulleys is increased, owing to the corresponding increase in friction.

Inclined planes, screws, etc., have low efficiencies owing to the great amount of friction present.

In some types of machine, particularly those incorporating some form of screw, the amount of friction present is sufficient to prevent what is known as "overhauling", i.e. it prevents the load from falling under the action of gravity if the effort should be removed.

If the efficiency of a machine is less than 50 per cent, more than half the effort is used to overcome friction, and less than half to support the load. Consequently, as the friction is more than sufficient to support the load, such a machine will not overhaul.

5. Limiting value of the efficiency of a machine

Knowing the law of a machine, the limiting value of its efficiency can be estimated as follows:

$$\text{Mechanical efficiency } (\eta) = \frac{\text{M.A.}}{\text{V.R.}} = \frac{L}{E \times \text{V.R.}}$$

Substituting for E ,

$$\begin{aligned} \eta &= \frac{L}{(mL + c) \times \text{V.R.}} = \frac{L}{(\text{V.R.} \times mL) + (\text{V.R.} \times c)} \\ &= \frac{1}{m \times \text{V.R.} + \frac{c \times \text{V.R.}}{L}} \end{aligned}$$

Thus, as the load increases, the value of $c \times \text{V.R.} \div L$ becomes less and less, whilst the efficiency rises and tends towards the limiting value of $1 \div (m \times \text{V.R.})$. This value would only be reached if the load were infinite; it cannot be exceeded.

Example 1.—What is the limiting value of the efficiency of the wheel and differential axle considered above?

$$\text{Limiting efficiency} = \frac{1}{m \times \text{V.R.}} = \frac{1}{0.162 \times 6.75} = 91.4 \text{ per cent.}$$

EXERCISES ON CHAPTER XVII

Section A

1. The following results were obtained from an experiment on a set of pulleys.

W denotes the weight raised, and E the effort applied. Plot these results on squared paper, and obtain the law connecting E and W .

W lb.	14	21	28	35	42	
E lb.	2.5	3.3	4.25	5.1	6	(U.E.I.)

2. Define “mechanical advantage” and “velocity ratio” of a machine. A machine gave the following results:

Load, lb.	0	40	80	120	160
Effort, lb.	2	4.5	7.1	9.4	12

(i) Draw a graph of load against effort. (ii) From the graph find the effort required to lift a load of 100 lb. (U.E.I.)

3. A test of a hoisting apparatus having a velocity ratio of 30 gave the following results:

No. of experiment:	1	2	3	4	5
Load, lb.	128	336	544	688	880
Effort, lb.	7	15	23	28.5	36

Draw the load v. effort graph and the load v. efficiency graph. (U.L.C.I.)

4. Explain how you would determine the actual velocity ratio of a Weston differential pulley block if you were experimenting with it in the laboratory. The efficiency of such a lifting appliance is 35 per cent and the velocity ratio 24; find what pull should be exerted on the chain of the block in order to raise a load of $\frac{1}{2}$ ton. (U.L.C.I.)

Section B

5. The following results were obtained in an experiment on a Weston differential pulley block of velocity ratio 16.

Load, lb.	0	14	28	42	56	70	84
Effort, lb.	2	4	6.5	8.75	11	13.5	15.5

Plot graphs of load against effort, and load against efficiency. State what you can deduce from these graphs.

6. It is found by trial that to raise weights of 14, 28, 42, and 56 lb. by means of a lifting appliance, efforts of 4.75, 7.5, 10.75, and 13 lb. weight are required. The velocity ratio of the tackle is 16. From these results determine: (a) the law connecting the actual effort E with the load W , and (b) the effort, mechanical advantage, and efficiency when a weight of 34 lb. is being raised. (U.L.C.I.)

7. A force of 3.5 lb. applied to the handle of a small lifting machine is just sufficient to raise a load of 50 lb., and a force of 9.9 lb. is sufficient for a load of 150 lb. Find the equation giving the relationship between the effort and the load, assuming the straight-line law. Also, calculate the efficiency of the machine when lifting a load of 220 lb., the velocity ratio being 18. (U.L.C.I.)

8. What do you understand by "overhauling"? Under what conditions would you expect it to occur?

In a Weston differential pulley block one pulley has 13 teeth and the other 12. A load of 56 lb. is raised by an effort of 9 lb. Will it "overhaul" when this effort is removed?

9. Describe how you would determine experimentally the velocity ratio of any lifting machine with which you are familiar.

The following results were obtained in a laboratory test of a lifting tackle, the velocity ratio of which was 25 to 1.

Load raised, lb.	0	50	100	150	200
Effort applied, lb.	5	10	15	21	28

Draw a graph connecting effort applied and load raised. From the graph determine the effort required to raise a load of 120 lb. What is the mechanical efficiency when raising this load? (S.W.E.T.C.)

10. Define the terms mechanical efficiency, velocity ratio, and mechanical advantage of a machine.

In a wheel and axle the diameter of the wheel is $23\frac{1}{2}$ in. and the diameter of the axle is $7\frac{1}{4}$ in. The thickness of the cord on the wheel is $\frac{1}{2}$ in. and on the drum is $\frac{3}{4}$ in. Find the velocity ratio of the machine. If the efficiency when lifting a load of 3 cwt. with a velocity of 1 ft. per sec. is 80 per cent, how many foot-pounds of work must be supplied to the machine per minute? (S.W.E.T.C.)

11. (a) Define mechanical advantage, velocity ratio, and efficiency of a machine.

(b) An experimental worm and worm-wheel under test gives the following results:

Load in lb.	10	20	30	40	50
Effort in lb.	0.85	1.60	2.30	2.95	3.65

If the V.R. is 45, make a table of load, effort, M.A., and efficiency, and draw graphs of load, effort, mechanical advantage, and efficiency. What is the limiting efficiency of the machine?

12. A system of pulleys is arranged as in the figure. What is the velocity ratio? If the efficiency is 60 per cent when the load is 100 lb., and 70 per cent when the load is 200 lb., find the efforts required to raise (i) 100 lb. and (ii) 200 lb.

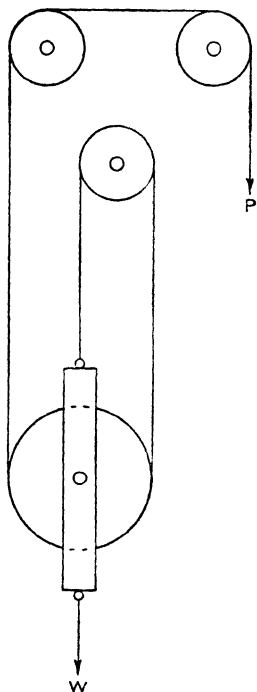


Fig. 17.3

The relationship between load and effort is of the form $P = aW \times C$; determine the values of the constants a and C . (S.W.E.T.C.)

13. A travelling crane has a velocity ratio of 300. The efforts required to raise various loads are given in the table:

Load, lb.	1000	2000	3000	4000	5000
Effort, lb.	19	28	37.5	46	54.5

Determine the law of the machine (the equation connecting load and effort). From this calculate the upper limiting value of the efficiency of the crane. (S.W.E.T.C.)

CHAPTER XVIII

Mechanical Transmission of Power

1. Efficiency

The power developed by an engine or a motor has in general to be transmitted to the machine or device which it is required to drive. Both the force and the motion may be modified during this transmission. For instance, the speed of rotation of the engine may be different from that desired to drive the machine. Or again, it may be desired to convert a small force moving quickly into a much larger force moving slowly.

In all forms of power transmission, some power is lost due to the effects of friction. The ratio

$$\frac{\text{power delivered by system}}{\text{power put into system}} \quad \text{or} \quad \frac{\text{power output}}{\text{power input}}$$

is known as the **efficiency** of the system. This ratio can never be greater than unity. It is often expressed as a percentage, and this can never exceed 100 per cent.

$$\text{Thus percentage efficiency} = \frac{\text{power output}}{\text{power input}} \times 100.$$

Power may be transmitted mechanically by the following types of drive:

- (1) Belt.
- (2) Rope.
- (3) Chain.
- (4) Friction.
- (5) Gearing.

2. Belt driving

A leather or composition belt is arranged between pulley wheels situated some distance apart (see fig. 18.1). The power is transmitted by the action of friction between the belt and the pulleys. If the belt is slack, or if the load becomes excessive, the belt will slip. Tightening the belt increases its grip, but slipping cannot be entirely prevented. Consequently belts do not give a *positive* drive, and their use must be avoided if this is important.

They are used to a large extent in workshops for driving machine tools from overhead lines of shafting; the possibility of slip in this case can be advantageous, as it acts as a form of safety valve if a machine becomes grossly overloaded, and so

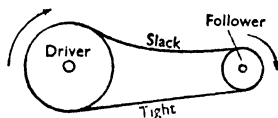


Fig. 18.1.—Belt drive

prevents the full effect of this overload being felt by the driving motor.

Two types of belt drive are used extensively. By far the commoner type is the flat belt, which works on the flat (or rather, slightly rounded) surfaces of the pulley wheels. This is the type seen in the majority of workshops. The second type is the V belt, usually of composition, which operates in a V groove on the pulley wheels. Frequently several V belts operating in a number of grooves are employed. Owing to the greater surface area available for friction in this type it does not slip so readily as a flat belt.

Some modern factories have done away with belt driving from overhead shafts by making use of "individual drive", in which each machine is fitted with a "built-in" electric motor; the friction losses in the line shafting and the unsightliness and complications of belt driving are thereby avoided. However, many examples of belt drives can be observed in everyday life.

3. Change of motion by belt drive

By using a large-diameter pulley as the **driver** and a smaller pulley as the **follower**, we can increase the speed of rotation of the driven machine. Conversely, a small driver and large

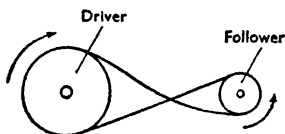


Fig. 18.2.—Reversing by crossed belt

follower will reduce the speed of rotation. The speed, in fact, is inversely proportional to the diameter of the pulleys. By crossing the belt (see fig. 18.2), we can reverse the direction of motion.

4. Velocity ratio of a belt drive

The ratio between the speeds of rotation of the driver and follower, often referred to as the **velocity ratio**, can be deduced as follows, assuming that there is no slip.

The belt and the outside surfaces of both pulleys will all move together at the same **linear** velocity.

In fig. 18.1, let the driver be of diameter D and the follower of diameter d . Then, when the driver makes N revolutions, a point on its rim will move through a distance $N\pi D$. In moving round it will move a length $N\pi D$ of belting, and this will cause a point on the rim of the follower to turn through this same distance $N\pi D$.

$\therefore$ Number of revolutions made by follower

$$= \frac{N\pi D}{\text{circumference of follower}} = \frac{N\pi D}{\pi d} = N \times \frac{D}{d}.$$

$\therefore$ Speed of follower = speed of driver $\times \frac{\text{diameter of driver}}{\text{diameter of follower}},$

or velocity ratio

$$= \frac{\text{speed of driver (r.p.m.)}}{\text{speed of follower (r.p.m.)}} = \frac{\text{diameter of follower}}{\text{diameter of driver}},$$

i.e. the ratio of the speeds is in inverse proportion to the ratio of the pulley diameters.

Sometimes the driving shaft, rotating at a constant speed, is required to drive a machine at several other different speeds, each of which may be selected at will, i.e. the velocity ratio is required to be variable. This can be arranged by what are known as **speed cones**. Fig. 18.3 shows the arrangement for securing three different velocity ratios, and is frequently met

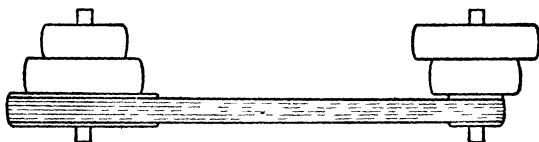


Fig. 18.3.—Speed cones

with in machine-tool drives. To change speeds, the belt is slipped from one set of cones to another, the size of these cones being such that the same length of belting suits each corresponding pair.

5. Power transmitted by belt drive

If a belt drive is examined whilst in operation, it will be noticed that one side of the belt is tight and the other slack (see fig. 18.1). The effective pull or turning force on a pulley will be the difference between the tensions in these two sides of the belt, i.e.

$$\text{effective turning force} = T_t - T_s,$$

where T_t and T_s represent the tension on the tight and slack sides respectively.

The speed of the belt in feet per minute will obviously be the same at all points on the belt,

∴ work done per minute by the belt (in ft.-lb.)

$$= (T_t - T_s) \text{ (lb.)} \times \text{speed of belt (ft./min.)},$$

and the horse-power transmitted will be

$$\frac{(T_T - T_S) \times \text{speed of belt}}{33,000}.$$

Example 1.—An electric motor running at a speed of 1500 r.p.m. is required to drive a line of shafting at 120 r.p.m. The motor pulley is of 4 in. diameter. Find the diameter of the pulley on the line shafting, neglecting slip in the belt. Find also the effective pull in the belt when it is transmitting 20 h.p.

$$\frac{\text{R.P.M. of driver}}{\text{R.P.M. of follower}} = \frac{\text{diameter of follower}}{\text{diameter of driver}},$$

$$\therefore \frac{1500}{120} = \frac{x}{4}.$$

$$\therefore \text{Diameter of pulley required on shafting} = \frac{1500 \times 4}{120} = 50 \text{ in.}$$

$$\text{H.P. transmitted} = \frac{\text{effective pull} \times \text{speed of belt}}{33,000},$$

$\therefore$ effective pull on belt

$$= \frac{20 \times 33,000}{\text{speed of belt}} = \frac{20 \times 33,000}{\pi \times 4 \times 120} = \frac{1375}{\pi} = 437.6 \text{ lb.}$$

Example 2.—The driving pulley of an electric motor is of 18 in. diameter and its speed 750 r.p.m. The tension in the tight and slack sides of a belt connected to this pulley are 210 lb. and 90 lb. respectively. Calculate the h.p. transmitted.

If the belt is 5 in. wide and $\frac{3}{8}$ in. thick, find the maximum intensity of stress produced in it.

$$\text{Effective pull in belt} = T_T - T_S = 210 - 90 = 120 \text{ lb.}$$

$$\text{Circumference of pulley} = \pi d = 1.5 \times \pi \text{ ft.}$$

$$\text{Speed of belt} = \text{circumference} \times \text{r.p.m.} = 1.5\pi \times 750 \text{ ft./min.}$$

$$\begin{aligned} \text{H.P. transmitted} &= \frac{\text{effective pull} \times \text{speed of belt}}{33,000} \\ &= \frac{120 \times 1.5\pi \times 750}{33,000} = 12.86 \text{ h.p.} \end{aligned}$$

$$\text{Maximum intensity of stress} = \frac{210}{\text{c.s. area}} = \frac{210}{5 \times \frac{3}{8}} = 112 \text{ lb./sq. in.}$$

6. Rope drive

Cotton, manila, hemp, or wire ropes are used between grooved pulley wheels. The power that can be transmitted is great, especially if a multiple drive, consisting of a large number of ropes working on a multiple grooved drum, is used. The V-shaped grooves are used in order to provide a greater frictional area for the rope to "bite" on; they therefore help to reduce slip, but, as with a belt drive, some slip is unavoidable.

7. Chain drive

In this form of drive slip is avoided by making use of cog or toothed wheels which engage with the links of a specially constructed chain. The bicycle provides a good example of this type of transmission.

The velocity ratio and the power transmitted by means of rope and chain drives can be calculated by the aid of similar methods to those used for belt drives. Care must be taken, however, that the wheel diameters used in the calculations are the effective diameters, i.e. the diameters corresponding to the centre line of the rope or chain (in most cases).

8. Friction drive

This form of drive is rarely met with to-day, except in certain specialized applications. For instance, fig. 18.4 shows a friction drive having a variable velocity ratio. The driver A consists of a disc, and the follower B transmits power at right angles to it. This follower can be adjusted in any position along the diameter of the driver, i.e. along XY. Its speed of rotation will increase from zero at the centre to a maximum at the periphery of the driver. If it is placed to the left of the centre of the driver, its direction of rotation will be reversed.

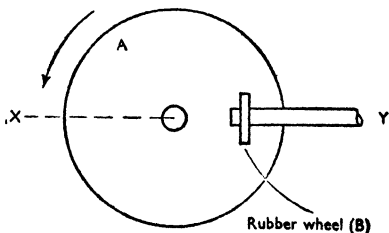


Fig. 18.4.—Friction drive

The amount of power that may be transmitted by a friction drive is limited to that corresponding to the pressure between the wheels at which slipping occurs. The velocity ratio will be

$$\frac{\text{r.p.m. of driver}}{\text{r.p.m. of follower}} = \frac{\text{diameter of follower}}{\text{effective diameter of driver}}$$

9. Geared drive

In this form of drive, teeth or cogs are cut into the periphery of cylindrical wheels, and are arranged to mesh with each other (fig. 18.5). Obviously these teeth must be equally

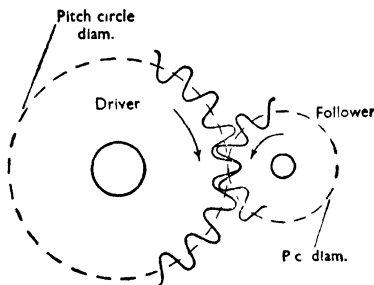


Fig. 18.5.—Geared drive

spaced around the circumferences of the two gear wheels to ensure smooth working, and there must be a whole number of complete teeth in each wheel. The velocity ratio between two gear wheels is that obtained by using the effective diameter of the two wheels, i.e.

$$\text{V.R.} = \frac{\text{r.p.m. of driver}}{\text{r.p.m. of follower}} = \frac{\text{effective diameter of follower}}{\text{effective diameter of driver}}.$$

The effective diameter of the wheels is the same as that of two cylinders in frictional contact which would give the same resulting motion.

This effective diameter of a gear wheel is known as its **pitch-circle diameter**; these **pitch circles** are shown dotted in fig. 18.5.

The distance between corresponding points on two consecutive teeth, measured along the pitch circle, is known as the **pitch**, or more correctly the **circular pitch** of the teeth.

Relationship between number of teeth and pitch

As there must be an exact number of teeth, then the ratio

$$\frac{\text{pitch-circle circumference}}{\text{circular pitch}} \quad \text{or} \quad \frac{\pi \times \text{pitch-circle diameter}}{\text{circular pitch}}$$

must equal the number of teeth.

As the number of teeth is thus proportional to the pitch-circle diameter we can write

$$\text{V.R.} = \frac{\text{r.p.m. of driver}}{\text{r.p.m. of follower}} = \frac{\text{number of teeth on follower}}{\text{number of teeth on driver}}.$$

This expression is often more useful than the previous one when making velocity ratio calculations.

10. Idle wheel

It is sometimes required that driver and follower shafts should both turn in the same direction. This can be secured

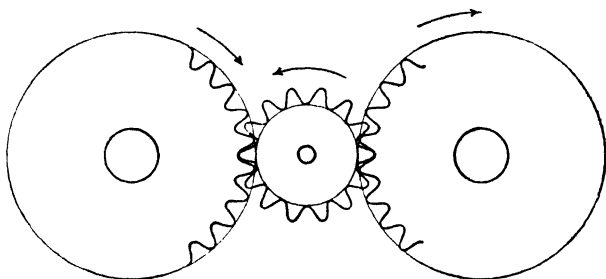


Fig. 18.6.—Use of idle wheel

by the use of an **idle wheel**, meshing with, and inserted between, driver and follower (see fig. 18.6).

This idle wheel has no effect on the velocity ratio, irrespective of the number of teeth it contains.

11. Gear trains

An arrangement of driver and follower, with or without an idle wheel, is known as a **simple gear train**. If the arrangement steps down the speed it is often called a **single-reduction gear**.

In order to secure a larger change of speed than can be conveniently made with a simple train, the driver can be arranged to turn a **layshaft** containing a second driver which meshes with the wheel on the **follower shaft** (see fig. 18.7).

Such an arrangement is termed a **compound train**, and if it steps down speed it is often called a **double-reduction gear**.

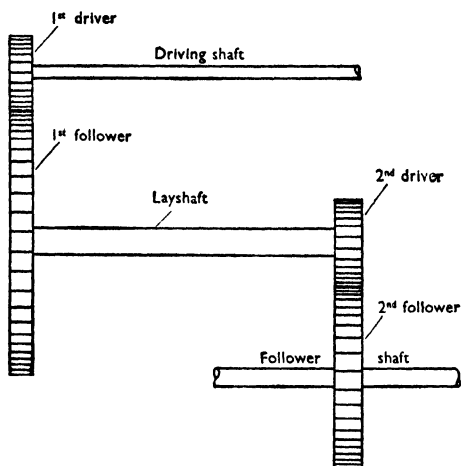


Fig. 18.7.—Compound reduction gear (double-reduction type)

If greater changes of speed are required, further layshafts may be added, the speed of the follower on each shaft becoming the speed of the driver for the next shaft.

The velocity ratio of a compound gear train may be worked out by the method already given, making one calculation for each stage, or the following formula may be used:

$$\begin{aligned} \text{V.R.} &= \frac{\text{r.p.m. of 1st shaft}}{\text{r.p.m. of last shaft}} \\ &= \frac{\text{no. of teeth on } F_1}{\text{no. of teeth on } D_1} \times \frac{\text{no. of teeth on } F_2}{\text{no. of teeth on } D_2} \times \text{etc.,} \end{aligned}$$

where F_1 , F_2 , D_1 , D_2 , etc., represent the various drivers and followers,

$$\therefore \text{V.R.} = \frac{\text{product of no. of teeth on followers}}{\text{product of no. of teeth on drivers}}$$

12. Power transmitted by geared drives

The effective turning force between gear wheels will be the pressure between the teeth, and this pressure will act tangentially to the pitch circle. Therefore the work done per minute = pressure between teeth (P lb.) $\times$ speed of pitch circle (ft. per min.).

$$\begin{aligned}\therefore \text{H.P. transmitted} &= \frac{P \text{ (lb.)} \times \text{speed of pitch circle (ft./min.)}}{33,000} \\ &= \frac{P \times \pi \times D \times N}{33,000},\end{aligned}$$

where D = pitch-circle diameter, N = no. of r.p.m.

Toothed wheels are capable of transmitting power up to the heaviest duties required in engineering. They give a positive

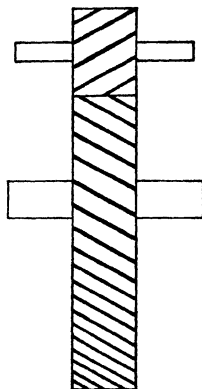


Fig. 18.8a.—Helical gears

drive, not suffering from the effects of slip. Modern design and production methods permit the manufacture of gears which are smooth and comparatively silent in operation, and the tooth shape is arranged to give the maximum strength. The pressure on the teeth in a heavy-duty drive can be quite considerable. To secure smooth running, the circular pitch is made small, and the teeth may be cut in a curve forming part of a helix across the edge of the gear wheel. This is known as a **helical gear** (fig. 18.8a).

This type of gear produces an end-thrust along the shaft in one direction; this thrust may be taken up by a suitable thrust bearing, or, if the amount of such thrust is large, it may be balanced out by using a **double helical gear** (fig. 18.8*b*), in which the teeth are arranged herring-bone fashion. Such gears are used extensively for speed reduction with electric motors, and for turbine reduction gears, particularly in marine power plants, where the most economical turbine speed is

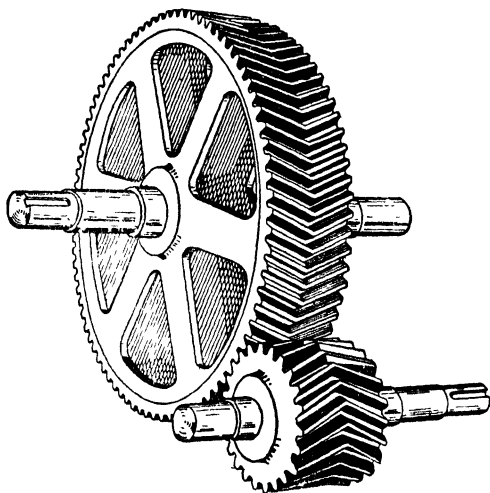


Fig. 18.8*b*.—Double helical gear
(David Brown & Sons (Huddersfield) Ltd.)

much higher than the most effective propeller speed. These gears are enclosed in suitable casings, and lubricated under pressure. High efficiencies, of the order of 98 per cent, may be obtained with them. Single-reduction gears are normally used for speed reductions of up to about 8 or 10 to 1. Above this, a double-reduction gear may be used for ratios up to about 60 to 1.

Example 3.—A ship is fitted with a single-reduction geared turbine drive of 2000 h.p. The turbine speed is normally 1200 r.p.m., and the propeller speed is required to be 140 r.p.m. The gear wheel on the turbine shaft has 42 teeth. Find the number of teeth required

on the propeller-shaft gear wheel, and the pressure between teeth at full power, if circular pitch is $\frac{1}{2}$ in. Double helical gears are used.

$$\frac{\text{R.P.M. driver}}{\text{R.P.M. follower}} = \frac{\text{no. of teeth on follower}}{\text{no. of teeth on driver}},$$

$$\therefore \frac{1200}{140} = \frac{\text{no. of teeth on follower}}{42}.$$

$$\therefore \text{Number of teeth required} = \frac{1200 \times 42}{140} = 360.$$

$$\begin{aligned} \text{Pressure between teeth} &= \frac{\text{h.p.} \times 33,000}{\text{pitch-circle circumference} \times N} \\ &= \frac{2000 \times 33,000}{(42 \times \frac{1}{2} \text{ in.} \times \frac{1}{12}) \times 1200} = 31,428 \text{ lb.} \end{aligned}$$

The actual pressure between teeth will be half this value, as it will be shared by two sets of wheels.

N.B. It is immaterial which pitch-circle circumference is used, provided that the correct value of N is used with it.

13. The screw

The screw can be used as a special form of geared drive for transmitting rotary motion of comparatively little force into slow linear motion of considerable force. Typical examples of its use are found in the lead screw of a lathe and the screw jack. When used for transmitting power, screw threads are usually made of square form. The pitch of a screw may be defined as the distance between corresponding points on adjacent threads. Consequently, in the case of a single-start thread, one revolution of the screw will move the load through a distance equal to the pitch.

An adaptation of the screw is found in the worm and worm wheel, in which the screw or worm engages with the teeth of a worm wheel, causing it to rotate at a slow speed compared to that of the worm. Worm reduction gears are made for heavy-duty work, and can give speed reduction ratios of the order of up to 10,000 to 1, by using a double-reduction arrangement.

Their efficiency, however, is generally low, owing to the great amount of friction present between worm and wheel.

The effect of this friction may be greater than the load, so the efficiency may never rise above 50 per cent.

In the case of a double-reduction gear, the efficiencies of the two units would have to be multiplied together to obtain the overall efficiency. This is true of all machines which are a combination of two or more simple machines, i.e.

Overall efficiency of a compound machine = $\frac{\text{product of individual efficiencies}}{\text{of component machines.}}$

Example 4.—A compound gear train with four stages is fitted with drivers having 40, 60, 80, and 120 teeth. The followers on the layshafts have 25, 50, 30 and 70 teeth. If the speed of the first driver is 120 r.p.m., find the speed of the final shaft.

If the efficiencies of each stage are 80, 85, 75 and 80 per cent respectively, find the overall efficiency.

$$\begin{aligned} \text{V.R.} &= \frac{\text{product of no. of teeth on followers}}{\text{product of no. of teeth on drivers}} \\ &= \frac{25 \times 50 \times 30 \times 70}{40 \times 60 \times 80 \times 120} = \frac{1}{8.77}. \end{aligned}$$

Speed of final shaft:

$$\text{V.R.} = \frac{\text{r.p.m. of first shaft}}{\text{r.p.m. of last shaft'}}$$

$$\begin{aligned} \therefore \text{speed of final shaft} &= \frac{\text{r.p.m. of first shaft}}{\text{V.R.}} \\ &= 120 \div \frac{1}{8.77} = 1052.4 \text{ r.p.m.} \end{aligned}$$

$$\begin{aligned} \text{Overall efficiency} &= 0.80 \times 0.85 \times 0.75 \times 0.80 \\ &= 0.408 \text{ or } 40.8 \text{ per cent.} \end{aligned}$$

EXERCISES ON CHAPTER XVIII

Section A

1. Show that if two toothed wheels are in gear their speeds are inversely proportional to their diameters or their numbers of teeth.

Three spur wheels A, B, and C on parallel shafts are in gear. A has 12 teeth, B 40 teeth, and C 48 teeth. Find the speed of C when A makes 80 r.p.m. What is the purpose of wheel B? (U.L.C.I.)

2. What do you understand by the "velocity ratio" of a lifting machine? The length of the handle of a winch is 16 in., and the diameter of the barrel is 8 in. The pinion on the handle axis has 16 teeth, and the spur wheel on the barrel axis has 90 teeth. Find the velocity ratio of the winch.

If the winch were without friction, what force at the handle would lift 675 lb.? In an experiment a force of 40 lb. at the handle was necessary to lift 675 lb. Account for the difference between the actual force and that necessary in a perfect machine. (U.L.C.I.)

3. A machine is driven by a belt drive from an electric motor. The linear speed of the belt is 400 in./sec. The diameter of the pulley on the motor shaft is 10 in., and the machine shaft runs at $\frac{3}{8}$ the speed of the motor shaft. What is the diameter of the pulley on the machine shaft, and the speeds of the motor and machine shafts in r.p.m.? (S.W.E.T.C.)

4. The figure shows the gearing of a machine tool. A is the motor shaft which runs at 480 r.p.m. The numbers of the teeth on the wheels

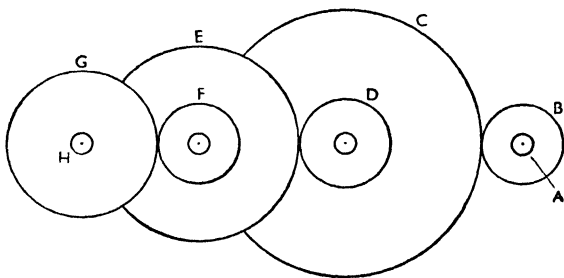


Fig. 18.9

are as follows: B, 20; C, 80; D, 25; E, 75; F, 20; G, 60. G is keyed to the shaft H. Determine the speed of the shaft H.

(U.L.C.I.)

(G 835)

5. The largest and smallest diameters of the pulleys of a speed cone fitted to a machine mandrel are 14 in. and 5 in. respectively. This speed cone is driven by a belt from a similar speed cone keyed to a countershaft which makes 220 r.p.m. Find the greatest and least speeds, in r.p.m., at which the machine mandrel may be driven.

(U.L.C.I.)

6. A pulley A drives a pulley B by means of a belt. The speed of the belt is 30 ft./sec. If the pulley A is to rotate at 200 r.p.m., what must be its diameter? If the diameter of pulley B is 30 in., what is its speed of rotation in r.p.m.?

(S.W.E.T.C.)

Section B

7. In the train of wheels shown below, A has 20 teeth, B has 30 teeth, D has 20 teeth, E has 95 teeth, and F has 25 teeth. A makes

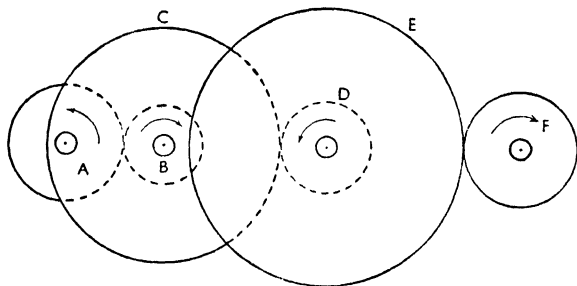


Fig. 18.10

30 and F 456 r.p.m. How many teeth must the wheel C have?

(S.W.E.T.C.)

8. Energy is transmitted to a shaft by a belt passing over a pulley 2 ft. in diameter. The speed is 300 r.p.m.; the pull in the slack side of the belt is 1000 lb., and in the tight side 2300 lb. Determine (a) the horse-power transmitted, (b) the horse-power absorbed in friction at the shaft bearings, if these are 2 in. in diameter and the coefficient of bearing friction is 0.08. Neglect the weight of the pulley and shaft.

(U.E.I.)

9. A belt is required to transmit 12 h.p. It passes over a pulley 2 ft. in diameter which is rotating at 250 r.p.m. Find the effective pull in the belt. If the pull in the tight side of the belt is 2.1 times

the pull in the slack side, find the maximum pull in the belt. If one square inch of the belt section will safely withstand a pull of 300 lb., what sectional area of belt will be required. (U.E.I.)

10. A pulley 8 ft. in diameter transmits power by means of 10 cotton ropes, the effective driving force of each rope being 160 lb. Determine the horse-power transmitted when the pulley is running at a speed of 220 r.p.m. (U.L.C.I.)

11. A gear wheel rotates at 150 r.p.m. and contains 70 teeth spaced on a pitch-circle diameter of 16.7 in. It is required to transmit 11.4 h.p. when driving a similar wheel. Calculate the pressure set up between the teeth.

CHAPTER XIX

More Complex Structures

1. Introductory

In this chapter we will consider the forces acting in more complicated structures than were dealt with in Chap. XIII. Attention will be confined, however, to cases in which the forces all act in the same plane, i.e. to coplanar forces.

Firstly, we will continue with some more problems in Graphic Statics, involving the equilibrium of concurrent coplanar forces.

2. Forces in structures containing more than three members

To determine the forces in a structure of this kind, the same general method is followed as was outlined in Chap. XIII. A reciprocal figure is drawn, starting by selecting a node where not more than two forces are unknown.

Example 1.—Find the forces in the overhanging framework shown in fig. 19.1a. Distinguish between ties and struts. (All acute angles are 45° .)

Method of procedure.

- (1) Draw *space diagram* and letter by *Bow's notation*.
- (2) Select node ABD where one force (AB) is known out of three.
- (3) Draw the *vector diagram* for the forces acting at this node, inserting arrowheads, starting with the known sense of vector *ab* (fig. 19.1b).
- (4) Take the node CDB, where the force in member BD has already been found, and draw the vector diagram for it (fig. 19.1c), so obtaining the forces in the two unknown members BC and CD. Insert arrowheads, noting that, as the force in member BD thrusts at node ABD, it must also thrust at node CDB, and therefore, in the vector diagram for this latter node, the vector *db* acts from *d* to *b*.

(5) Combine the above two vector diagrams into one *reciprocal figure* (fig. 19.1d), or, alternatively, build the second vector diagram (c) on top of the first (b).

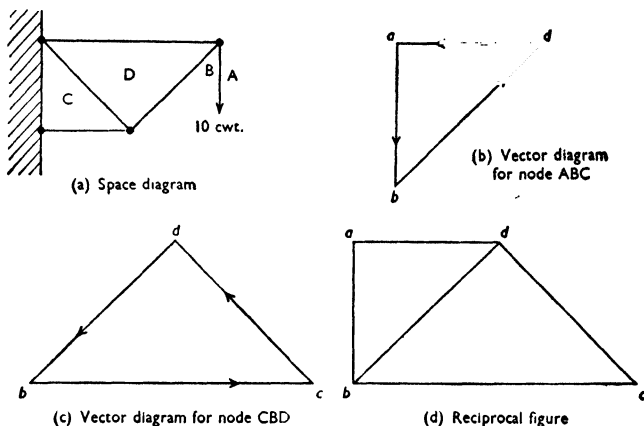


Fig. 19.1

Ties and struts may be distinguished readily by transferring the arrowheads to the space diagram, if required.

For the loading and configuration given, the results are tabulated below.

Member	Force, cwt.	Nature
BD	14	Strut
DA	10	Tie
BC	20	Strut
DC	14	Tie

3. The reciprocal figure

In the case of more complex structures than the one considered above, the reciprocal figure is built up of a large number of triangles and polygons of forces, one for each node.

Should the load be applied at more than one point, then, after applying Bow's notation, vectors of each load (or applied

force) are drawn to form a **load line**, and the reciprocal figure is built up on this.

It should be noted that arrowheads should not appear on a reciprocal figure, as some of the vectors in it refer to forces acting at more than one node, and their sense will be different in each case.

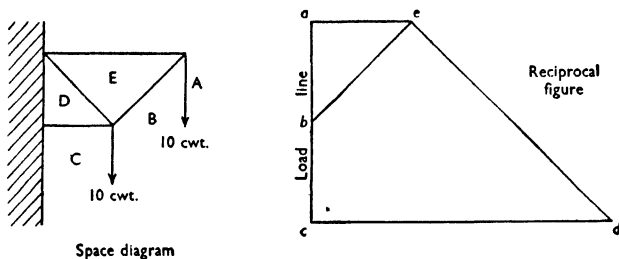


Fig. 19.2

If, in the previous example, an additional load of 10 cwt. had been applied at the lower node, then the reciprocal figure would be as shown in fig. 19.2.

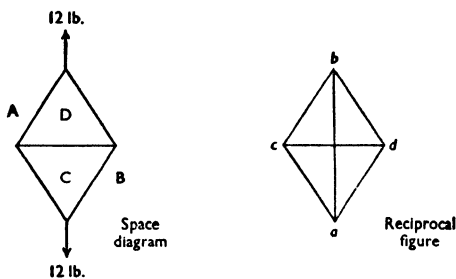


Fig. 19.3

In some structures the loading may be shown as two equal and opposite pulls, instead of as a load and its reaction/s. Using Bow's notation, however, the reciprocal figure can be obtained without difficulty, as in the example shown in fig. 19.3.

Example 2.—Find the reactions at the supports for the roof truss shown in fig. 19.4, and then determine graphically the load in each

member. Tabulate the results, and state whether the members are struts or ties.

The Method of Procedure is as follows (see fig. 19.4a and b):

- (1) Annotate by means of Bow's notation.
- (2) Find the reactions at the supports. (As in this case the load is at the apex, and the truss is symmetrical, the reactions R and S will be each equal to half the load, i.e. $R = S = 10\frac{1}{2}$ cwt.)
- (3) Start the *reciprocal figure* by drawing the *load-line* to some suitable scale, showing the load vector ab acting downwards, and the reaction vectors bc and ca acting upwards. (These forces are all completely specified.)
- (4) Consider the forces at the left-hand support, and draw the vectors ca , ad , and de parallel to the corresponding members on the space diagram. (N.B. ca is the known reaction R already drawn.)
- (5) Repeat for the forces at the node DEC, starting with the vector cd found in (4), and adding the vectors de and ec to the reciprocal figure.
- (6) Continue by dealing with the forces at each of the other nodes in turn, and adding their vectors to the reciprocal figure. (Take care to consider the forces in the clockwise order originally taken.)
- (7) Insert arrowheads at each of the nodes on the space diagram. Starting with a force of known direction, the direction of these arrowheads can be inserted in rotation on the vector diagram for each node *as the reciprocal figure is drawn*, the arrowheads then being transferred to their corresponding nodes on the space diagram. When the reciprocal figure is completed, however, care must be taken not to confuse the different arrowheads on each vector referring to the direction at different nodes.

As already mentioned, a reciprocal figure should not really have any arrowheads on it, but in this case they can be regarded as part of the construction necessary for sense finding.

- (8) Measure the vectors to scale, and tabulate the results, showing the nature of the stress in each member as determined by the arrowheads.

For the particular loading and design of roof truss given above, the space diagram, reciprocal figure, and table of results are shown in fig. 19.4a and b.

Note 1. As the figure and loading is symmetrical it is only strictly necessary to consider half of the diagram.

Note 2. For convenience, although not strictly correct, arrowheads have been inserted on some of the vectors in the reciprocal figure. The arrowhead nearest to a letter represents the sense of the vector ending with that letter, when forces are taken clockwise about a node. e.g. the arrowhead near "e" shows the sense of the vector to be "de" (not "ed") about the node CDE.

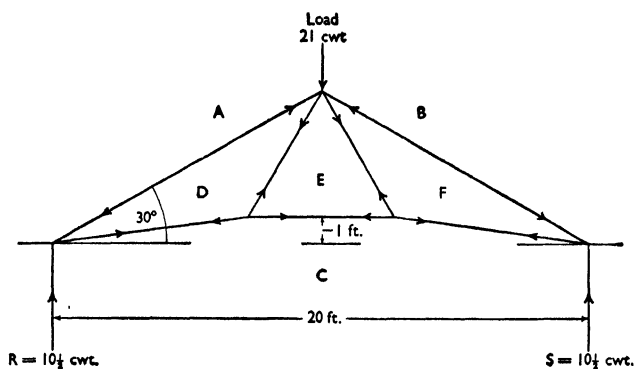
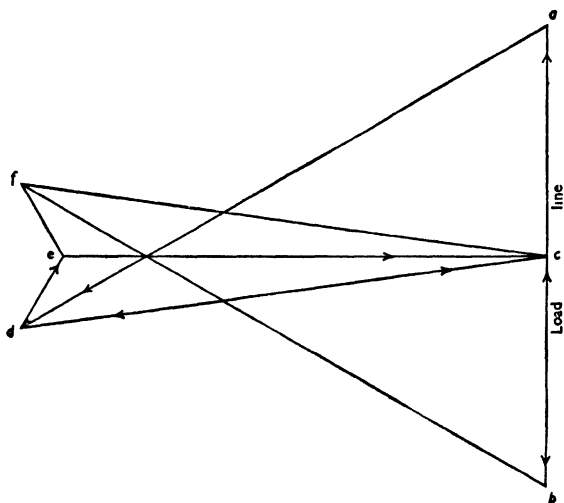


Fig. 19.4a.—Space diagram



Member	Stress, cwt.	Nature
AD and BF	27·6	Compression
DE and EF	3·8	Tension
EC	22	Tension
FC and CD	24·2	Tension

Fig. 19.4b.—Force diagram or reciprocal figure

4. Conditions of equilibrium of a particle

When dealing with any number of coplanar forces meeting at a point, these forces are regarded as acting on a single particle located at that point, and are termed **concurrent coplanar forces**.

We have seen that problems involving them may be solved graphically by drawing a vector diagram. If the vector polygon closes, then the particle is in equilibrium under the action of these forces.

Alternatively, such problems may be solved by calculation.

We will now deal with forces which do not meet at a point.

5. Non-concurrent coplanar forces

Coplanar forces do not always act at the same point on a body. They may act at several different points, either obliquely or in straight parallel lines. Obviously in such cases they cannot be regarded as acting on the same particle. They are therefore considered to be acting on a body of extended size known as a **rigid body**. Such a body always remains the same size and shape, irrespective of the forces applied to it (in practice no absolutely rigid body exists).

Forces acting on a rigid body in this manner are termed **non-concurrent coplanar forces**.

6. Conditions of equilibrium of a rigid body

(a) *Equilibrium under the action of two forces*

Fig. 19.5a shows two forces P and Q acting on a rigid body at points a and b respectively. It is clear that there is a ten-

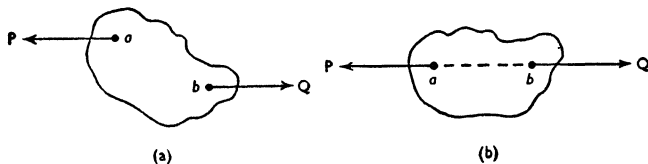


Fig. 19.5

dency for the body to turn (**rotate**) until the lines of action P and Q are in the same straight line (see fig. 19.5b). Further-

more, if P and Q are not equal in magnitude, there will also be a tendency for movement (**translation**) of the body in the direction of the greater force.

Consequently we may state:

“If two forces act on a rigid body at two different points, they will be in equilibrium only when these forces are equal and opposite in magnitude, and act in the same straight line.”

(b) *Equilibrium under the action of three non-parallel forces*

Fig. 19.6 shows three non-parallel forces acting on a rigid body. Consider any two of these forces, say A and B , acting at points a and b . If the lines of action of these forces are produced, they must meet at some point, and do so in this

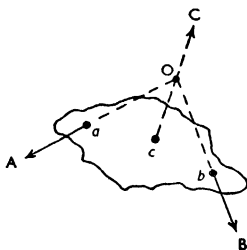


Fig. 19.6

case at point O . In order to produce equilibrium, the line of action of the third force C must also pass through this point O , and its magnitude and direction must be such that the vector triangle of forces closes.

N.B. The lines of action of the forces may meet outside the body.

Example 3.—A roof structure ABC of pitch 30° and span 19 ft. carries a load of 500 lb. concentrated at D and acting at right angles to AB . D is 4 ft. along the slope from A . The roof is hinged at C and rests on a smooth support at A . Find the magnitudes of the reactions at A and C , and the inclination of the reaction at C to the vertical.

Method.—Consider the structure as a rigid body. For it to be in equilibrium, the lines of action of the three forces involved must meet at a point, and their vector polygon must close. As support A

is smooth its reaction must be vertical (see Chap XIII, § 4), whilst the reaction at the hinge C is oblique.

Draw space diagram to scale (fig. 19.7).

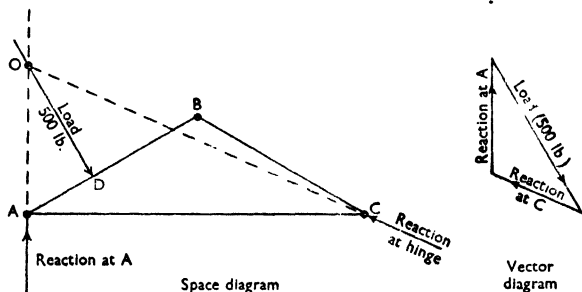


Fig. 19.7

Produce known lines of action (load and reaction at A) to meet at a point (O). Then line of action of reaction at C must pass through this point. Draw this line (CO).

Draw vector diagram (load is fully specified, directions of reactions are known).

Results (by measurement).

Reaction at A = 325 lb. Reaction at C = 275 lb.

Inclination of reaction at C to the vertical = 66.5° .

It should be noted that the above conditions for equilibrium are really the converse of the triangle of forces law.

“If three non-parallel forces are in equilibrium, then their lines of action must intersect at one point.”

Example 4.—A uniform ladder of weight 55 lb. and length 20 ft. rests against a smooth wall. Its foot stands on horizontal rough ground 7 ft. from the wall. Find the pressure exerted on the wall, and the magnitude and direction of the reaction set up on the ground.

Firstly, as the wall is perfectly smooth, the reaction exerted by it will be at right angles to the wall. The forces acting on the ladder will be:

- (1) Its weight W acting downwards through its centre of gravity.
- (2) The reaction R to the pressure on the wall, acting perpendicular to the wall.
- (3) The reaction to the thrust on the ground (T).

As the ladder is in equilibrium, then the lines of action of these three forces must meet at a point.

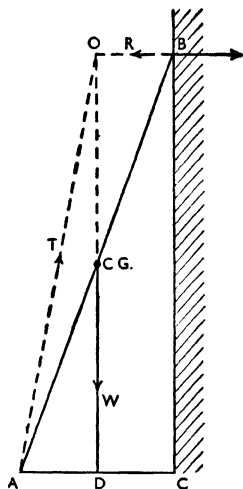


Fig. 19.8

In fig. 19.8 produce the line of action of W upwards to meet R at O . Join AO . Then AO represents the line of action of T , and the sides of the triangle ADO are proportional to forces W , R , and T .

If DO is made equal to 55 lb. (W), then

AD to this same scale will measure R ,

and

AO " " " " " T .

By measurement (a separate vector diagram of the forces at O could be drawn to scale).

$AD = 10\frac{1}{2}$ lb.

$AO = 56$ lb. (at 10° from the vertical).

These results may be checked by trigonometry, or alternatively the problem may be solved by using moments (see Chap XIII, § 5).

It may be noted here that the angle between OA and the vertical is the angle of friction between the ladder and the ground (see Chap. XIV, § 4).

(c) *Equilibrium under the action of three parallel forces*

In order to produce equilibrium in this case (see fig. 19.9), we know that W must equal $(R + S)$, and also that the clock-

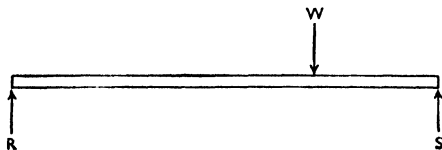


Fig. 19.9

wise moments of the forces taken about any point must equal the anticlockwise moments (see Chap. IX).

(d) *Equilibrium under the action of any number of parallel forces*

Similarly in this case, in order to produce equilibrium, we know that the algebraic sum of the forces must be zero, and the algebraic sum of the moments of these forces, taken about any point, must also be zero.

(e) *General conditions of equilibrium of a rigid body under the action of any number of coplanar forces*

In order to prevent movement (**translation**), or a tendency for movement, of the body in the direction of any resultant force,

- (i) **the vector sum of all the forces must be zero** (i.e. the vector polygon must close);

and, in order to prevent **rotation**, or a tendency for rotation, of the body in the direction of any resultant moment

- (ii) **the algebraic sum of all the moments of the forces, taken about any point in the plane, must be zero.**

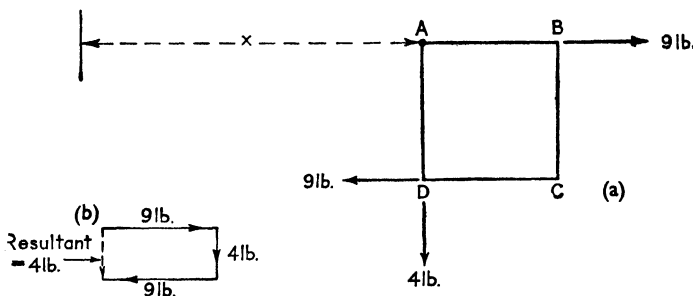


Fig. 19.10

Example 5.—(a) State the general conditions of equilibrium for any number of forces in one plane acting on a body. (b) ABCD is a square of $2\frac{1}{2}$ in. side. Two forces of 9 lb. each act from A to B and from C to D. A third force of 4 lb. acts from A to D. Find the resultant of the forces in magnitude, direction and position. (U.E.I.)

Method.—Draw the space diagram (see fig. 19.10a).

(i) To prevent translation the vector sum of the forces must be zero. Draw the vector diagram (fig. 19.10b), the closing line of which must represent the equilibrant in magnitude and direction.

By measurement: Resultant = 4 lb. directed vertically downwards.

(ii) To prevent rotation the algebraic sum of the moments about any point in the figure must be zero.

Take moments about A. (Note that forces AB and AD have no moment about A, as their lines of action pass through A.)

$$\begin{aligned}\text{Clockwise moment} &= \text{force CD} \times \text{distance AD} \\ &= 9 \times 2\frac{1}{2} = 22\frac{1}{2} \text{ lb.-in.}\end{aligned}$$

To produce equilibrium this moment must be balanced by the anticlockwise moment of the equilibrant about A.

Let x = perpendicular distance of line of action of equilibrant from A.

$$\text{Then} \quad \text{anticlockwise moment} = 4x.$$

$$\therefore 4x = 22\frac{1}{2} \quad \text{and} \quad x = 5\frac{5}{8} \text{ in.}$$

Therefore the position of the equilibrant (and of the resultant) must be $5\frac{5}{8}$ in. to the left of A along AB produced.

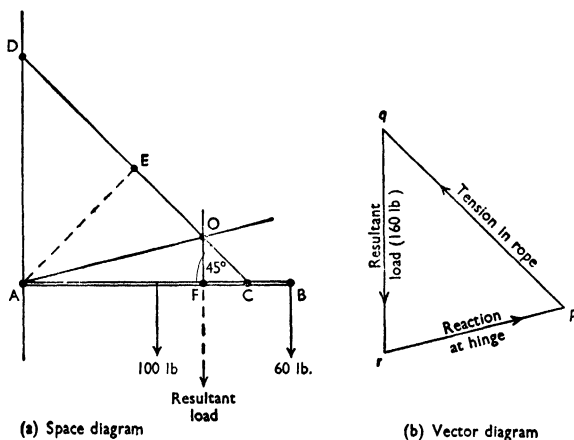


Fig. 19.11

Example 6.—A light beam AB, 12 ft. long, is hinged at A and supported in a horizontal position by a rope attached to a point C in the beam 2 ft. from B. The rope is inclined at 45 degrees to the beam, and is fastened to a bolt vertically above A. Vertical loads of 100 lb. and 60 lb. are carried by the beam at the mid-point and at the end B respectively. Find the tension in the rope, and the reaction at the hinge A. (C.P.E.)

Method.—This problem can be solved mathematically, or with the aid of graphical construction. First draw a diagram (to scale for the graphical method, see fig. 19.11a).

To prevent rotation of the beam about A, the moments of its

loads must be balanced by the moment of the pull in the rope (all moments being considered about A).

$$100 \times 6 + 60 \times 12 = \text{pull in rope } (P) \times \text{distance AE.}$$

$$\therefore P = \frac{600 + 720}{\text{AE}}.$$

By measurement: Distance AE = 7.1 ft.

By calculation: Distance AE = $10 \sin 45^\circ = 7.071$ ft.

$$\therefore \text{Tension in rope } (P) = 1320 \div 7.071 = 186.8 \text{ lb.}$$

To prevent translation of the beam under the action of the loads and the pull in the rope, the reaction at the hinge A must be such that the vector polygon closes.

In the vector diagram (fig. 19.11b), draw

pq representing the tension in the rope, and

qr representing the total load (160 lb.).

then *rp* will represent the reaction at the hinge A.

By measurement: *rp* = 135 lb. inclined at 12° to the beam.

By calculation: As $a^2 = b^2 + c^2 - 2bc \cos A$,

$$(rp)^2 = 160^2 + 186.8^2 - 2 \times 160 \times 186.8 \times 0.7071.$$

$$\therefore (rp) = 134.5 \text{ lb.}$$

As

$$\frac{a}{\sin A} = \frac{b}{\sin B},$$

$$\sin r = \frac{186.8 \times 0.7071}{134.5} = 0.983, \text{ and angle at } r = 79^\circ \text{ (approx.).}$$

$$\therefore \text{angle of inclination to beam} = 90^\circ - 79^\circ = 11^\circ \text{ (approx.).}$$

The line of action of the reaction at the hinge A, and its angle of inclination to the beam, can also be shown graphically on the space diagram (fig. 19.11a).

First find the position of the resultant load.

Let *x* be its distance from A.

$$\text{Then } 160x = 600 + 720,$$

$$\text{and } x = 1320 \div 160 = 8\frac{1}{4} \text{ ft. from A.}$$

Mark this position F on the space diagram, and project its line of action vertically upwards until it cuts the line of action of the pull in the rope at O.

Then AO must represent the line of action of the reaction at the hinge A.

N.B. Equilibrium of moments may be checked graphically by drawing a figure known as a *funicular*, *link*, or *moment polygon*. Closure of this figure is a proof that this condition of equilibrium is satisfied. The bending-moment diagrams in Chap. XX are examples of moment polygons.

EXERCISES ON CHAPTER XIX

1. The figure shows a mechanism for making mouldings. The hydraulic ram has a diameter of 4 in. and the fluid pressure on it is

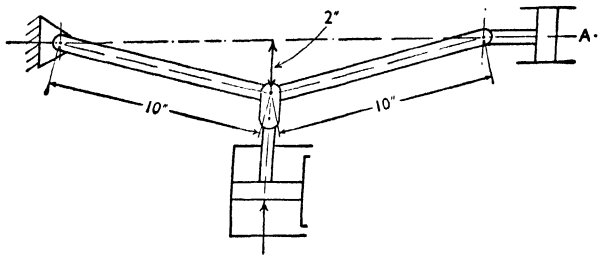


Fig. 19.12

800 lb. per sq. in. With the toggle links in the position shown, find graphically the force in the links and on the plunger A. (U.E.I.)

2. ABCD is a quadrilateral formed by four light rods freely jointed at their extremities. The angles at A and B are each right angles;

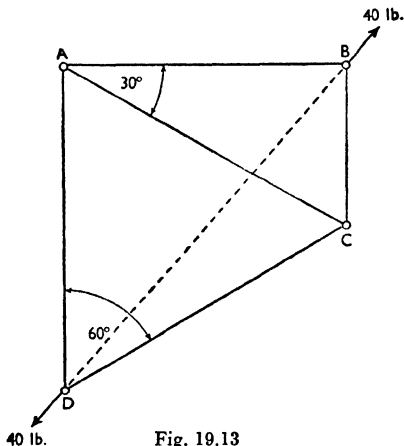


Fig. 19.13

the angle ADC is 60° and $AD = DC$. It is stiffened by the rod AC, and forces of 40 lb. weight act at D and B in such a manner that the frame is in equilibrium. Find the tensions or thrusts in the rods.

(S.W.E.T.C.)

3. A pin-jointed structure is loaded as shown in the diagram. Find the forces in the members AB and BC. (S.W.E.T.C.)

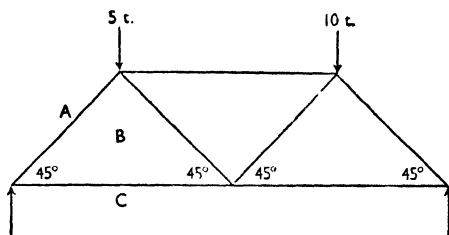


Fig. 19.14

4. Determine graphically the forces acting in the various members of the frame shown below without first obtaining the reactions at the supports. (S.W.E.T.C.)

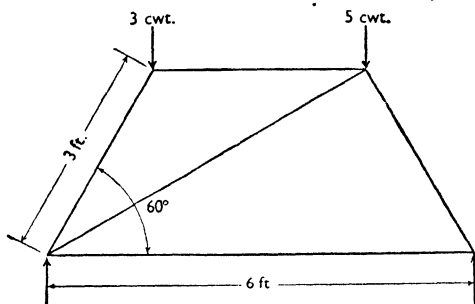


Fig. 19.15

5. The structure ABC is supported at the points A and C. The reaction at A is vertical, and the reaction at C is oblique. Find the

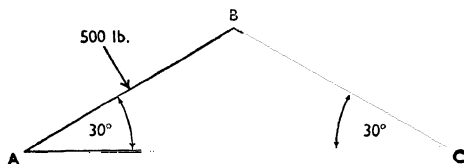


Fig. 19.16

magnitudes of the reactions at A and C, and the inclination of the reaction at C to the vertical. The 500-lb. force acts at the centre point of AB, and at right angles to AB. (S.W.E.T.C.)

6. The pin-jointed structure shown is simply supported vertically at A and C and carries a load of 4000 lb. at B. Determine the forces

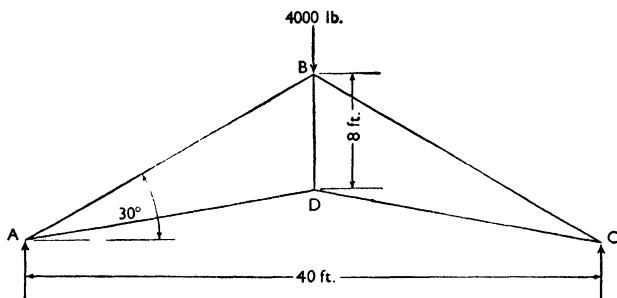


Fig. 19.17

in all the members, stating whether tension or compression for each one, and tabulate the results. (C.P.E.)

7. The forces P and W act on the bell-crank lever as shown in the diagram. If P is 28 lb., what is W ?

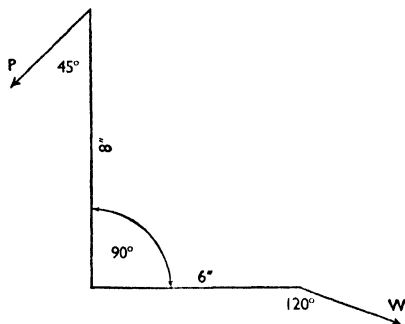


Fig. 19.18

What is the magnitude and direction of the reaction at the fulcrum? (S.W.E.T.C.)

8. A ladder 40 ft. long is resting with the lower end on horizontal rough ground and 15 ft. away from a smooth vertical wall against which the upper end is resting. The combined weight of a man and the ladder is W and the centre of gravity is 30 ft. along the ladder measured from the lower end. Determine, in terms of W , the horizontal friction force acting at the lower end of the ladder. (C.P.E.)

9. Two points A and B, 10 ft. apart, lie in the same plane as a force of 12 lb. which has a counter-clockwise moment of 20 lb.-ft. about A and a clockwise moment of 30 lb.-ft. about B. Find the inclination of the line of action of the force to AB and determine the position of the point C where the line of action of the force cuts AB.

(C.P.E.)

10. The jib of a crane, hinged at one end, is being raised into position by means of a rope attached to the free end and in the same vertical plane as the jib. The jib is 50 ft. long, weighs 4 tons, and its centre of gravity is 20 ft. from the hinged end. When the jib is inclined at 45° to the ground, the rope is inclined at 60° to the centre line of the jib. Find, by graphical means, the pull in the rope and the reaction at the hinge in magnitude and direction.

(U.E.I.)

11. A uniform bar AB is 30 in. long and weighs 100 lb. It is suspended by two cords AC and BD from points C and D in the same horizontal line, so that A and B are 12 in. and 6 in. respectively below the horizontal line through CD. The inclination of the cord AC is 45° , i.e. the angle DCA is 45° . Find, using a graphical solution, the inclination of the cord BD to the horizontal, and the pulls in both cords.

(U.E.I.)

12. A pole AB, 30 ft. long, is hinged at A, and a chain attached to B keeps the end B 20 ft. above the level of A. The chain at B is at right angles to the pole. Find the pull in the chain and the magnitude and direction of the reaction of the hinge at A when a load of 1 ton is suspended from the middle of the pole.

(U.L.C.I.)

13. Find the forces, and the nature of the forces, in the members of the framed structure shown.

(U.L.C.I.)

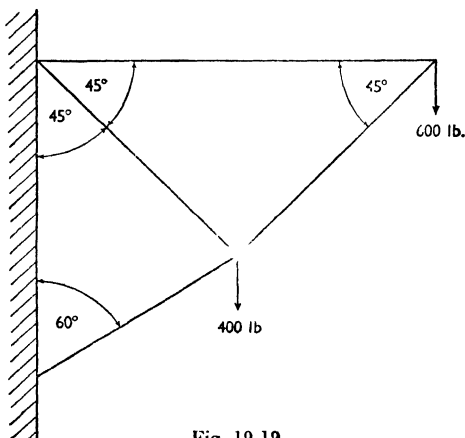


Fig. 19.19

14. A ladder rests with one end on rough horizontal ground, the other end resting against a smooth vertical wall. Show by means of a diagram the forces acting on the ladder. Denote carefully the directions of the forces and describe the forces, e.g. weight, etc.

If the length of the ladder is 60 ft., and its centre of gravity is 25 ft. from the end resting on the ground, find by means of a scale drawing the reaction between the ladder and the wall when the foot of the ladder is 30 ft. from the wall. The weight of the ladder is 100 lb.

(U.E.I.)

15. A uniform bar AB weighing 100 lb. is inclined at 15° to the horizontal plane, and is supported by cords AC and BD. The cord BD is vertical, and AC is inclined at 30° to the vertical, and the angles DBA and CAB are each 105° . The bar is held in this position by a horizontal force P applied at B, P being in the same vertical plane as the bar and the cords. Find the value of the force P . (C.P.E.)

16. A uniform rod AB, 4 ft. long and weighing 10 lb., is inclined at 60° to a smooth vertical wall. The end A is in contact with the wall and B is below the level of A. The rod is maintained in this position by a cord BC, one end attached to B and the other end connected to a point C on the wall vertically above A. Find the length of the cord BC and the tension in it. (C.P.E.)

17. Light pin-jointed rods AB, BC and CA form a triangular frame ABC with sides 6, 10 and 8 in. long respectively. The frame is suspended from A and weights are attached at B and C such that BC is horizontal. If the larger weight is 20 lb., find the magnitude of the smaller weight and determine the forces in the rods.

(C.P.E.)

18. What are the conditions of equilibrium of three forces acting in a plane and meeting at a point? A uniform ladder weighing 100 lb. rests with its upper end against a smooth vertical wall and makes an angle of 30° to the vertical. Find the direction and magnitude of the force exerted on the foot of the ladder by the ground.

(C.P.E.)

CHAPTER XX

Beams and Bending

1. Beams

Any part of a structure which is acted upon by forces at right angles to its length is called a **beam**. This includes cases where the load is applied obliquely (as in fig. 20.1); the normal component of the load then constitutes the force acting at right angles to the beam.

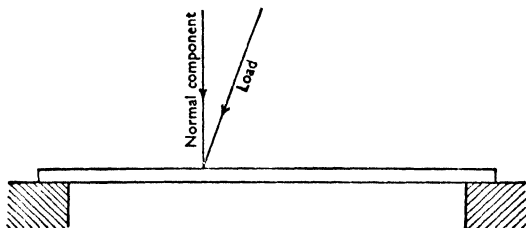


Fig. 20.1.—Loaded beam

The section of a beam may vary from a simple rectangle $\square$, as in the case of timber beams, to various shaped sections such as I, L, T, H, made in steel or other metals, or to various forms of built-up girder.

It should be realized that, since the load is applied at right angles to, and not along the length of, the member, the intensity of stress in any section of a loaded beam is proportional to the *moment* of its load (or loads) about that section, and not to the load, as is the case with ties and struts.

2. Types of beams

Three types of beams are shown in fig. 20.2, together with some indication of the type of deflection produced when under load. Both the cantilever and the built-in beam are firmly fixed

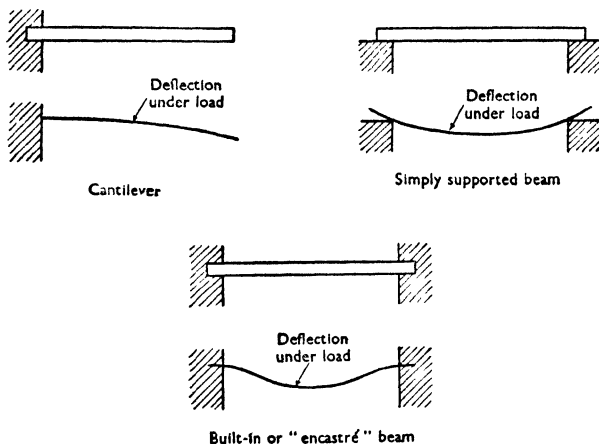


Fig. 20.2.—Types of beam

into the wall, the cantilever being fixed at one end only. The ends of the simply supported beam are not restrained, and tend to curl upwards under load. The built-in beam will not be dealt with in this chapter.

3. Methods of loading

The external load on a beam may act continuously, when it forms a **dead load**, or it may act intermittently, as in the case of a moving vehicle, when it forms a **moving or live load**.

Loads may be concentrated at a point on the beam (or may be regarded as such if their contact area with the beam is comparatively small), or they may be **uniformly distributed** (as in the case of a floor weighing P lb. per sq. ft.).

Often in practice the load on a beam may consist of several different types.

For a beam to be in a state of equilibrium, it should be realized that the external loads and their supporting forces must be equal and opposite, and the moments of these loads and forces must balance (i.e. the vector sum of the forces, and the algebraic sum of the moments must both be zero, see Chap. XIX, § 6).

4. Loaded cantilever. Bending moment and shearing force

Consider a cantilever loaded with a concentrated load W at its free end (fig. 20.3). Deflection would occur as already indicated. If failure were to occur at any section XX (owing, perhaps, to a sawcut at this point), it would be the result of two distinct actions:

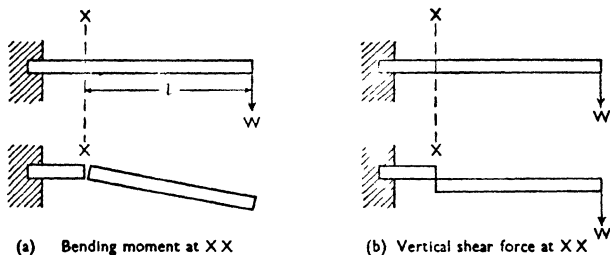


Fig. 20.3

1. The load W acting at a distance l from the section XX producing a moment Wl which the material at this section is unable to withstand.

2. The load W acting vertically downward across the section XX, causing the portion to the right of this section to slide vertically past the portion to the left, the material being unable to withstand this action.

It should be realized that both these actions occur simultaneously.

The moment Wl is termed the **bending moment (B.M.)** at the section XX, whilst the load W is termed the **shear (or shearing) force (S.F.)** at this section (see fig. 20.3a and b).

In order to prevent failure at any section such as XX, the

material and section of the beam must be strong enough to set up internally reacting forces which balance *independently* both the bending moment and shearing force at that section. This balancing moment and balancing force constitute the **moment of resistance** and the **shear resistance** respectively.

It will be seen that each particular section will have its own bending moment, depending on the distance of this section from the load, but the position of the load does not affect the shearing force, which will be of value W at any section of the cantilever.

If there is more than one load, then the bending moment at any section will be equal to the algebraic sum of the moments of the loads taken about one side of that section.

Similarly the shearing force at any section will be the algebraic sum of the forces on one side of that section.

Example 1.—Find the bending moment and shearing force at points A and B in the loaded cantilever (fig. 20.4).

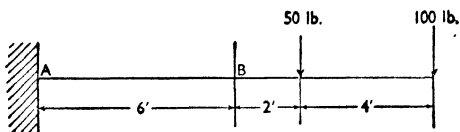


Fig. 20.4

As two loads act on the cantilever their total effect must be considered.

B.M. at A (take moments about A as fulcrum)

$$= 100 \times 12 + 50 \times 8 = 1600 \text{ lb.-ft.}$$

B.M. at B (take moments about B as fulcrum)

$$= 100 \times 6 + 50 \times 2 = 700 \text{ lb.-ft.}$$

$$\text{S.F. at A} = 100 + 50 = 150 \text{ lb.}$$

$$\text{S.F. at B} = 100 + 50 = 150 \text{ lb.}$$

(Note that the shearing force drops from 150 lb. to 100 lb. as we proceed to the right past the 50-lb. load point.)

5. Bending moment and shearing force of a simply supported beam

Take the case of a loaded beam supported at both ends as shown in fig. 20.5. It is required to find the bending moment and shearing force at section AB.

The reactions R and S must first be calculated by the principle of moments, and will be such that

$$R + S = W_1 + W_2.$$

To find the bending moment at section AB, proceed as follows: Take moments of R and W_1 about AB.

These are $R \times x$ (clockwise), and $W_1 \times (x - y)$ (anticlockwise).

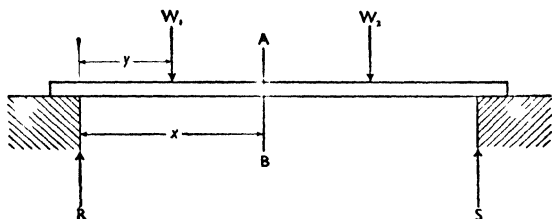


Fig. 20.5

As these moments act in opposite directions, the total bending moment at AB will be their difference (i.e. their algebraic sum), i.e.

$$\text{B.M. at AB} = \text{difference between } Rx \text{ and } W_1(x - y).$$

Note that moments have been taken on one side of the section only. It is immaterial which side.

The vertical shearing force, considered at section AB, will be due to W_1 acting downwards, and R acting upwards, i.e. it will be the algebraic sum of these forces. Hence

$$\text{Shearing force} = \text{difference between } R \text{ and } W_1.$$

(Note that forces have been considered on one side of the section only.)

By taking a numerical example we will see that it is immaterial which side of the section is considered.

Example 2.—Find the bending moment and shearing force at section AB of the beam shown in fig. 20.6.

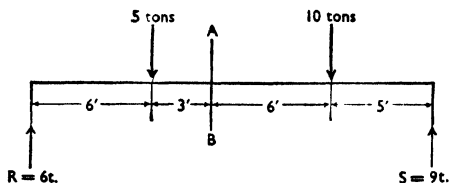


Fig. 20.6

$W_1 = 5$ tons, $W_2 = 10$ tons, $y = 6$ ft., $x = 9$ ft., span = 20 ft., W_2 is 5 ft. from S .

First find the reactions R and S by taking moments about R .

$$(5 \times 6) + (10 \times 15) = 20S,$$

$$\therefore 20S = 180, \quad \therefore S = 9 \text{ tons.}$$

$$R = 15 - 9 = 6 \text{ tons.}$$

Now taking forces to the left of the section (the right-hand side can conveniently be covered up to avoid confusion),

$$\text{B.M. about section AB is } 9R - (5 \times 3) = 54 - 15 = 39 \text{ ton-ft.}$$

(Clockwise moments have been considered positive.)

Now taking forces to the right of the section (the left-hand side being kept covered up),

$$\text{B.M. about section AB is } (10 \times 6) - 11S = 60 - 99 = -39 \text{ ton-ft.}$$

It is thus seen that the same numerical result is obtained, but the sign is opposite. This must be so because, as the beam is in equilibrium, the moments on one side must be equal and opposite to those on the other. The minus sign has no other significance than indicating opposition. If clockwise moments had been regarded as negative (as they sometimes are), the two results would have changed place.

S.F. at section AB (considering forces to left of section)

$$= 6 - 5 = 1 \text{ ton acting upward.}$$

S.F. at section AB (considering forces to right of section)

$$= 10 - 9 = 1 \text{ ton acting downward.}$$

The forces on either side of the section are thus equal and opposite, which must be so as the beam is in equilibrium.

6. Summary

The **bending moment** at any section of a beam is the algebraic sum of the moments of the external forces acting on any one side of the section.

The **moment of resistance** at any section of a beam is the moment provided by the internal resisting forces of the beam fibres tending to overcome bending; provided that the elastic limit of the material is not exceeded, it is equal and opposite to the bending moment.

The **shearing force** at any section of a beam is the algebraic sum of all the vertical forces acting on any one side of the section.

It is immaterial which side of the section is considered, the result will be the same.

With cantilevers, it is simpler to consider the forces which act on the free-end side of the section.

7. Sign convention

The sign convention to be adopted to distinguish between positive and negative bending moments, and between positive and negative shear, is a matter of some argument. However, it is largely immaterial, as one type of bending or shear must always be balanced by the opposite type in order to produce equilibrium.

One such convention is to regard all clockwise bending moments as positive. For shearing forces, when considering those to the right of a section, downward forces are regarded as positive, and upward forces as negative; when considering those to the left of a section, the reverse holds.

8. Bending-moment and shear-force diagrams

It is often convenient to show the bending moment and the shear force along a loaded beam by graphical construction.

Diagrams of bending moment and shear force for some common forms of loaded beam are given below.

1. *Cantilever loaded at its end* (fig. 20.7)

In this case the shearing force at any section of the beam, being the algebraic sum of forces on any side of the section, will be equal to W throughout the entire length, and the

shearing-force diagram, of width W to scale, can be drawn as shown.

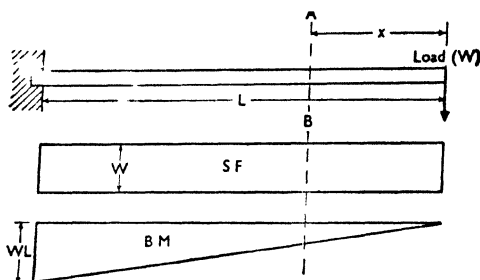


Fig. 20.7

The bending moment at any section will be equal to the load multiplied by the distance from that section, and will increase uniformly from zero at the end to a maximum of WL at the support.

The bending moment at section AB will be Wx .

2. *Cantilever with a uniformly distributed load of P lb. per ft. (fig. 20.8)*

Here the shearing force at any section will be equal to the weight acting on any one side of that section, and will increase

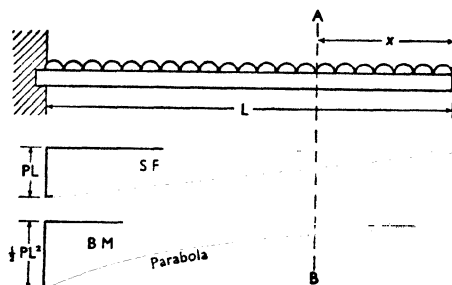


Fig. 20.8

uniformly from zero at the end to a maximum of PL at the support. The shearing force at section AB will be Px .

The bending moment at any section will be equal to the load

multiplied by the distance of its centre of gravity from that section. The bending moment at any section AB will be $Px \times \frac{1}{2}x = \frac{1}{2}Px^2$ (i.e. B.M. $\propto$ distance²). The bending-moment graph will thus be a parabola increasing from zero at the end to a maximum of $\frac{1}{2}PL^2$ at the support.

3. Beam supported at both ends. Centrally loaded (fig. 20.9)

In this case, as $R + S = W$, and $R = S$, then $R = S = \frac{1}{2}W$. The shearing force at any section of the beam will therefore be $\frac{1}{2}W$, but it will change sign at the centre.

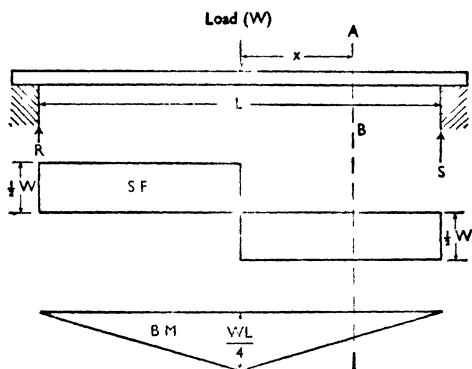


FIG. 20.9

The bending moment will increase from zero at R and S to a maximum value of $\frac{1}{2}W \times \frac{1}{2}L = \frac{1}{4}WL$ at the centre.

The bending moment at section AB will be

$$\frac{1}{2}W(\frac{1}{2}L - x), \text{ which also equals } S(\frac{1}{2}L - x).$$

4. Beam supported at both ends. Not centrally loaded (fig. 20.10)

If the load is not centrally located, but situated at a distance A from R , and B from S , then it will be shared by the supports in such a way that

$$R = \frac{WB}{L} \quad \text{and} \quad S = \frac{WA}{L}.$$

The shearing-force diagram will thus commence at value R , change sign at the load, and continue at value S .

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The bending-moment diagram will rise from zero at R to a maximum of $RA = \frac{WBA}{L}$ at the load.

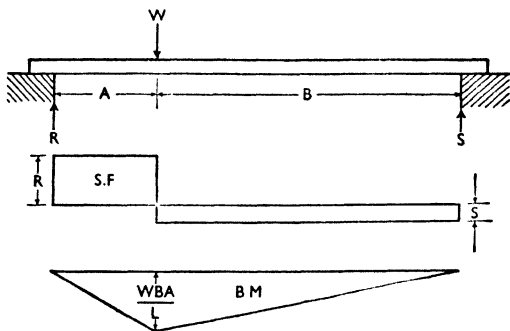


Fig. 20.10

5. *Beam supported at both ends. Uniformly distributed load of P lb. per ft. (fig. 20.11)*

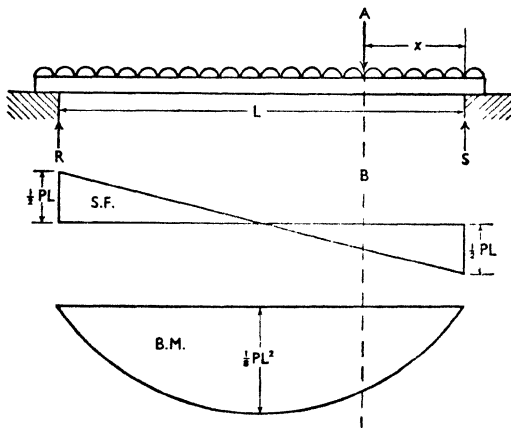


Fig. 20.11

As the load will be equally shared by the supports, then

$$R = S = \frac{1}{2}PL.$$

The shearing-force diagram will thus decrease from $\frac{1}{2}PL$ at the left-hand support to zero at the centre; there it will change sign, and increase to $\frac{1}{2}PL$ at the right-hand support.

The shear force at any section AB (distant x from support) will be

$$\begin{aligned}\text{Algebraic sum of forces on one side of section} \\ = \frac{1}{2}PL - Px = P(\frac{1}{2}L - x).\end{aligned}$$

When x is at the centre (and therefore $x = \frac{1}{2}L$),

$$\text{S.F.} = \frac{1}{2}PL - P \times \frac{1}{2}L = 0.$$

As shear force is proportional to L , the shear-force diagram will consequently be a straight line. It will pass through zero at the centre.

The bending moment at any section, such as at AB, will be (considering forces to right-hand side of section)

$$Px \times \frac{1}{2}x - Sx = \frac{1}{2}Px^2 - \frac{1}{2}PLx = \frac{1}{2}Px(x - L).$$

At the centre, where $x = \frac{1}{2}L$,

$$\text{B.M.} = \frac{1}{2}Px(x - L) = \frac{1}{2}P \times \frac{1}{2}L(\frac{1}{2}L - L) = \frac{1}{4}PL(-\frac{1}{2}L) = -\frac{1}{8}PL^2.$$

Consequently, as the bending moment is proportional to L^2 , the bending-moment diagram will be a parabola. It will rise from zero at the supports to a maximum value of $\frac{1}{8}PL^2$ at the centre.

Example 3.—A locomotive is stationary on a bridge of span 40 ft. with the leading wheels 10 ft. from the left-hand support. Commencing with the leading wheels, the loads are 4, 4, 10, 10, and 10 tons, and the distances between the wheels in the same order are 4, 6, 7, and 7 ft. Calculate the reactions at the supports and draw the shearing-force and bending-moment diagrams. State the values of the maximum S.F. and B.M., indicating where they occur.

(U.E.I.)

The arrangement is shown diagrammatically in fig. 20.12.

First find the reactions at the supports R and S .

Taking moments about R ,

$$4 \times 10 + 4 \times 14 + 10 \times 20 + 10 \times 27 + 10 \times 34 = 40S,$$

i.e.

$$906 = 40S.$$

$$\therefore S = 906 \div 40 = 22.65 \text{ tons}$$

$$R = (4 + 4 + 10 + 10 + 10) - 22.65 = 15.35 \text{ tons.}$$

B.M. and S.F. values for each load-point may be calculated and are tabulated opposite.

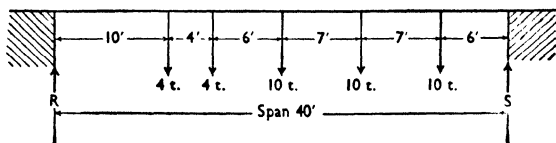


Fig. 20.12

The diagrams are then plotted from these values, and are shown in figs. 20.13 and 20.14.

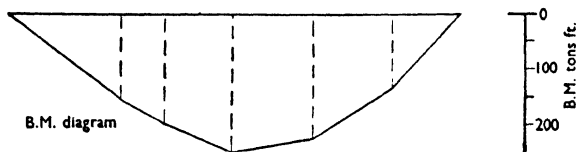


Fig. 20.13

Maximum B.M. is 243 ton-ft. occurring at mid-span.

Maximum S.F. is 22.65 tons occurring between rear wheels and right-hand support. (Note S.F. changes abruptly at each load.)

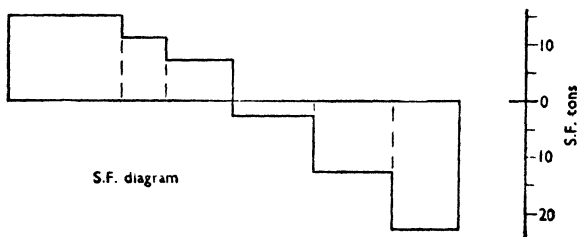


Fig. 20.14

9. Behaviour of beams of similar section

Knowing the behaviour of a beam under load, the behaviour of another beam of the same material, but with differing dimensions, can be estimated if its manner of loading and support is similar.

Load-point distance from R , ft.	B.M. calculation (R.H. side of load-point)	B.M., ton-ft.	S.F. calculation (R.H. side)	S.F., tons
0	$4 \times 10 + 4 \times 14 + 10 \times 20 + 10 \times 27 + 10 \times 34 - S \times 40$	0	$-S + 10 + 10 + 10 + 4 + 4 = R$	+15.35
10	$4 \times 4 + 10 \times 10 + 10 \times 17 + 10 \times 24 - S \times 30$	-153.5	$-S + 10 - 10 + 10 + 4$	+11.35
14	$10 \times 6 + 10 \times 13 + 10 \times 20 - S \times 26$	-198.9	$-S + 10 + 10 + 10$	+7.35
20	$10 \times 7 + 10 \times 14 - S \times 20$	-243	$-S + 10 + 10$	-2.65
27	$10 \times 7 - S \times 13$	-224.45	$-S + 10$	-12.65
34	$-S \times 6$	-135.9	$-S$	-22.65
40	0	0		

The **strength** of a beam, measured by the load it will carry, is directly proportional to the breadth and the square of its depth, and inversely proportional to its length.

It may be shown that the deflection produced in a beam is directly proportional to the bending moment. The **stiffness** of the beam is inversely proportional to the deflection. The deflection produced in a simply supported beam of rectangular section is directly proportional to the total load and the cube of its length, and inversely proportional to its breadth and the cube of its depth.

Example 4.—A simply supported beam of 3 ft. span, 1 in. depth and 1 in. breadth, breaks when a load applied at its centre reaches 960 lb. Estimate the load which must be applied at the centre of a similar beam of 4 ft. span, $1\frac{1}{2}$ in. breadth and 3 in. depth, in order to break it.

Expressing the strengths of the two beams in proportional form:

$$W_1 : W_2 = \frac{b_1 d_1^2}{l_1} : \frac{b_2 d_2^2}{l_2}.$$

$$\therefore 960 : W_2 = \frac{1 \times 1^2}{36} : \frac{1\frac{1}{2} \times 3^2}{48}.$$

$$\therefore \text{Breaking load } (W_2) = \frac{960 \times 3 \times 9 \times 36}{2 \times 48} = 9720 \text{ lb.}$$

Example 5.—A simply supported beam of 2 ft. span, 0.5 in. broad and 1 in. deep, deflects 0.2 in. when a load of 56 lb. is placed at its centre. Estimate the deflection produced in a beam of similar material of 8 ft. span, 4 in. breadth and 6 in. depth, under a centrally applied load of 3600 lb.

Expressing the deflections of the beams in proportional form:

$$\text{deflection (1) : deflection (2)} = \frac{W_1 l_1^3}{b_1 d_1^3} : \frac{W_2 l_2^3}{b_2 d_2^3}$$

$$0.2 : \text{deflection (2)} = 56 \times 2^3 \div 0.5 \times 1^3 : 3600 \times 8^3 \div 4 \times 6^3.$$

$$\therefore \text{Deflection produced} = \frac{0.2 \times 3600 \times 512}{56 \times 8 \times 2 \times 4 \times 216} = 0.476 \text{ in.}$$

EXERCISES ON CHAPTER XX

1. A horizontal beam AB, 27 ft. long, is simply supported at points C and E, distant 5 and 20 ft. respectively from A. A vertical concentrated load of 1 ton is applied at A, a vertical concentrated load of 2 tons at D, distant 15 ft. from A, and a uniformly distributed vertical load of $\frac{1}{4}$ ton per ft. is applied between D and B. Determine the reactions at the supports.

2. A girder, 50 ft. span, supports three side wheels of a six-wheel motor-lorry. The fore wheel is 10 ft. from the left-hand support, and the distances between the axles, in order, are 12 and 4 ft., and the loads transmitted to the girder are, in order, $4\frac{1}{4}$, $3\frac{1}{2}$, and 4 tons respectively. Calculate (a) the reactions at the supports, (b) the bending moment, and (c) the shearing force at a section 12 ft. from the left-hand support. (U.E.I.)

3. A beam is 20 ft. long and is supported at its ends. It carries loads of 200, 100, and 300 lb. at 5, 8, and 12 ft. respectively from the left-hand end. Find the magnitude of the maximum bending moment on the beam, and state where this occurs. (I.E.E. Grad.)

4. An electric motor weighing 4 tons is carried by two parallel beams which rest on supports 12 ft. apart. The centre line of the motor is at a distance of 8 ft. from the left-hand support. Assuming the load to be a concentrated load, draw curves of bending moment and shearing force for one of the loaded beams, each beam carrying the same load. (U.E.I.)

5. A beam of wood rests on supports 10 ft. apart. At a point 3 ft. from the left-hand support, pulley blocks are suspended in order to raise a machine weighing 2000 lb. The effort, applied vertically downwards, to raise the machine is 100 lb. Calculate the bending moment and shearing force on the beam at a section 4 ft. from the left-hand support, and draw to scale the curves of bending moment and shearing force. (U.E.I.)

6. A rod, 40 in. long, is supported at its ends, and carries loads of 10 and 8 lb. at distances of 12 and 30 in. respectively from the left-hand support. (a) Calculate the reactions at the supports, and the bending moment at each point of load. (b) Draw the bending-moment diagram, making use of the values obtained in (a). (Neglect the weight of the rod.) (U.E.I.)

7. A horizontal beam AB, 15 ft. long, is simply supported at the ends A and B and carries loads as follows: 10 tons 3 ft. from A, and 21 tons 10 ft. from A. The weight of the beam is 3 tons and may

be assumed to act midway between A and B. Calculate the reactions at the supports, and draw the shearing-force and bending-moment diagrams. State the values of maximum B.M. and S.F., indicating clearly where each occurs. (U.E.I.)

8. A beam 20 ft. long is supported at its ends. It carries loads of 100, 240, and 350 lb. at points 5, 12, and 16 ft. respectively from the left-hand support. Calculate the bending moments and shearing forces at the points of application of the loads, and draw bending-moment and shearing-force diagrams. What is the area of the shearing-force diagram from the left-hand support to the point of application of the 100-lb. load? What does this result suggest? (U.E.I.)

9. (a) Define shearing force and bending moment.

(b) Sketch the shearing-force and bending-moment diagrams for the following simply supported beam showing all relevant values and units. (S.W.E.T.C.)

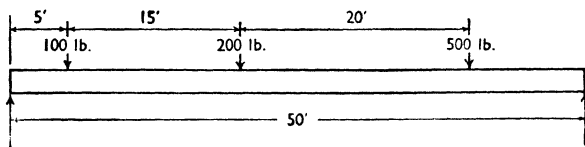


Fig. 20.15

10. (a) Explain the terms (i) bending moment, (ii) shear force.

(b) A beam is 20 ft. long and is simply supported at each end. It carries loads of 2 tons, 4 tons, and 6 tons, at points 5 ft., 10 ft., and 15 ft. respectively from the left-hand support. (i) Draw to scale a shear-force diagram. (ii) Draw to scale the bending-moment diagram. (S.W.E.T.C.)

11. A beam 20 ft. long is simply supported at each end. A load of 5 tons is applied at 6 ft. from the left-hand end, and a load of 10 tons applied at 12 ft. from the left-hand end. Draw the bending-moment diagram for the beam. State the value and position of the maximum bending moment. (S.W.E.T.C.)

12. State Hooke's law.

A beam 20 ft. long rests on supports at each end, and carries a load of $\frac{1}{2}$ ton per ft. run, and an additional load of $1\frac{1}{2}$ ton per ft. run for 12 ft. from the left-hand end. Draw the shearing-force and bending-moment diagrams, and find the magnitude of the maximum bending moment. (S.W.E.T.C.)

13. A cantilever projects 8 ft. from a wall and carries a load of 8 cwt. at 2 ft. from the free end and another load of 6 cwt. at 2 ft. from the wall. Draw the diagrams of bending moment and shearing force, and state the values of the bending moment and shearing force: (a) at the wall, (b) 3 ft. from the wall. (U.E.I.)

14. How does the deflection of a simply supported beam of rectangular section, loaded at its centre, depend upon its dimensions? A timber beam 1 in. wide, $1\frac{1}{2}$ in. deep rests on supports 3 ft. apart, and a central load of 1 cwt. produces a deflection of $\frac{1}{2}$ in. What would be the probable deflection of a beam of the same material 4 in. wide, 6 in. deep, and span 10 ft. under a central load of 20 cwt. (U.E.I.)

CHAPTER XXI

Kinematics

1. Motion

The study of movement, or motion, without reference to the forces causing it, or the masses moved, is known as **kinematics**.

A body occupying a fixed position for any length of time is said to be **at rest** during that period of time. When its position is varying it is said to be **in motion**. Its change of position is sometimes referred to as **displacement**.

The **speed** of a body may be defined as its **rate of change of position**, or, more simply, as its "rate of movement" through space. Obviously to specify "rate of movement" requires a knowledge of the time taken for the movement to occur. Therefore we may write:

$$\text{speed} = \frac{\text{distance moved}}{\text{time taken}} = \frac{\text{displacement}}{\text{time}}.$$

Any convenient units may be used; thus we can talk of speed in feet per second, miles per hour, kilometres per hour, etc.

Speed is said to be "uniform" or "constant" if equal distances are passed over in equal time intervals. Thus a train travelling at a "uniform speed of 60 m.p.h." would pass a quarter-mile post every 15 sec. Its speed could also be expressed as being

$$\frac{1}{4} \times 5280 \text{ ft.} \div 15 \text{ sec.} = 1320 \div 15 = 88 \text{ ft. per sec.}$$

It is useful in calculations to know that:

$$60 \text{ m.p.h.} = 88 \text{ ft. per sec.,}$$

$$30 \text{ m.p.h.} = 44 \text{ ft. per sec.,}$$

$$15 \text{ m.p.h.} = 22 \text{ ft. per sec.}$$

In actual practice, a train rarely maintains a constant speed for any appreciable length of time, owing to changes of gradient,

curves, signal checks, stops, etc. It is usual, however, to refer to the **average speed** of the train over a given distance. For instance, the "Coronation" of the L.N.E.R. did the journey from King's Cross to Edinburgh in 6 hours non-stop. Its average speed was therefore 65.5 m.p.h. for the 393-mile journey. To maintain this average speed, the train had to travel considerably faster over certain parts of the route. Speeds of over 100 m.p.h. were quite common over short distances in order to make up for the times when the speed was below the average, owing to compulsory speed checks, etc.

Velocity is the term used to signify speed in a certain direction. It is therefore a **vector quantity**. In everyday life the terms "speed" and "velocity" are often used indiscriminately, but the distinction should be remembered. Thus we should talk of a wind speed of 15 m.p.h., but a wind velocity of 15 m.p.h. S.E.

Uniform velocity is a velocity which does not vary, however small the interval of time taken.

If velocity is not uniform, then **acceleration** occurs whilst the velocity is increasing, and **retardation** whilst it is decreasing. Retardation is sometimes called negative acceleration or deceleration. **Acceleration** may be defined as the rate of change of velocity, i.e.

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time taken}}.$$

Strictly speaking it also is a vector quantity, and therefore the direction in which acceleration is occurring should be mentioned.

Acceleration may be either uniform or variable, but in a preliminary study we will only consider uniform acceleration. It may be expressed in any units of distance and time, e.g. if a body increases its speed at the rate of 2 ft. per sec. during every second, its acceleration is 2 ft. per sec. per sec. (usually abbreviated to 2 ft./sec.²).

Example 1.—A train travelling at 30 m.p.h. increases speed to 45 m.p.h. in 5 min. Calculate its acceleration in ft. per sec. per sec. (assuming it to be uniform).

Remember 30 m.p.h. = 44 ft./sec. and 45 m.p.h. = 66 ft./sec.

$$\begin{aligned}\text{Then } \text{acceleration} &= \frac{\text{change of velocity}}{\text{time taken}} = \frac{66 - 44}{5 \times 60} = \frac{22}{300} \\ &= 0.0733 \text{ ft./sec.}^2\end{aligned}$$

2. Acceleration due to gravity

An important case of acceleration is that due to gravity. A body dropped under the influence of gravity will increase its velocity at the rate of approximately 32.2 ft. per sec. every second, *provided that there is no other influence present to alter this increase*. This acceleration due to gravity is known as g , and it varies slightly at different places on the earth's surface, owing to variations in the distance from the earth's centre. Thus in London it is 32.18 ft./sec.², or 981 cm./sec.² In approximate calculations a value of 32 ft./sec.² is often used.

It will be seen that, in the absence of other influences, *all* bodies of any shape or weight will fall with the same acceleration (g). Owing, however, to air friction, wind, etc., having a greater effect on light bodies, a feather, for instance, will take longer to fall than a stone. Similarly, a streamlined bomb will fall with greater acceleration than a cylindrical one, owing to the smaller amount of air resistance encountered.

All bodies of whatever shape or weight will fall with the same acceleration in a vacuum.

Sometimes, as with aircraft, a test has to be carried out to determine behaviour at accelerations greater than g . A test at $4g$ might, for instance, be required.

3. Speed-time diagrams

The speed (or velocity) of a body at successive time intervals can be plotted on a graph. If time is plotted horizontally, then the graph is said to be plotted on a *time base*.

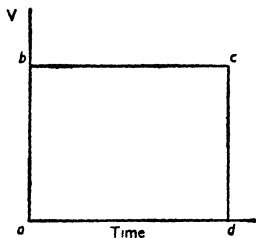


Fig. 21.1

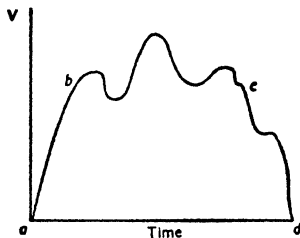


Fig. 21.2

Fig. 21.1 shows a graph for a body moving with uniform velocity, whereas fig. 21.2 is for a body moving with varying

velocity. In fig. 21.1, the velocity is uniform, so

$$\text{space } (s) \text{ covered in time } t = \text{velocity } (v) \times \text{time } (t),$$

i.e. the area under the curve *abcd* represents the distance (space) over which the body has moved. This is true for fig. 21.2 or any speed-time diagram.

In fig. 21.2, however, as the velocity is varying, its average value will be

$$\text{average velocity} = \frac{\text{area under curve } (s)}{\text{time taken } (t)}.$$

In fig. 21.3, the velocity is increasing uniformly (i.e. uniform acceleration). The average velocity can be found as before, or, alternatively, as *acceleration is uniform*,

$$\text{average velocity} = \frac{\text{initial velocity } (u) + \text{final velocity } (v)}{2}.$$

Thus, for uniform acceleration,

$$s = \frac{1}{2}(u + v)t.$$

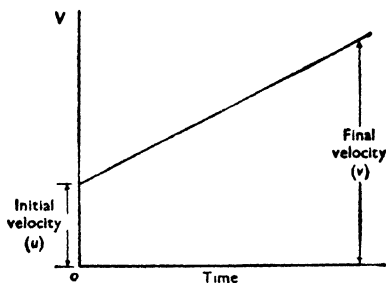


Fig. 21.3

It should be noted that the gradient (slope) of a velocity-time graph represents the acceleration at the point considered.

$$\text{Slope} = \frac{\text{change of velocity}}{\text{time taken}} = \text{acceleration}.$$

Example 2.—Find the distance travelled by the train in Example 1, whilst accelerating to 45 m.p.h.

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}(44 + 66) \times 5 \times 60 = 55 \times 300 = 16,500 \text{ ft.}$$

4. Equations for uniformly accelerated motion

Problems such as those already considered, and many others involving acceleration, etc., can be readily solved by remembering certain algebraic formulæ.

Let s = space passed over (ft.),
 u = initial velocity (ft. per sec.),
 v = final velocity (ft. per sec.),
 a = acceleration (ft./sec.²),
 t = time (sec.).

Then, if a body is accelerating uniformly,

its velocity after 1 sec. = $u + 1a$,
 „ „ „ 2 sec. = $u + 2a$,
 „ „ „ t sec. = $u + at$.

Therefore final velocity (v) after t sec. is

$$\underline{v = u + at.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As distance = average speed $\times$ time,

$$s = \frac{1}{2}(u + v) \times t; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and, as $v = u + at$, we may substitute for v in equation (2).

$$\begin{aligned} s &= \frac{1}{2}[u + (u + at)] \times t = \frac{1}{2}(2u + at) \times t \\ &= \frac{1}{2}(2ut + at^2) = ut + \frac{1}{2}at^2, \end{aligned}$$

$$\text{i.e.} \quad \underline{s = ut + \frac{1}{2}at^2.} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is apparent from fig. 21.4.

$$\begin{aligned} s &= \text{area under graph} \\ &= \text{area of rect. OABC} + \text{area of triangle ABD} \\ &= ut + \frac{1}{2}(at \times t) \\ &= ut + \frac{1}{2}at^2. \end{aligned}$$

A fourth equation is obtained as follows:

$$v = u + at.$$

* The symbol f is sometimes used for acceleration.

$$\begin{aligned}
 \text{Square both sides: } v^2 &= (u + at)^2 \\
 &= u^2 + 2uat + a^2t^2 \\
 &= u^2 + 2a\left(ut + \frac{1}{2}at^2\right).
 \end{aligned}$$

But equation (3) already states that $s = ut + \frac{1}{2}at^2$,

$$\therefore v^2 = u^2 + 2as. \quad \dots \dots (4)$$

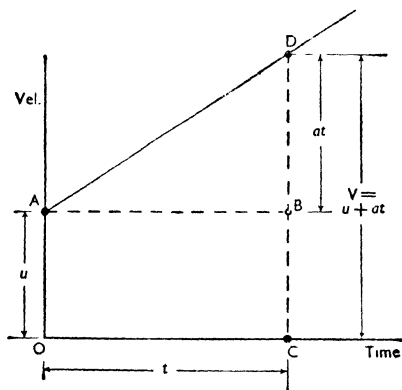


Fig. 21.4

These four equations should be carefully memorized.

When dealing with falling bodies, the above equations may be used with g substituted for a . (When bodies are projected vertically upwards $-g$ is used.) A number of examples of the use of acceleration formulæ follow.

Example 3.—A train accelerates uniformly from rest to 30 m.p.h. in 20 sec. Find its acceleration, and the distance covered during acceleration.

First lay out the information given as follows:

$$u = 0, \quad v = 44 \text{ ft./sec.}, \quad t = 20 \text{ sec.}$$

It is required to find a and s .

Select a formula containing the given quantities and that required.

Thus to find a , $v = u + at$.

Substituting given quantities

$$44 = 0 + a \times 20,$$

i.e.

$$44 = 20a$$

$$\text{and } a = 44 \div 20 = 2.2.$$

$$\therefore \text{acceleration} = 2.2 \text{ ft./sec.}^2$$

To find s ,

$$s = \frac{1}{2}(u + v)t.$$

$$\therefore s = \frac{1}{2}(0 + 44) \times 20 = 22 \times 20 = 440 \text{ ft.}$$

Example 4.—A shell is fired vertically upwards with a muzzle velocity of 2000 ft. per sec. Find the height to which it will rise (neglecting air resistance).

$u = 2000 \text{ ft./sec.}$, $v = 0$, $a = -g = -32.2 \text{ ft./sec.}$ s is required. Selecting $v^2 = u^2 + 2as$, this becomes $v^2 = u^2 - 2gs$,

$$\text{i.e.} \quad 0^2 = 2000^2 - 64.4s, \quad \therefore 64.4s = 2000^2.$$

$$\therefore \text{height } (s) = \frac{v^2}{2g} = \frac{4,000,000}{64.4} = 62,200 \text{ ft.}$$

N.B. The shell rises upwards *against* the acceleration due to gravity, which gradually reduces its velocity to zero. Hence $a = -g$, and the greatest height is reached when $v = 0$. The shell then begins to fall, and accelerates under the action of $+g$, reaching its original muzzle velocity (neglecting the effects of air friction) upon striking the ground.

As $v = u + at$, then the time of ascent will be equal to the time of descent.

Example 5.—(a) Find the time of ascent of the shell previously considered. (b) How long will it take to reach a height of 40,000 ft? (Take $g = 32 \text{ ft./sec.}^2$)

(a) $u = 2000 \text{ ft./sec.}$, $v = 0$, $a = -g$. t is required.

$$v = u - gt.$$

$$0 = 2000 - 32t,$$

$$\therefore t = 2000 \div 32 = 62.5 \text{ sec.}$$

(b) $u = 2000$, $s = 40,000$, $a = -g$. t is required.

$$s = ut - \frac{1}{2}gt^2.$$

$$40,000 = 2000t - \frac{1}{2} \times 32t^2,$$

i.e.

$$16t^2 - 2000t + 40,000 = 0.$$

Factorizing, $(4t - 400)(4t - 100) = 0$,

from which

$$t = 100, \text{ or } t = 25 \text{ sec.}$$

This means that the shell reaches a height of 40,000 ft. after 25 sec. *on its way up*, and after 100 sec. from firing *on its way down*.

N.B. When finding t from the equation $s = ut + \frac{1}{2}at^2$, which contains both t and t^2 , it will be noticed that a quadratic equation may have to be solved. This can be done, as above, by factorizing, if possible, or it may be necessary to use the general formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where the quadratic equation is of the form

$$ax^2 + bx + c = 0.$$

Example 6.—A bullet is dropped from a height of 6000 ft. Find (a) its velocity on striking the ground, and (b) the time taken in falling (neglecting air resistance).

If the bullet had been fired vertically downwards at 1600 ft./sec., find (c) its velocity on striking the ground, and (d) the time taken.

(a) $s = 6000$, $u = 0$, $a = g$. v is required.

$$v^2 = u^2 + 2gs = 0^2 + 2g \times 6000 = 386,400.$$

$$\therefore v = \sqrt{386,400} = 622 \text{ ft./sec.}$$

(b) t is required.

$$s = ut + \frac{1}{2}gt^2.$$

$$6000 = 0 + \frac{1}{2}gt^2 = 16 \cdot 1t^2.$$

$$\therefore t^2 = 6000 \div 16 \cdot 1 = 372 \cdot 7,$$

$$\therefore t = \sqrt{372 \cdot 7} = 19 \cdot 3 \text{ sec.}$$

(c) u is now 1600. v is required.

$$v^2 = u^2 + 2gs = 1600^2 + 386,400 = 2,946,400.$$

$$\therefore v = \sqrt{(2,946,400)} = 1716 \text{ ft./sec.}$$

(d) t is required. (Take g as 32.)

$$s = ut + \frac{1}{2}gt^2.$$

$$6000 = 1600t + 16t^2.$$

$$16t^2 + 1600t - 6000 = 0.$$

$$t^2 + 100t - 375 = 0.$$

$$\therefore t = \frac{1}{2}\{-100 \pm \sqrt{[10,000 - (4 \times 1 \times -375)]}\}$$

$$= \frac{1}{2}(-100 \pm \sqrt{11,500}) = \frac{1}{2}(-100 \pm 107 \cdot 2)$$

$$= -103 \cdot 6 \text{ sec. or } 3 \cdot 6 \text{ sec.}$$

The answer is 3·6 sec., as the negative value is obviously not required. It represents the time taken for the bullet to reach the

ground if fired vertically upwards. (Check that the time of ascent in this case is 50 sec. and the time of descent to its original level a further 50 sec. The velocity of the bullet will then be back to its original value of 1600 ft./sec.)

Example 7.—The brakes of a train are able to produce a retardation of 3.5 ft./sec./sec. If the train is travelling at 60 m.p.h., at what distance from a station should the brakes be applied if it is desired to stop at the station?

If the brakes are put on at half this distance, with what speed will the train pass the station? (S.W.E.T.C.)

$$u = 88 \text{ ft./sec.}, v = 0, a = -3.5 \text{ ft./sec}^2.$$

$$v = u - at.$$

$$0 = 88 - 3.5t.$$

$$\therefore t = 88 \div 3.5 = 25\frac{1}{7} \text{ sec.}$$

$$\text{and distance } (s) = \frac{1}{2}(u + v)t = 44 \times 25\frac{1}{7} = 1106\frac{2}{7} \text{ ft.}$$

(Alternatively, s can be found in one operation by using $v^2 = u^2 - 2as$.)

If brakes are applied at half this distance, then $s = 553\frac{1}{7}$ ft.

To find v (speed on passing station),

$$v^2 = u^2 - 2as.$$

$$v^2 = 88^2 - 7 \times 553\frac{1}{7} = 7744 - 3872 = 3872.$$

$$\therefore v = \sqrt{3872} = 63 \text{ ft./sec.}, \text{ or } \frac{63}{88} \times 60 = 43 \text{ m.p.h.}$$

5. Vector representation of velocities

As velocities are vector quantities, problems involving them can often be solved by drawing a vector diagram (see Chap. VIII).

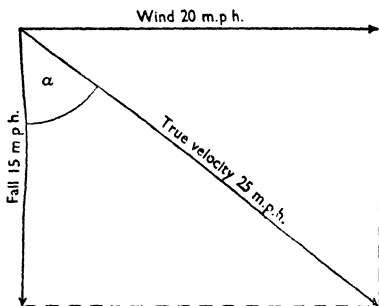


Fig. 21.5

Example 8.—A parachutist is falling at 15 m.p.h. and encounters a horizontal wind blowing at 20 m.p.h. Find the parachutist's true velocity and angle of descent.

Vectors are drawn representing the wind velocity and the parachutist's downward velocity (fig. 21.5).

The resultant true velocity is then found by measurement to be 25 m.p.h. The angle of descent (α) can also be found by measurement.

Alternatively: $(\text{True velocity})^2 = 20^2 + 15^2 = 400 + 225 = 625$.

$$\therefore \text{True velocity} = \sqrt{625} = 25 \text{ m.p.h.}$$

Also $\cos \alpha = 15 \div 25 = 0.6$, $\therefore \alpha = 53^\circ$ approx.

6. Relative velocity

A little consideration should be sufficient to show that all motion is relative to some body which is regarded as fixed. Normally we regard the earth as fixed, and then state velocities with respect to it. This, however, is not true, for the earth is rotating at 1 revolution per day (the speed of a body at the equator being about 1000 m.p.h.), and also it is moving in an orbit around the sun. We do not feel these velocities because they are constant, and we are only capable of feeling *changes* of velocity. (We would not particularly notice the speed of a fast, smooth-running vehicle if no windows were provided, but we would notice a sudden application of the brakes.)

It is sometimes necessary to determine the velocity of one body relative to some other moving body (other than the earth). The term **relative velocity** is used to describe the velocity of one body when viewed from another moving body. When a straight line joining two points changes in magnitude and/or direction, the points are said to have relative velocity.

Consider two trains on parallel tracks.

Train A is going at 60 m.p.h.

Train B „ „ „ 50 m.p.h.

If the trains are going in opposite directions (fig. 21.6a), the relative velocity, as seen by passengers on the trains, will be the sum of the two velocities, i.e. $60 + 50 = 110$ m.p.h. They will flash past each other at this speed. If, however, the trains are going in the same direction (fig. b), a passenger on

train A will see his train overtake train B with a velocity of $60 - 50 = 10$ m.p.h. A passenger in train B (the slower train) can get the impression that he is travelling backward at 10 m.p.h. if he gazes at the faster train.

The relative velocity of A to B will be 10 m.p.h.

“ “ “ „ B to A „ „ -10 m.p.h.

Consequently, if two bodies are moving in the same straight line, their relative velocities with respect to each other will be the algebraic *difference* of the two velocities.

Thus, in fig. 21.6a,

relative velocity = $60 - (-50) = 60 + 50 = 110$ m.p.h.,

and in fig. b,

relative velocity = $60 - 50 = 10$ m.p.h.

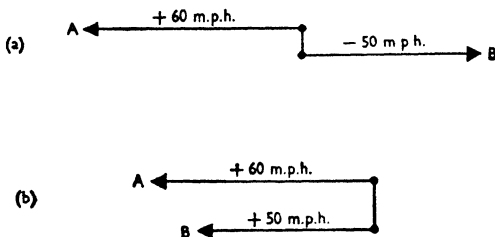


Fig. 21.6

Example 9.—A patrol boat of speed 20 knots sets out to catch a merchant ship of speed 12 knots. Initially the ships are 96 nautical miles apart. Determine how long it will be before the ships are level; also how many nautical miles each has travelled during the chase.

Relative velocity = $20 - 12 = 8$ knots.

Time taken = distance $\div$ relative velocity = $\frac{96}{8} = 12$ hours.

The patrol boat will have travelled 12 hours at 20 knots = 240 nautical miles.

The merchant ship will have travelled 12 hours at 12 knots = 144 nautical miles.

7. Relative velocities where the bodies are not moving in the same straight line

When the two velocities are inclined to each other, their relative velocity is found by taking their *vector* difference.

Vector subtraction can be carried out by drawing the vector diagram, reversing one vector, and then adding (i.e. finding the resultant).

Therefore to obtain the relative velocity of a body A with respect to a body B, reverse the velocity vector of B (i.e. in effect, bring it to rest), and then add this vectorially to the velocity vector of A.

Example 10.—Two aircraft cross each other's path at 45° . Fighter A is travelling at 350 m.p.h., and bomber B at 300 m.p.h. Find the velocity at which A's range is altering when viewed from B (i.e. find the relative velocity of A to B).

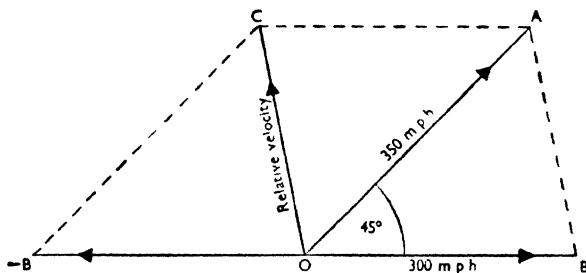


Fig. 21.7

We have to determine the direction in which A will lie when viewed by an observer on B.

Draw vectors OA and OB representing 350 and 300 m.p.h. respectively. The triangle may be completed by drawing BA, but it is better to draw a parallelogram, obtaining the vector difference as follows. (Refer to fig. 21.7.)

Reverse vector OB, so in effect bringing aircraft B to rest (an observer in it would be *relatively* at rest and would not notice his motion).

Now add vectors OA and OB reversed (i.e. find their resultant). This will give the vector *difference* of OA and OB.

The resultant OC now represents the relative velocity of A with respect to B.

By measurement: $OC = 250$ m.p.h. (approx.) at an angle of 103° to port of B. (A gun fired in this direction from B would, in general, hit A.)

Example 11.—A steamship is steaming due north at 11 knots, and a wind of 8 knots is blowing towards a point 30° E. of N. Find the appearance (direction and speed) of the smoke trail from the funnel when viewed from the deck of the ship.

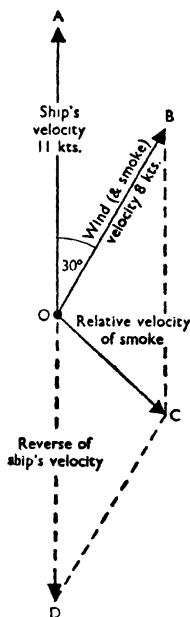


Fig. 21.8

The relative velocity of the smoke to the ship is required. To find it, draw OD representing the ship's velocity OA reversed (so in effect bringing the ship to rest, the apparent velocity relative to someone on its deck). Compound this with the wind velocity OB . The resultant OC will be the required result (fig. 21.8).

By measurement: $OC = 6$ knots at 46° S. of E.

Example 12.—The nozzle of a water wheel is set at an angle of 45° to the wheel. The water issues from the nozzle at a velocity of 160 ft./sec. The velocity of the wheel is 80 ft./sec. Find the relative velocity of the jet of water to the wheel.

Draw the vector diagram (see fig. 21.9).

Bring the wheel to rest by reversing its vector. Then add vectorially this reversed vector and the jet vector (i.e. find their resultant). This resultant is the required relative velocity.

By measurement: Relative velocity = 120 ft./sec. at 75° to line of motion of wheel.

N.B. In practice the water wheel vanes would be shaped at this angle, so that the water could enter without shock.

Example 13.—A ship A sailing due east at a speed of 12 m.p.h. passes a certain point at noon; a second ship, B, sailing north at the same speed passes the same point at 12.45 p.m.; at what time are they nearest together and how far are they then apart? (C.P.E.)

At 12 noon ship B will be $\frac{1}{2} \times 12 = 9$ miles due S. of ship A. Lay out the position of the ships at 12 noon to scale, showing their respective velocity vectors (fig. 21.10).

Find the relative velocity of the two ships by determining their vector difference.

$$\text{Relative velocity} = \overline{BR} - \overline{BP} = \overline{BQ}.$$

The shortest distance between point A and the line BQ represents to scale the distance between the ships when they are nearest to-

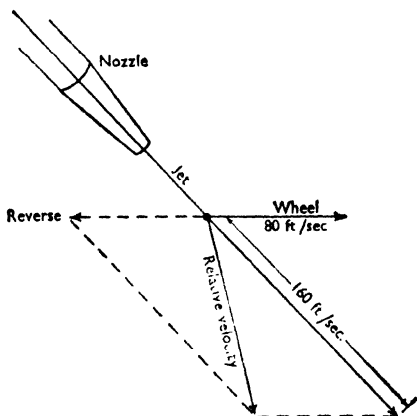


Fig. 21.9

gether. This shortest distance can be found by drawing a line AS through A and perpendicular to BQ, and measuring this to scale.

By measurement: $AS = 6.36$ miles.

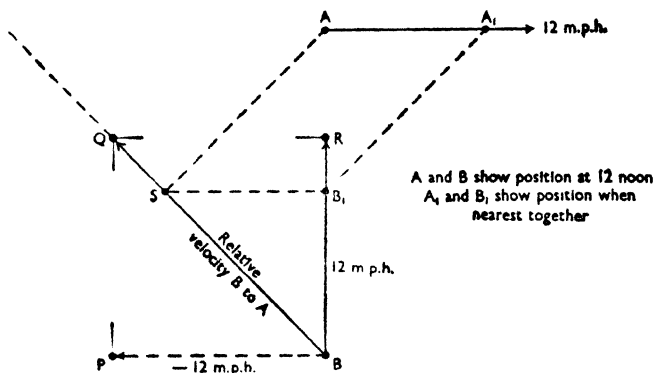


Fig. 21.10

The actual position of the ships when they are nearest together is represented to scale by points B₁ and A₁, point B₁ being obtained by drawing from S parallel to the direction of motion of A.

By measurement: distance $BB_1 = 4\frac{1}{2}$ miles.

$$\text{Time taken for this distance} = 4\frac{1}{2} \div \frac{12}{60} = 22.5 \text{ min.}$$

Therefore the ships are nearest together at $12.22\frac{1}{2}$ p.m., the distance between them being 6.36 miles.

8. Angular velocity

If a body moves in a circular path, its angular velocity is the rate of its angular movement about the centre of the circle. This angular velocity can be expressed in revolutions per minute (or per second), or in radians per minute (or per second).

Angular velocity, using radian measure, is often signified by the Greek letter omega (ω).

$$\text{Angular velocity } (\omega) = \frac{\text{angle traversed (radians)}}{\text{time taken } (t)},$$

$$\text{but} \quad \text{angle in radians} = \frac{\text{arc}}{\text{radius}},$$

$$\begin{aligned} \therefore \omega &= \frac{\text{length of arc traversed}}{\text{radius}} \div t \\ &= \frac{\text{length of arc traversed}}{t} \div \text{radius} = \frac{\text{linear velocity } (v)}{\text{radius } (r)}, \end{aligned}$$

$$\text{i.e.} \quad \omega = \frac{v}{r}.$$

Hence

$$v = \omega r.$$

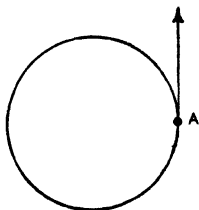


Fig. 21.11

A body revolving in a circle and describing equal angles in equal times (i.e. moving at constant speed) will be continually undergoing changes of velocity, owing to its direction being continually changing. To deal with such a variable velocity we can specify its velocity at a given instant of time, this instant being so short that neither factor has time to change. Thus in fig. 21.11 a body is traversing a circular path, but its velocity at instant A is depicted by the straight-line path it would take if the velocity remained uniform (this straight line will be at a tangent to the circle).

Example 14.—A locomotive driving wheel is rotating at 360 r.p.m. Find the linear velocity of its rim (in ft./sec.), and its angular velocity (in radians per second) if its diameter is 6 ft.

Linear speed of rim = $\pi \times (\text{diameter}) \times (\text{rev. per sec.})$

$$= \pi \times 6 \times \frac{360}{60} = 36\pi = 113.1 \text{ ft. per sec.}$$

$$\text{Angular velocity} = \frac{360 \times 2\pi}{60} = 12\pi = 37.7 \text{ radians per sec.,}$$

$$\text{or } \omega = \frac{\text{linear speed of periphery } (v)}{\text{radius } (r)} = \frac{113.1}{3} = 37.7 \text{ radians/sec.}$$

Example 15.—A locomotive travelling at 45 m.p.h. on a level track has driving wheels 6 ft. 6 in. in diameter. Assuming the wheels are not slipping, find:

(i) The angular velocity of the wheels.

(ii) The velocities, in magnitude and direction, of two points on the rim of a driving wheel at the opposite ends of a diameter which at that instant is parallel to the track. (C.P.E.)

(i) 45 m.p.h. = 66 ft./sec.

$$\text{Angular velocity } (\omega) = \frac{v}{r} = \frac{66}{3\frac{1}{2}} = 20.3 \text{ radians per sec.}$$

(ii) A point on the rim of a driving wheel has two velocities:

(a) That due to the forward motion of the locomotive (AC).

(b) That due to the rotation of the wheel (AE), the direction of this velocity always being at right angles to the radius AO.

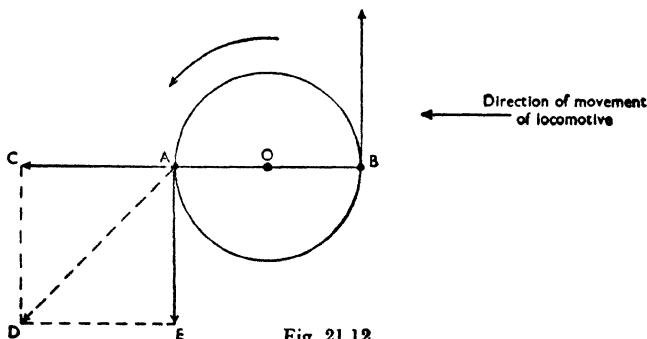


Fig. 21.12

The magnitudes of both velocities will be 66 ft./sec., and they will be disposed at 90° to each other at the instant mentioned in this particular problem (see fig. 21.12).

Consequently the resultant velocity over the ground of point A will be $AD = \sqrt{(66^2 + 66^2)} = 93.5 \text{ ft./sec.}$, and its direction will be pointing forwards and downwards at 45° to the horizontal. Similarly, the resultant velocity of point B will be of the same magnitude, but its direction will be inclined forwards and upwards at 45° to the horizontal.

9. Angular Acceleration

Problems involving angular motion and acceleration can be solved by means of the acceleration formulæ already dealt with, provided that care is taken to use the same units throughout.

Example 16.—An aeroplane propeller accelerates uniformly from rest, and reaches a speed of 3000 r.p.m. after 15 sec. Find its acceleration (a) in revolutions per second per second, and (b) in radians per second per second.

First lay out the information given and required.

(a) $u = 0$, $v = 3000 \text{ r.p.m.} = 50 \text{ rev. per sec.}$, $t = 15 \text{ sec.}$

It is required to find a .

Select an appropriate formula.

$$v = u + at.$$

$$50 = 0 + 15a.$$

$$\therefore 15a = 50,$$

$$\therefore a = 50 \div 15 = 3\frac{1}{3} \text{ rev./sec.}^2$$

(b) $u = 0$, $v = 50 \times 2\pi = 100\pi \text{ radn./sec.}$, $t = 15 \text{ sec.}$

$$v = u + at.$$

$$100\pi = 0 + 15a.$$

$$\therefore a = 100\pi \div 15 = 6\frac{2}{3}\pi = 20.95 \text{ radians/sec.}^2$$

Example 17.—In the previous example find: (a) the number of revolutions made whilst accelerating; (b) the final linear velocity of the propeller tip, if the propeller is 9 ft. in diameter; (c) the linear acceleration of the propeller tip.

(a) If velocities are in revolutions per second, then s will be in revolutions.

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}(0 + 50) \times 15 = 25 \times 15 = 375 \text{ rev.}$$

(b) As final angular velocity (ω) = 100π radians/sec., then final linear velocity of propeller tip = $v = \omega r = 314.2 \times 4\frac{1}{2} = 1413.9$ ft./sec.

(c) Linear acceleration of tip

$$= \frac{\text{change of } v}{t} = \frac{1413.9}{15} = 94.26 \text{ ft./sec.}^2$$

EXERCISES ON CHAPTER XXI

Section A

1. Give definitions of: (a) velocity; (b) speed; (c) displacement. Convert: (d) 185 m.p.h. to feet per second; (e) 440 yd. per min. to miles per hour.

2. A motor-car, starting from rest, moves with uniform acceleration, and after having travelled 200 yd., its speed is 36 m.p.h. What is the acceleration in ft./sec./sec., and what time in seconds has the car taken to travel the 200 yd.? (S.W.E.T.C.)

3. The speed of a train is reduced from 30 m.p.h. to 15 m.p.h. whilst it travels a distance of 120 yd. How much farther will it travel under the same retardation before it comes to rest? (S.W.E.T.C.)

4. Two stations A and B are half a mile apart. A train, which accelerates uniformly, passes A at 10 m.p.h., and one minute later passes B. At what speed in m.p.h. does it pass B, and what is its acceleration in ft./sec./sec.? (S.W.E.T.C.)

5. A train starts from a station A with an acceleration of 1 ft. per sec. per sec. which is maintained for 26 sec. The train runs at uniform velocity for 3 min., and then slows down to come to rest at station B with a uniform deceleration of 2 ft. per sec. per sec. What is the distance between station A and B? (S.W.E.T.C.)

✓6. Two telegraph poles A and B at the side of a railway track are 100 ft. apart. A train which is accelerating uniformly has a speed of 30 m.p.h. as it passes A. It takes 2 sec. to travel from A to B. What is its velocity in m.p.h. as it passes B, and what is its acceleration in ft./sec./sec.? (S.W.E.T.C.)

7. The second hand on a clock is $\frac{3}{4}$ in. long. What is the velocity of the tip of the second hand in m.p.h.? What distance in ft. is moved by the tip in one minute? (S.W.E.T.C.)

8. A pulley starting from rest and accelerating uniformly reaches a speed of 860 r.p.m. in 14 sec. What is its acceleration in rev./sec./sec., and how many revolutions has it made? (S.W.E.T.C.)

9. Two trains A and B are running in the same direction on parallel tracks. A is travelling at 20 m.p.h. and is 1 mile in front of B which

is travelling at 32 m.p.h. What time will elapse before they are running side by side? How far will B have travelled before they are running side by side? (S.W.E.T.C.)

10. If a body is projected vertically upward from the surface of the moon with a velocity of 40 ft. per sec., find the height to which it would rise if the force of gravity there is one-sixth of its value on the earth's surface.

11. A flywheel on an engine is 10 ft. in diameter and rotates at 100 r.p.m. What is the speed of the rim of the wheel in ft. per sec., and through what distance does a point on the rim move in one minute? (S.W.E.T.C.)

12. A ship steaming due north at 10 m.p.h. sights another ship 5 ml. away in a north-easterly direction steaming at 15 m.p.h. in a direction due west. How near will the two steamers approach one another? (S.W.E.T.C.)

Section B

13. (a) What is (i) velocity, (ii) acceleration?

(b) Establish the formula $s = ut + \frac{1}{2}at^2$, where s = distance travelled, u = initial velocity, a = acceleration, and t = time, for a body travelling in a straight line with uniform acceleration.

(c) A vehicle starts from rest with a uniform acceleration of 4 ft. per sec. per sec. for 20 yd. (i) What is the maximum speed attained? (ii) It comes to rest again under uniform retardation in 30 sec. What is the total distance covered? (S.W.E.T.C.)

14. A body is projected vertically upwards with an initial velocity of 1000 ft. per sec. (a) To what height will it rise? (b) How long will it take to reach the maximum height? (c) At what height will the body be when its velocity is 150 ft. per sec? $g = 32$ ft. per sec. per sec. (S.W.E.T.C.)

15. Power is transmitted from a motor shaft to a machine shaft by means of a belt drive. There is a 15-in. diameter pulley on the motor shaft, and a 24-in. diameter pulley on the machine shaft. (a) If the motor, starting from rest, reaches a speed of 360 r.p.m. in 30 sec., what is the linear acceleration of the belt in ft. per sec. per sec., assuming it to be uniform? (b) When the motor speed is 360 r.p.m., what is the machine shaft speed? (S.W.E.T.C.)

16. The velocity of a train is uniformly accelerated from 20 to 50 m.p.h. while travelling one mile. Draw a graph showing the velocity of the train in miles per hour at all intermediate places. What is the mean velocity and the time of describing the mile? What is the position of the train at the instant its velocity is 35 m.p.h.? (S.W.E.T.C.)

17. A train A, running on a straight track, passes a signal box at a uniform speed of 30 m.p.h. Ten seconds later a train B, on a parallel

line, passes the box at a speed of 36 m.p.h. and is accelerating at the rate of 0.5 ft. per sec. per sec. Find the time taken for each train and the distance from the signal box when the two trains are together. (C.P.E.)

18. A tram-car at a given instant has a speed of 12 m.p.h. It is given a uniform retardation of 2.5 ft./sec./sec. until its speed is reduced to 7 m.p.h. Determine the time taken in reducing the speed, and the distance in feet the car has travelled during retardation. (U.L.C.I.)

19. A tram-car, moving at 15 m.p.h., is brought to rest by the application of the brakes in a distance of 20 yd. on a level track. The diameter of the wheels is 2 ft. Find: (a) the average retardation of the car in feet and second units; (b) the time taken to come to rest from time of application of brakes; (c) the angular retardation of the wheels in radians per second per second. (U.E.I.)

20. Draw a time-displacement graph to illustrate the motion of a body in a straight line with uniform acceleration. A body starts from rest and travels in a straight line for 4 sec. with uniform acceleration of 10 cm./sec.², and continues to move with this velocity for a further 6 sec., after which it is brought to rest by a uniform acceleration in 5 sec. Find the total distance travelled either by graph or calculation. (C.P.E.)

21. What is meant by uniform acceleration? A ball is thrown vertically downwards with an initial velocity of 76 ft. per sec. from the top of a tower 120 ft. high. With what velocity does it strike the ground if its acceleration is 32 ft. per sec. per sec., and how long is it in the air? (C.P.E.)

22. A stone is dropped into a well and reaches the bottom with a velocity of 96 ft. per sec.; the sound of the splash on the water reaches the top of the well in $3\frac{2}{3}$ sec. from the instant at which the stone starts. Find the velocity of sound. (S.W.E.T.C.)

23. A body is projected vertically upwards with a velocity of 80 ft. per sec. Plot a velocity-time graph for the first 5 sec. of the motion and from it deduce (a) the time the body takes to reach its highest point, (b) the greatest height attained. (C.P.E.)

24. A vessel A, sailing in a south-easterly direction at a speed of 8 knots, sees another vessel B which always appears to be in a direction due east and to be going farther away. If the speed of B is 12 knots, find the direction in which B is sailing. (C.P.E.)

25. A car has a speed of 50 m.p.h. on a straight level road, and, from the instant the brakes are applied and the car brought to rest, the distance travelled is 180 yd. Assuming the retardation is uniform, find the time of action of the brakes.

If the time of action of the brakes is reduced by 25 per cent, find the percentage increase in the uniform retardation. (C.P.E.)

CHAPTER XXII

Kinetics

So far we have considered motion without reference to the forces involved.

The study of motion and the forces involved in moving bodies is called **kinetics**. We will now deal briefly with the fundamental ideas of kinetics, ideas which were first laid down by Sir Isaac Newton (1642–1727), and have since become known as Newton's Laws of Motion.

1. Momentum and inertia

A body in motion is said to possess a *quantity of motion* the magnitude of which depends upon the mass of the body and its velocity. The product of mass and velocity is termed **momentum**.

$$\text{Momentum} = \text{mass } (m) \times \text{velocity } (v) = mv.$$

Inertia is the property of sluggishness, or resistance to change of motion, possessed by all bodies. The amount of inertia possessed by a body may be measured in terms of its mass.

2. Newton's Laws of Motion

First Law

This law, sometimes referred to as the law of inertia, may be stated as follows:

“A body remains in a state of rest, or of uniform motion in a straight line, unless it is acted upon by external forces.”

This means that an inanimate body can do nothing of its own accord. If at rest, it remains at rest; if in motion, its velocity (speed and direction) remains constant, unless some external force is applied, i.e. its momentum (mv) remains constant.

It will be remembered that force was defined with reference to this law (see Chap. III).

“Force is that which changes, or tends to change, a state of rest, or of uniform motion in a straight line.”

Second Law

This law, sometimes referred to as the law of change of momentum, may be stated as follows:

“The rate of change of momentum is proportional to the force producing the change, this change taking place along the straight line in which the force acts.”

This law provides a method of measuring force. As the mass of a body is constant, then a change of momentum means a change of velocity (which constitutes acceleration). Therefore force may be measured by the acceleration it imparts to a given body.

Rate of change of momentum

Suppose a body of mass m changes its velocity from v_1 to v_2 in t seconds.

Its original momentum will be mv_1 ,

„ final „ „ mv_2 ,

„ change of „ „ $m(v_1 - v_2)$,

and its rate of change of momentum will be $\frac{m(v_1 - v_2)}{t}$,

But acceleration $(a) = \frac{\text{change of velocity}}{\text{time taken}} = \frac{v_1 - v_2}{t}$.

$\therefore$ rate of change of momentum = mass $(m) \times$ acceleration (a) .

But Newton's Second Law tells us that rate of change of momentum is proportional to the force producing it.

$\therefore$ Force $(F) \propto$ mass $(m) \times$ acceleration (a) ,

or $F = kma$ (where k is a constant).

This expression is known as the fundamental kinetic equation.

Third Law

This law, the law of reaction, may be stated as follows:

*"To every action there is always an equal and opposite reaction."**

Many examples of this law will have been noted. Forces always occur in pairs. Thus the weight (force) of someone sitting on a chair is opposed by an equal and opposite upward force. With bodies in motion, the same law is also true. Thus an engine pulling a train at constant speed on a level track exerts a tractive effort (pull) just equal to the backward pull due to tractive resistance. If a greater pull is exerted, acceleration occurs, this extra pull being counterbalanced by the backward pull due to the inertia of the train (its reluctance to change its state of motion).

3. Relations between force, mass and acceleration

The actual numerical relationship between the quantities expressed in the fundamental kinetic equation depends upon the system of units employed.

Two systems of units are in use, the **absolute** and the **gravitational**. Absolute units are always the same, irrespective of position in the universe. Gravitational units are not constant, as they depend upon the acceleration due to gravity (and therefore upon weight), which varies with position on the earth, may be non-existent in free space, and has a totally different value in the various planets and other heavenly bodies (see Chap. III).

4. Absolute systems of units

By choosing our units so that unit force causes unit acceleration upon unit mass, then force will be equal to mass multiplied by acceleration.

1. In the **metric absolute (C.G.S.) system of units**, the unit of force is the **dyne**. It is defined as that force which will impart an acceleration of one centimetre per second per second to a mass of one gram, i.e.

$$\text{Force (dynes)} = \text{mass (grams)} \times \text{acceleration (cm./sec.}^2\text{)}.$$

* "Force" and "reacting force" may be substituted for "action" and "reaction" if desired.

2. In the **metric absolute (M.K.S.) system of units**, the unit of force is the **newton**. It is defined as that force which will impart an acceleration of one metre per second per second to a mass of one kilogram, i.e.

$$\text{Force (newtons)} = \text{mass (kg.)} \times \text{acceleration (metres/sec.}^2\text{)}.$$

3. A similar absolute system making use of **F.P.S. units** can be used. In this system the unit of distance is the foot, the unit of mass is the pound, and the unit of time is the second. To produce unit acceleration (1 ft./sec.²) on unit mass (1 lb.) would require a unit of force (F) such that

$$F = \text{mass (1 lb.)} \times \text{acceleration (1 ft./sec.}^2\text{)}.$$

This unit of force is called the **poundal**.

Therefore, in the **British absolute (F.P.S.) system of units**, the unit of force is the **poundal**. It is defined as that force which will impart an acceleration of one foot per second per second to a mass of one pound, i.e.

$$\text{Force (poundals)} = \text{mass (lb.)} \times \text{acceleration (ft./sec.}^2\text{)}.$$

5. Gravitational systems of units

1. In ordinary engineering practice absolute units of force are seldom employed, although the first two mentioned above are of great importance in scientific work.

The engineer generally likes to employ as his unit of force that force which is exerted by gravity on one pound of matter, i.e. he uses as his unit "a force of one pound weight (lb. wt.)" —usually abbreviated to "a force of 1 lb.". Also, he seldom refers to mass, but prefers to speak of the weight of a body, which he also measures in pounds. To avoid confusion it is better to eliminate m from the fundamental kinetic equation.

Consider one pound of matter falling freely. The force acting on it will be its weight (W), and it will accelerate under the action of gravity at the rate of " g " ft./sec.², where $g = 32.2$ ft./sec.² approx.

Hence, from the fundamental kinetic equation

$$F = m \times a.$$

$$\text{In this case} \quad W = m \times g,$$

$$\text{from which} \quad m = \frac{W}{g}.$$

If this expression (W/g) is now inserted instead of m in the fundamental kinetic equation, we obtain

$$F = \frac{W}{g} \times a,$$

i.e. Force exerted (lb. wt.)

$$= \text{mass} \left(\frac{W \text{ (lb. wt.)}}{g \text{ (ft./sec.}^2\text{)}} \right) \times \text{acceleration (ft./sec.}^2\text{)}.$$

Thus m is replaced by W/g , and in this form the customary engineering units may be employed in the equation. The value [$W(\text{lb. wt.}) \div g$] is sometimes called the mass of a body in **engineers' units of mass** (or **slugs**), and the force of a one pound weight is sometimes called the **engineers' unit of force**. Consequently, in the **British gravitational system of units**, the unit of force is the **pound weight**. It may be defined as that force which will impart an acceleration of g ft. per sec. per sec. to a weight of one pound, or an acceleration of 1 ft. per sec. per sec. to one engineers' unit of mass or one **slug** (i.e. a weight of 32.2 lb.).

2. Similarly, in the **metric gravitational system of units**, the unit of force is the **gram weight**. It may be defined as that force which will impart an acceleration of g cm. per sec. per sec. to a weight of one gram.

6. Relationship between absolute and gravitational units

The value of g in F.P.S. units being 32.2 ft./sec.² (approx.), a force of 1 lb. wt. will be equal to 32.2 poundals.

The value of g in C.G.S. units being 981 cm./sec.² (approx.), a force of 1 gm. wt. will be equal to 981 dynes.

The value of g in M.K.S. units being 9.81 metres/sec.² (approx.), a force of 1 kg. wt. will be equal to 9.81 newtons. (N.B. 1 newton = 10⁵ dynes.)

As $1 \text{ newton} = \frac{1}{9.81} = 0.102 \text{ kg. wt. of force (approx.)}$, and
 $1 \text{ kg.} = 2.205 \text{ lb.}$, then

$$1 \text{ newton} = 0.102 \times 2.205 = 0.225 \text{ lb. wt. (approx.)}$$

40 newtons are thus equal to $40 \times 0.225 = 9 \text{ lb. wt. (approx.)}$,
 and $1 \text{ lb. wt.} = 40 \div 9 = 4.444 \text{ newtons (approx.)}$.

7. Examples involving Newton's Second Law

If in any problem gravitational (F.P.S.) units are used, then force will be in W pounds weight, and mass will be W/g lb.

If absolute (F.P.S.) units are used, then force will be $W \times g$ poundals, and mass will be W pounds.

Some worked examples showing both methods follow. (g has been taken as 32 ft./sec.² for convenience.)

Example 1.—A force of 6 lb. wt. acts on a mass of 48 lb. Find the acceleration produced.

$$F = m \times a.$$

$$(1) \text{ Using gravitational units: } F = 6, m = \frac{48}{g}.$$

$$\text{Then} \quad 6 = \frac{48}{g} \times a = \frac{48}{32} a = 1\frac{1}{2}a.$$

$$\therefore a = \frac{6}{1\frac{1}{2}} = 4 \text{ ft./sec.}^2$$

$$(2) \text{ Using absolute units: } F = 6 \times g, m = 48.$$

$$\text{Then} \quad 6 \times 32 = 48 \times a,$$

$$\therefore a = \frac{192}{48} = 4 \text{ ft./sec.}^2 \text{ as before.}$$

Example 2.—A force produces an acceleration of 16 ft. per sec. per sec. on a mass of 50 lb. Find its magnitude.

$$F = m \times a.$$

$$(1) \text{ Using gravitational units: } F \text{ will be in pounds,}$$

$$m = \frac{50}{g}, a = 16.$$

$$\text{Then} \quad F = \frac{50}{g} \times 16 = \frac{50}{32} \times 16 = 25 \text{ lb.}$$

$$(2) \text{ Using absolute units: } F \text{ will be in poundals, } m = 50, a = 16.$$

$$\text{Then} \quad F = 50 \times 16 = 800 \text{ poundals.} \quad \#$$

Taking 32 poundals = 1 lb. wt., then this is equivalent to $\frac{800}{32} = 25$ lb. as before.

Example 3.—A train of 300 tons weight starts from rest and reaches a speed of 30 m.p.h. after 1 min. If the frictional resistance of the track is 12 lb. per ton of the train's weight, find the tractive effort (force) developed by the engine.

Tractive effort is always expressed in pounds, so we can use gravitational units.

$$F = m \times a.$$

$$m = \frac{300 \times 2240}{g}, \text{ and } a = \frac{v - u}{t} = \frac{44 - 0}{60} \text{ (as 30 m.p.h.} = 44 \text{ ft./sec.)}.$$

$$\text{Then } F = \frac{300 \times 2240}{32} \times \frac{44}{60} = 15,400 \text{ lb.}$$

In addition to this the engine must develop a force to overcome rail friction of $12 \times 300 = 3600$ lb.

$$\therefore \text{ Tractive effort developed} = 15,400 + 3600 = 19,000 \text{ lb.}$$

Example 4.—A 10-ton truck increases speed from 15 to 30 m.p.h. in 40 sec. Find the average force acting on it during this period.

$$\text{Change of velocity} = 44 - 22 = 22 \text{ ft./sec.}$$

Change of momentum

$$= \text{mass} \times \text{change of velocity} = \frac{10 \times 2240}{32 \cdot 2} \times 22 = 15,300 \text{ units.}$$

$$\text{Average force} = \frac{15,300}{40} = 382 \cdot 5 \text{ lb.}$$

The following two examples involve metric units.

Example 5.—A mass of 12 kg. is suspended from a rope. Find the tension in this rope in dynes, and in newtons.

A body falling freely has a force exerted on it due to gravity of

$$F = m \times a.$$

This will be the tension set up in the rope when the body is suspended from it.

$$m = 12,000 \text{ gm., } a = g = 981 \text{ cm./sec.}^2$$

$$\text{Then } F = 12,000 \times 981 = 11,772,000 \text{ dynes,}$$

$$\text{or } 117 \cdot 72 \text{ newtons (} 10^6 \text{ dynes} = 1 \text{ newton).}$$

Example 6.—A force of 10 newtons acts on a mass of 2 kg. for 15 sec. Find the acceleration produced. Also find the velocity and

the distance covered after 15 sec., if the body starts from rest.

$$F = m \times a.$$

$$10 = 2a, \quad \therefore a = 5 \text{ metres/sec.}^2$$

Final velocity after 15 sec.,

$$v = u + at = 0 + 5 \times 15 = 75 \text{ metres/sec.}$$

Distance covered,

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}(0 + 75) \times 15 = 37.5 \times 15 = 562.5 \text{ metres.}$$

8. Impulse

A force acting on a body for a very brief duration of time produces what is known as an **impulse**, meaning a sudden knock or hammer blow.

The magnitude of this impulse depends upon the force and the time for which it acts. If the force is not constant, then the average force acting during the time should be taken, i.e. the **time average** of the force.

$$\text{Impulse} \propto \text{force } (F) \times \text{time } (t).$$

The fundamental kinetic equation is $F = m \times a$,

$$\therefore \text{impulse} = Ft = m \times a \times t.$$

But change of velocity $= a \times t$,

$$\therefore \text{impulse} = m \times (\text{change of velocity}) = m(u - v).$$

As $m(u - v) = \text{change of momentum}$, then

$$\text{impulse} = \text{change of momentum,}$$

i.e.

$$Ft = m(u - v).$$

When using this equation either gravitational or absolute units may be employed.

1. Using gravitational units,

$$Ft(\text{lb.-sec.}) = \frac{W(\text{lb. wt.})}{g} (u - v).$$

2. Using absolute units,

$$Ft(\text{poundals-sec.}) = m(\text{lb.}) (u - v).$$

It should be realized that this equation is really only another way of expressing the fundamental kinetic equation. Problems may be worked out by either method.

Example 7.—A pile-driver hammer weighs 500 lb. and falls from a height of 20 ft. Find the impulsive force of the blow delivered if the hammer is brought to rest in $\frac{1}{100}$ sec.

Method 1.—First find the velocity (v) immediately before impact.

$$u = 0, a = 32 \text{ ft./sec.}^2, s = 20 \text{ ft.}$$

$$v^2 = u^2 + 2as = 0^2 + 2 \times 32 \times 20 = 1280.$$

$$\therefore v = \sqrt{1280} = 35.8.$$

$$Ft = \frac{W}{g} (u - v) = \frac{500}{32} \times \sqrt{1280},$$

$$\therefore F = \frac{500}{32} \times \sqrt{(1280)} \div \frac{1}{100} = 55,900 \text{ lb.}$$

Method 2.—Using $F = m \times a$,

$$F = \frac{500}{32} \times a.$$

Deceleration produced when hammer is brought to rest

$$= \frac{\text{velocity at bottom of drop} - 0}{\text{time taken}}.$$

The velocity immediately before impact has already been found in Method 1 from $v^2 = u^2 + 2as$, and is $\sqrt{1280}$.

$$\text{Deceleration} = a = \frac{\sqrt{(1280)} - 0}{\frac{1}{100}}.$$

$$\therefore \text{Force of blow} = \frac{500}{32} \times \sqrt{1280} \div \frac{1}{100} = 55,900 \text{ lb.,}$$

which is the same result as before.

9. Conservation of momentum

As force is equal to rate of change of momentum, it follows from the Third Law of Motion that there is no loss of momentum in any action between two bodies. The momentum lost by one body is equal to that gained by the other. This constitutes the **Principle of Conservation of Momentum**, which may be stated as follows:

“In any system the sum of the momenta is constant.”

It should be noted that, as momentum is a vector quantity, the vector sum must be taken.

It follows that when two inelastic bodies collide, the total momentum after collision is equal to the sum of the momenta before collision, and is in the direction of the motion of the body with the greater momentum.

Example 8.—Two balls coming from opposite directions collide. They weigh 2 lb. and 3 lb. and have velocities of 12 and 5 ft. per sec. respectively. After colliding they roll along together. Determine their common velocity.

Before collision.

$$\text{Momentum of 2-lb. ball} = (2 \times 12) \div g = 24/g.$$

$$\text{Momentum of 3-lb. ball} = (3 \times 5) \div g = 15/g.$$

Combined momentum after collision $= (24 - 15) \div g = 9/g$ (the momenta being in opposite directions we must subtract).

$$\text{Combined mass after collision} = \frac{3 + 2}{g}.$$

Let v = common velocity.

$$\text{Then} \quad \left(\frac{3 + 2}{g} \right) v = \frac{9}{g},$$

and $v = \frac{9}{5} = 1.8$ ft. per sec. in the original direction of the 2-lb. ball.

Consider a projectile fired from a gun which is free to move. Since both are under the action of the same force for the same time, the momentum given to the projectile in one direction is equal and opposite to the momentum of the gun as it recoils.

The projectile, being of comparatively small mass (m_1), acquires a high velocity (v_1), whilst the gun of large mass (m_2) acquires a comparatively small velocity (v_2) in the opposite direction,

$$\therefore m_1 v_1 = -m_2 v_2.$$

Example 9.—A 15-in. shell weighs 1920 lb., and it is fired with a muzzle velocity of 2450 ft./sec. If the gun barrel weighs 98 tons, find its initial velocity of recoil on firing.

By the Third Law,

momentum of shell forward = momentum of gun backward

(because the force of explosion produces the same momentum in each).

$$m_1 v_1 = m_2 v_2.$$

$$1920 \times 2450 = 98 \times 2240 \times v.$$

where

$v_2 = \text{velocity of recoil.}$

$$\therefore v_2 = \frac{1920 \times 2450}{98 \times 2240} = 21.43 \text{ ft./sec.}$$

N.B. Mass can be expressed in lb. or (lb. $\div$ g). In the latter case g would cancel out.

In practice the recoil of a gun is checked by some form of recoil mechanism. In a simple case the carriage is fixed, whereas in other cases it is combined with a hydraulic compression cylinder. The action of recoil causes glycerine to be forced through a small opening in the piston; this produces a gradually increasing resistance to the recoil, and so stops it within a short distance.

N.B. A more accurate determination of the velocity of recoil would take into account the momentum of the weight of the charge. It will act forward (in gaseous form) at the instant of discharge, and its velocity may be estimated at half that of the projectile. Also, the total forward momentum is only equal to the backward momentum if the gun is free to move in *exactly* the opposite direction. If the gun was fixed to a wheeled carriage on horizontal ground, and the barrel was elevated, the vertical component of its momentum would be destroyed by impulsive pressure on the ground. In this case the horizontal component of the momentum of the shell, etc., forward would be equal to the horizontal momentum of the carriage backward.

10. Distinction between time average and space average of a force

When a force is not constant, an average value is often quoted for purposes of calculation.

This value may be either a "time average" or a "space average". Fig. 22.1 shows a graph of a variable force plotted against time. The average height of this curve will represent the time average of the force, i.e. the average value of F during time t . The area under the curve will represent $F \times t$, which is equal to "impulse", or change of momentum.

Fig. 22.2 shows a graph of a variable force plotted against distance. The average height of this curve will represent the space average of the force, i.e. the average value of F during the distance s . The area under the curve in this case represents

“force $\times$ distance”, which is equal to “work done”. This matter has been considered in Chap. XV.

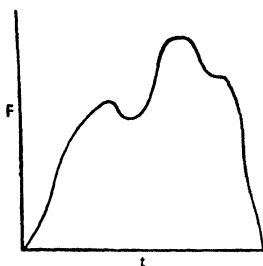


Fig. 22.1

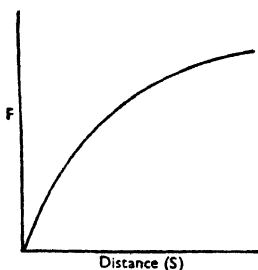


Fig. 22.2

Example 10.—If the recoil mechanism of the 15-in. gun previously considered produces a resistance which increases uniformly with time from zero to 120 tons, find how long the recoil will take, and how far the gun will travel backwards.

The time average of the force produced by the recoil mechanism will be $\frac{1}{2}(0 + 120) = 60$ tons (see fig. 22.3). This retarding force will be equal to mass of gun $\times$ deceleration produced, i.e.

$$F = m \times a.$$

$$60 \times 2240 = \frac{98 \times 2240}{32} \times a,$$

$$\therefore a = \frac{60 \times 2240 \times 32}{98 \times 2240} = \frac{960}{49} = 19.59 \text{ ft./sec.}^2$$

$v = u + at$ (or $v = u - at$ if the acceleration is negative), u = initial velocity of recoil as found in Example 9.

$$\begin{aligned} \therefore t &= \frac{v - u}{a} = (0 - 21\frac{1}{2}) \div \left(-\frac{960}{49}\right) \\ &= \frac{150}{7} \times \frac{49}{960} = \frac{35}{32} = 1\frac{3}{8} \text{ sec.} = \text{time of recoil.} \end{aligned}$$

Distance covered during recoil,

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}\left(\frac{150}{7} + 0\right) \times \frac{35}{32} = \frac{375}{32} = 11\frac{3}{8} = 11 \text{ ft. } 8\frac{1}{2} \text{ in.}$$

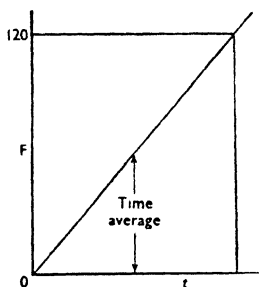


Fig. 22.3

11. Tension in supporting strings, etc.

When at rest, or in a state of uniform motion, the tension in a vertical string supporting a mass is due to the weight of this mass.

In any system where a mass is accelerated, the tension in the attached string, etc., will be the algebraic difference of the weight of the mass and the accelerating force.

Consider a mass of weight W attached to a string, but falling freely under the action of gravity.

The accelerating force will be

$$F = m \times a = \frac{W}{g} \times g = W.$$

$$\begin{aligned} \text{Tension in string} &= \text{weight of mass (acting down)} \\ &\quad - \text{accelerating force (acting down)} \\ &= W - W = 0. \end{aligned}$$

If, on the other hand, the weight is being accelerated upward, then

$$\begin{aligned} \text{tension in string} &= \text{wt. (acting down)} \\ &\quad - \text{accelerating force (acting up)} \\ &= \text{wt.} + \text{accelerating force.} \end{aligned}$$

It will thus be seen why a string which can be safely used to raise a load steadily may break under the action of a pull applied with rapid acceleration.

Example 11.—A lift cage weighs 17 cwt. Find the tension in the supporting rope when the cage is being (a) raised, and (b) lowered, with an acceleration of 4 ft./sec.²

(a) When being raised, the tension in the rope will exceed the weight of the cage by an amount equal to the force required to produce acceleration.

$$\text{Accelerating force} = \frac{17}{g} \times 4 = 2.106 \text{ cwt.}$$

$$\begin{aligned} \text{Tension in rope} &= \text{wt.} + \text{accelerating force} = 17 + 2.106 \\ &= 19.106 \text{ cwt.} \end{aligned}$$

(b) When being lowered, the tension in the rope will be less than the weight of the cage.

$$\begin{aligned} \text{Tension in rope} &= \text{wt.} - \text{accelerating force} = 17 - 2.106 \\ &= 14.894 \text{ cwt.} \end{aligned}$$

If two masses are attached to the ends of a light string, and are then suspended on either side of a pulley which is assumed to be weightless and frictionless, then the tension in this string will be the same throughout its length. The weight of any unbalanced mass will represent a force available for accelerating the total mass. As the masses are connected together, the acceleration produced will be the same in any part of the system. This acceleration can be calculated by means of the fundamental kinetic equation.

Example 12.—Two equal weights of 1 lb. are suspended at the ends of a cord which passes over a light frictionless pulley. An

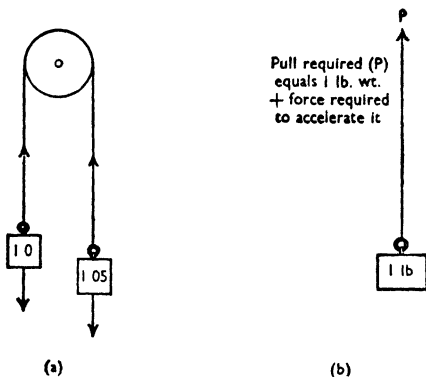


Fig. 22.4

additional weight of 0.05 lb. is attached to one of them to start the system in motion. Find (a) the pull in the cord, (b) the time for the weights to move 5 ft. (U.E.I.)

First find the acceleration produced.

$$\text{Accelerating force} = F = 0.05 \text{ lb.}$$

$$\text{Mass moved} = \frac{W}{g} = \frac{1 + 1 + 0.05}{g}.$$

$$F = m \times a.$$

$$0.05 = \frac{2.05}{g} \times a,$$

$$\therefore a = \frac{0.05 \times g}{2.05} = 0.7854 \text{ ft./sec.}^2$$

(a) One method of finding the pull in the cord is to consider the 1 lb. wt. being lifted and accelerated at a ft./sec.² (see fig. 22.4b).

$$\begin{aligned}\text{The total pull required} &= 1 \text{ lb.} + \left(\frac{1 \text{ lb.}}{g} \times 0.7854 \right) \\ &= 1 + 0.0244 = 1.0244 \text{ lb.}\end{aligned}$$

(b) To find t , given $s = 5$ ft. and $a = 0.7854$ ft./sec.²

$$s = ut + \frac{1}{2}at^2.$$

$$5 = 0 + \frac{1}{2} \times 0.7854t^2.$$

$$\therefore t^2 = \frac{5}{\frac{1}{2} \times 0.7854} = 12.72,$$

$$\therefore t = 3.57 \text{ sec.}$$

N.B. The arrangement shown in fig. 22.4 represents the essentials of Atwood's machine used in the laboratory for showing that $F \propto ma$, determining g , etc.

EXERCISES ON CHAPTER XXII

1. A light railway car weighs 6 tons and is capable of exerting a tractive effort of 580 lb. Find the acceleration obtainable on level track, if the tractive resistance is 15 lb. per ton of the car's weight. (Take g as 32.)

2. A force of 10 newtons acts on a body of mass of 2 kg. for 15 sec. If the body starts from rest, find (a) the acceleration produced, (b) its final velocity, and (c) the distance covered.

3. A train of weight 400 tons starts from rest on level track and 4 min. later reaches a speed of 60 m.p.h. after accelerating uniformly. If the tractive resistance is 20 lb. per ton of its weight, find (a) the tractive effort which the locomotive must develop, and (b) the horse-power at the end of this time.

4. State Newton's Second Law of Motion. Use it to explain the derivation of the unit of force in the F.P.S. system of units. Given that 1 lb. = 453.6 gm., how would you convert poundals in the above system of units into dynes in the C.G.S. system. (U.L.C.I.)

5. A motor-car weighs 24 cwt., of which two-thirds is carried by the rear wheels. The car is travelling on a level road at 35 m.p.h. when the brakes, which are fitted to the back wheels, are suddenly applied and lock the wheels. The coefficient of friction between the tyres and road is 0.65. Find (a) the momentum of the car at 35 m.p.h., (b) the time taken in bringing the car to rest. (U.L.C.I.)

6. Masses of 20 and 30 gm. are suspended at the ends of a cord which passes over a light frictionless pulley. Find (a) the acceleration produced, (b) the tension in the cord, and (c) the distance moved by the masses in 2 sec.

7. Define "acceleration". A load of 3 tons is raised by a chain and is given a starting acceleration of $\frac{1}{2}$ ft. per sec. per sec. Determine the initial pull in the chain.

If the acceleration be continued for 6 sec., find the final velocity and the distance travelled in this time. (U.L.C.I.)

8. Find the force required (in poundals and then in pounds weight) to give a mass of 15 pounds an acceleration of 45 feet per second per second. (S.W.E.T.C.)

9. During a laboratory experiment it was found that a body moved over distances of 3 in. and 5 in. respectively in two consecutive periods of $\frac{1}{10}$ sec. each. Find the acceleration of the body in ft. per sec. per sec. If the body weighed 15 lb., what would be the magnitude of the accelerating force? (U.L.C.I.)

10. A rope passes over a frictionless pulley. A weight of 75 lb. is attached to one end of the rope, and a weight of 105 lb. to the other. The 105-lb. weight is held stationary, then released. What vertical distance will it fall in 3 sec.? (S.W.E.T.C.)

11. Explain what is meant by the coefficient of friction. A mass weighing 20 lb. and resting on a horizontal table is connected by a string passing over a smooth pulley at the edge of the table to another mass weighing 3 lb. and hanging vertically. Initially the friction between the 20-lb. mass and the table is just sufficient to prevent motion. If an additional mass weighing 1 lb. is added to the 3-lb. mass, find the acceleration of the two masses. (C.P.E.)

12. A body which is being retarded at the rate of 4 ft./sec.² is observed to travel a distance of 40 ft. during a certain second. How long will it take to travel the next 40 ft.?

If the body weighs 25 lb., what is the force acting on it?

(C.P.E.)

13. Find the horse-power required to haul a train weighing 500 tons at 55 m.p.h. along a level track if the resistance to motion is 18 lb. per ton.

If the power is shut off and the brakes applied, what resistance, in lb. weight per ton, would bring the train to rest in 600 yd.?

(C.P.E.)

14. Describe a method for determining the coefficient of friction for two materials with surfaces in contact.

A block of wood slides along a horizontal table and its speed is reduced from 10 to 4 ft. per sec. whilst moving over a distance of 5 ft. Determine the coefficient of friction for the block and table.

(C.P.E.)

15. A force acting on a mass weighing 12 lb. produces a velocity of 8 ft. per sec. in 4 sec. starting from rest. If a force of the same magnitude acts on a mass weighing 2 lb. and initially at rest, how long will it take to move through a distance of 96 ft.? (C.P.E.)

16. An electric train starts from rest on a level track and reaches a speed of 30 m.p.h. in 25 sec. against frictional resistance of 16 lb. per ton wt. of train. Find the tractive force required per ton weight of train.

If the efficiency of the gearing and motors is 70 per cent, find the horse-power per ton weight of train that must be supplied to the motors when the speed of the train has just reached 30 m.p.h.

(C.P.E.)

17. An elevator of 5000 lb. wt. and initially at rest moves vertically downward with constant acceleration through a distance of 60 ft. in 8 sec. Find the tension in the cable attached to the elevator.

(C.P.E.)

18. The frictional resistance of a train on a horizontal track is 15 lb. wt. per ton. If the driving force on the train is 18 lb. wt. per ton, find the distance travelled on a horizontal track while the speed of the train increases from 30 to 45 m.p.h.

What is the horse-power per ton required to drive the train when the speed is 40 m.p.h.?

(C.P.E.)

CHAPTER XXIII

Energy

1. Forms of energy

Although energy is intangible, we may recognize it by its effects. It is present whenever a change of motion, or a transformation of matter, occurs. It has been defined as that agent which is capable of doing work. It follows that it can appear in many inter-related forms, such as mechanical, electrical, chemical, heat and atomic energy.

Thus a shell is fired from a gun. The chemical matter of the explosive charge is transformed suddenly into a different form, and heat energy is given out. The expansion due to this "explosion" performs work on the shell, which leaves the gun with considerable velocity. The energy possessed by the shell due to this velocity is termed **kinetic energy**. The shell gradually loses some of this energy and velocity (i.e. changes its motion) in overcoming the friction of the air, and after travelling for some miles eventually falls to earth. If, however, it hits a target en route, a very sudden change of motion occurs, and considerable energy is imparted to the target, sufficient to do the work of punching a hole through several inches of steel. Also, owing to the high speed at which this work is done, the steel becomes extremely hot, and may set fire to nearby inflammable articles.

For a more peaceful example of the different forms in which energy may occur, let us consider the production and use of electricity.

Electricity may be generated by burning fuel in some kind of heat engine, which converts the chemical energy hidden in the fuel into heat energy and then into mechanical energy. For instance, coal may be burned in a furnace, and its heat energy used to convert water into steam at high temperature and pressure inside a boiler. This steam is then allowed to expand to a lower temperature and pressure inside the cylinders of an engine, or through a turbine. In doing so it gives up some

heat and performs mechanical work, thereby producing rotation of a shaft. An electric generator coupled to this shaft converts this mechanical energy into electrical energy.

Alternatively, the electric generator may be driven by means of a water wheel or water turbine. Water, when at a height above some datum level, possesses **potential energy** due to this height and its weight. If allowed to fall to the datum level, this energy is imparted to the moving water (and is then termed kinetic energy), and this moving water can then be arranged to rotate a water wheel or turbine placed at that level.

The electrical energy, however produced, is passed along suitable cables to where it is required. It may then be passed through a resistance wire which becomes very hot. The electrical energy is thus converted into heat energy, and we are able to use it for heating buildings, cooking, etc. If passed through a filament lamp, heat is produced at such a high temperature that light (radiation energy) is given off. If passed through a gaseous discharge or a fluorescent lamp, comparatively little heat is given off, but the electrical energy is used in various ways to disturb the molecular structure of the matter in the lamp, and so to produce electromagnetic radiations of various kinds which give rise to light.

Our electrical energy from the generator may be used to drive an electric motor, in which case it is converted into mechanical energy. It may be passed through an electroplating vat, in which case it is converted into chemical energy, resulting in certain of the matter in the vat being deposited on the articles to be plated. It may be used to produce sound energy by means of a motor-driven siren. It may be used to produce electromagnetic radiation energy by means of a radio transmitter, this energy being used to convey speech, music, vision, or other information to a distance.

2. Mechanical energy

Mechanical energy may take the forms of potential and kinetic energy.

The *potential energy* possessed by a body due to its position is a measure of the amount of work it can do in falling to some datum level. Thus, a one-pound weight held at a distance of one foot above the datum level is capable of doing one foot-pound of work in falling to this level. (Notice that exactly

this amount of work is required in the first instance in order to raise the weight to its height of one foot.)

Therefore, in this case,

Potential energy available
(ft.-lb.)

$$= \text{weight of body} \times \text{height above datum level.}$$

$$(W \text{ lb.}) \qquad (h \text{ ft.})$$

In some cases potential energy is stored mechanically in a body by means other than gravity (e.g. the strain energy stored in a spring).

The *kinetic energy* possessed by a body due to its movement can be given up by bringing it to rest. If its motion is slowed down, part of its kinetic energy is given up. Owing to the Principle of Conservation of Energy, the total kinetic energy cannot exceed the original potential energy of the body, and will exactly equal this if frictional losses, etc., due to motion are disregarded. Thus

$$\text{Kinetic energy} = \text{potential energy} = Wh.$$

Example 1.—A lake, situated at a height of 400 ft. above sea-level, can supply 40 million gallons of water per day. It is proposed to use this to operate a water-turbine-driven electric generator situated at sea-level. Calculate the potential energy available daily in foot-pounds, and in horse-power-hours. If the efficiency of the water turbine is 80 per cent, and of the generator 90 per cent, determine the amount of electrical energy which can be generated daily. Also determine the kilowatt rating of the generator, assuming that the available water supply is all utilized in running the plant for 20 out of every 24 hours.

As 1 gal. of water weighs 10 lb.,

Potential energy available per day

$$= \text{weight of water available} \times \text{height above sea-level}$$

$$= 40,000,000 \times 10 \times 400$$

$$= 160,000 \text{ million ft.-lb.} = 16 \times 10^{10} \text{ ft.-lb.}$$

$$\text{or} \quad \frac{16 \times 10^{10}}{33,000 \times 60} = 80,800 \text{ h.p.-hours.}$$

As 1 h.p. = 0.746 kW,

$$80,800 \text{ h.p.-hours} = 0.746 \times 80,800 = 60,280 \text{ kWh.}$$

$$\begin{aligned} \text{Electrical energy available daily} &= 0.8 \times 0.9 \times 60,280 \\ &= 43,400 \text{ kilowatt-hours.} \end{aligned}$$

$$\text{Rating of generator} = 43,400 \div 20 = 2170 \text{ kW.}$$

3. Conversion of mechanical and electrical energy into heat energy

Heat energy is readily obtainable from mechanical energy when the latter is expended in overcoming friction of some form or other. Thus, bringing a vehicle to rest by braking results in heat being generated in the brake blocks, etc.; mechanical energy used to compress a gas results in the temperature of the gas being raised. The relationship between mechanical and heat energy is known as the **mechanical equivalent of heat**, or **Joule's equivalent**, after J. P. Joule who investigated the problem (see Chap. XXVII).

778 ft.-lb. of mechanical energy are needed to produce 1 British Thermal Unit (B.Th.U.) of heat energy,

or $1400 \text{ ft.-lb.} = 1 \text{ Centigrade Heat Unit (C.H.U.)}$

or $4.2 \text{ joules} = 1 \text{ gram-calorie} = 3.1 \text{ ft.-lb.}$

If electrical energy is passed through a suitable conducting medium, such as a resistance wire, this medium will become hot, i.e. the electrical energy becomes converted into heat energy.

1 kilowatt-hour of electrical energy is capable of producing 3412 B.Th.U. (see Chap. XXVII, § 8).

Example 2.—A 10-ton tram is brought to rest in 20 ft. by the application of a constant braking force of 500 lb. per ton. Find the amount of heat generated by the brake mechanism.

Energy given up = $F \times \text{distance} = (500 \times 10) \times 20 = 100,000 \text{ ft.-lb.}$

$$\text{Heat generated by brake} = \frac{100,000}{778} = 128.5 \text{ B.Th.U.}$$

4. Kinetic energy

The kinetic energy of a body is the capacity it has for doing work by virtue of its motion. It can be given out on changing this motion.

Consider a body of weight W lb. which falls from a height of h ft. (starting from rest). Its velocity at the bottom of its fall can be found from the formula

$$v^2 = u^2 + 2as,$$

where $u = 0$, $a = g = 32.2 \text{ ft./sec.}^2$, and $s = h \text{ ft.}$

Then $v^2 = 2gh$, and $v = \sqrt{2gh}$.

The distance fallen (h) will therefore be $v^2 \div (2g)$. But

kinetic energy (K.E.) = potential energy (P.E.) = $W \times h$,

$$\therefore \text{K.E.} = \frac{W \times v^2}{2g} \text{ ft.-lb.}$$

This amount of energy will be given out on hitting the ground.

Again, consider a body of mass m brought to rest in a distance s by a constant force F . Then the fundamental kinetic equation tells us that

$$F = m \times a.$$

But K.E. = work done on bringing a body to rest,

and work done = force acting $\times$ distance moved (s)

$$= (m \times a) \times s.$$

The distance moved = $s = \frac{1}{2}(u + v)t = \text{average velocity} \times t$.

As final velocity = 0, then $s = \frac{1}{2}vt$ (where v = original velocity), and

$$\text{K.E. given out} = (m \times a) \times \frac{1}{2}vt.$$

As $v = u + at$, then $a = v \div t$, and

$$\text{K.E.} = m \times \frac{v}{t} \times \frac{1}{2}v \times t = \frac{1}{2}mv^2.$$

If the body is not brought to rest, then

K.E. given out = $\frac{1}{2}m(v^2 - u^2)$ (where u = final velocity).

When using gravitational units, $\frac{W}{g}$ is substituted for m .

Example 3.—A 100-lb. bomb is dropped from a height of 144 ft. Determine its potential energy before dropping, the velocity with which it strikes the ground, and its kinetic energy on striking.

Using gravitational units,

P.E. = work done in lifting = $Wh = 100 \times 144 = 14,400 \text{ ft.-lb.}$

As $v^2 = u^2 + 2gs$, $u = 0$, and $s = h$,

$$v = \sqrt{(2gh)} = \sqrt{(2 \times 32.2 \times 144)} = 96.3 \text{ ft./sec.}$$

$$\text{K.E.} = \frac{Wv^2}{2g} = \frac{100 \times 64.4 \times 144}{64.4} = 14,400 \text{ ft.-lb.}$$

Hence P.E. lost = K.E. gained, also work done on striking = change of energy.

N.B. At any point during the bomb's fall it will possess both potential and kinetic energy. Their sum must still equal Wh , provided that no work has been done during the fall.

This can be stated in the form of what is called the energy equation:

$$\text{P.E.} + \text{K.E.} + \text{work done} = \text{a constant.}$$

Example 4.—A 15-in. shell weighs 1920 lb. and is fired with a muzzle velocity of 2450 ft. per sec. Find its K.E. on leaving the muzzle of the gun.

If it hits a fixed target at this velocity and is stopped after penetrating 27 in. of metal, calculate the average resistance set up in tons. Hence deduce the average value of the ultimate shear stress of the metal.

$$\text{K.E.} = \frac{Wv^2}{2g} = \frac{1920 \times 2450^2}{64 \cdot 4 \times 2240} = 79,910 \text{ ft.-tons.}$$

Work done on armour = average resistance overcome $\times$ distance moved,

$$\therefore \text{average resistance} = \frac{79,910 \text{ ft.-tons}}{2\frac{1}{4} \text{ ft.}} = 35,515 \text{ tons.}$$

$$\begin{aligned} \text{Area resisting shear} &= \text{perimeter of hole} \times \text{thickness} \\ &= 15\pi \times 27 = 1272 \text{ sq. in.} \end{aligned}$$

$$\therefore \text{Ultimate shear stress of armour} = \frac{35,515}{1272} = 27.9 \text{ tons/in.}^2$$

Example 5.—A mass of 10 kg. moving at 5 metres/sec. hits a body and has its speed reduced to 2 metres/sec. Find the K.E. given up to this body.

Using absolute units (M.K.S.),

$$\begin{aligned} \text{K.E. given up} &= \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 10(5^2 - 2^2) \\ &= 5(25 - 4) = 5 \times 21 = 105 \text{ joules.} \end{aligned}$$

Some problems, not necessarily requiring the calculation of kinetic energy as part of the answer, can be solved by making use of its principles.

Consider the following case:

Example 6.—Find the horse-power required to haul a train weighing 500 tons at 55 m.p.h. along a level track if the resistance to motion is 18 lb. per ton.

If the power is shut off and the brakes applied, what resistance, in lb. weight per ton, would bring the train to rest in 600 yd.?
(C.P.E.)

The first part of the question does not involve any new principles.

$$\text{Resistance to motion} = 18 \times 500 = 9000 \text{ lb.,}$$

$$55 \text{ m.p.h.} = \frac{55}{60} \times 88 = 80\frac{1}{3} \text{ ft./sec.,}$$

$$\text{h.p.} = \frac{9000 \times 80\frac{1}{3}}{550} = 1320.$$

The second part of the question may be solved from considerations of kinetic energy.

$$\text{K.E. of train} = \frac{Wv^2}{2g} = \frac{500 \times (80\frac{1}{3})^2}{64.4} = 50,400 \text{ ft.-tons.}$$

All this energy will be absorbed by the brakes and rail friction in bringing the train to rest in 600 yd.

$$\text{Work done} = \text{resistance overcome} \times \text{distance,}$$

$$\therefore \text{resistance} = \frac{\text{work done (ft.-tons)}}{1800 \text{ ft.}} = \frac{50,400}{1800} = 28 \text{ tons} = 62,720 \text{ lb.}$$

But tractive resistance is 9000 lb.,

$$\therefore \text{brake resistance} = 62,720 - 9000 = 53,720 \text{ lb.}$$

$$\text{or} \quad \frac{53,720}{500} = 107.44 \text{ lb. wt. per ton.}$$

$$\begin{aligned} \text{Total resistance} &= \text{brake resistance} + \text{rail resistance} \\ &= 107.44 + 18 = 125.44 \text{ lb. wt. per ton.} \end{aligned}$$

Alternatively, the second part of this question may be solved by making use of the fundamental kinetic equation, as outlined in Chap. XXII, the retardation produced, multiplied by the mass moved, giving the force (resistance) overcome.

5. Kinetic energy of rotation

Consider a particle of weight W rotating at a linear velocity v around a centre O at a radius r (fig. 23.1). Its kinetic energy will be

$$\text{K.E.} = \frac{Wv^2}{2g}.$$

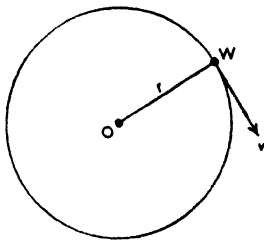


Fig. 23.1

As v = angular velocity (ω radians) $\times r$, then

$$\text{K.E.} = \frac{Wv^2}{2g} = \frac{W}{2g} \times (\omega r)^2 = \frac{1}{2}\omega^2 \times \frac{W}{g}r^2.$$

For any given weight and radius it will be seen that the factor $\frac{W}{g}r^2$ is a constant.

Now consider a body made up of several particles of weight W_1, W_2, W_3 , etc., which collectively total W , these particles being at radii r_1, r_2, r_3 , etc., from the centre. Then

Total K.E. = sum of K.E. of each particle

$$= \frac{1}{2}\omega^2 \left(\frac{W_1}{g}r_1^2 + \frac{W_2}{g}r_2^2 + \frac{W_3}{g}r_3^2 + \dots \right).$$

Here again, for a given body, the quantity in the brackets is a constant, and is known as the **moment of inertia** (I) of the body.

This moment of inertia can be re-written as $\frac{W}{g}k^2$, where k is that radius at which the whole of the rotating mass of the body can be regarded as being concentrated without affecting its moment of inertia. k is known as the **radius of gyration** of the body, and the moment of inertia may be considered as the capacity of a rotating body to possess momentum.

For a thin-rimmed body, such as a flywheel, the radius of gyration may be taken as being at the centre of the rim, but for other rotating bodies such as the rotors of turbines, and electrical machines, its position may be found by calculation or experiment.

K.E. of rotation = $\frac{1}{2}\omega^2 \times$ moment of inertia

$$= \frac{1}{2}\omega^2 \times \frac{W}{g}k^2 = \frac{1}{2}I\omega^2.$$

N.B. The above expressions reduce to the fundamental $\frac{1}{2}mv^2$, where v replaces ωk . Any of these expressions may be used as convenient, and may be deduced from the fundamental formulæ.

Example 7.—A flywheel of weight 112 lb. has a radius of gyration of 5 ft. Find its kinetic energy at 120 r.p.m.

$$\text{Angular velocity } (\omega) = \frac{120}{60} \times 2\pi = 4\pi \text{ radians/sec.}$$

$$\therefore \text{K.E.} = \frac{1}{2}\omega^2 \times \frac{W}{g} k^2 = \frac{1}{2} \times (4\pi)^2 \times \frac{112}{g} \times 5^2 = \frac{56 \times 400 \times \pi^2}{g},$$

$$\text{or K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{112}{g} \times \left(2\pi \times 5 \times \frac{120}{60}\right)^2 = \frac{56 \times 400 \times \pi^2}{g} \\ = 6870 \text{ ft.-lb.}$$

If the flywheel in this example had its speed reduced to 100 r.p.m., its kinetic energy at this speed would be less than before. The excess of kinetic energy would be given up in an attempt to maintain the original speed of rotation.

$$\text{K.E. at 100 r.p.m.} = \frac{56 \times (16.66\pi)^2}{g} = 4760 \text{ ft.-lb.}$$

$$\text{Excess of energy given up} = 6870 - 4760 = 2110 \text{ ft.-lb.}$$

Example 7 shows the reason for using a heavy flywheel on machines for which a constant speed of rotation is desirable. It acts as a reservoir of kinetic energy, taking in additional energy when the speed of rotation of the machine is above average, and giving it out when this speed falls below average. It is essentially a controller of short-period variations in angular velocity, such as those occurring during the period of, say, the four-stroke cycle of a single-cylinder internal-combustion engine. Here energy is given out to the flywheel by the engine only once in every four strokes, and, even during the one power-stroke, this energy is supplied far from smoothly.* With such a machine the stored energy in the flywheel maintains rotation and endeavours to smooth out the fluctuations in angular velocity.

A flywheel cannot prevent a gradual increase or decrease in angular velocity from building up over a long period, such as might be due to a variation in the fuel supply of an engine. In order to maintain an appreciably constant speed of rotation over a long period, it is customary to provide an engine with some form of centrifugal governor; it may be arranged to increase the fuel or steam supply when the speed falls, and to decrease it when the speed rises (see Chap. XXIV, § 7).

* Owing to variations in cylinder pressure and in leverage of the piston on the crank.

Example 8.—Weights of 3, 4 and 5 lb. are distributed at radii of 6, 12 and 18 in. respectively from the axis of rotation of a body. Determine its radius of gyration. What amount of kinetic energy will it store when rotated at 240 r.p.m.?

$$\frac{W}{g} k^2 = \frac{W_1}{g} r_1^2 + \frac{W_2}{g} r_2^2 + \frac{W_3}{g} r_3^2,$$

$$\therefore Wk^2 = W_1 r_1^2 + W_2 r_2^2 + W_3 r_3^2.$$

$$(3 + 4 + 5)k^2 = 3 \times \left(\frac{1}{2}\right)^2 + 4 \times 1^2 + 5 \times \left(1\frac{1}{2}\right)^2.$$

$$12k^2 = \left(3 \times \frac{1}{4}\right) + 4 + \left(5 \times 2\frac{1}{4}\right) = 16.$$

$$k^2 = \frac{16}{12} = 1\frac{1}{3}.$$

$$k = \sqrt{1.333} = 1.154 \text{ ft.}$$

$$\text{K.E.} = \frac{1}{2} \omega^2 \times \frac{W}{g} k^2 = \frac{1}{2} (8\pi)^2 \times \frac{12}{g} \times 1.154^2 = 156.8 \text{ ft.-lb.}$$

6. The Izod impact test

In the Izod and similar impact testing machines, the kinetic energy possessed by a weight falling through a circular path is

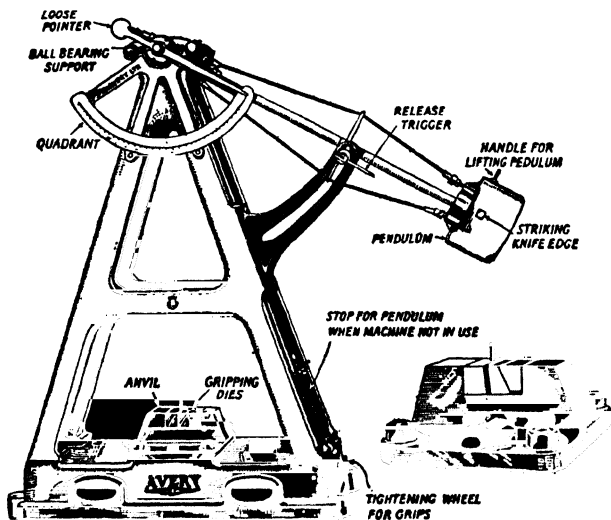


Fig. 23.2.—Izod impact testing machine (*W. & T. Avery, Ltd.*)

used to break a specimen of material, and so test its behaviour to shock. Such a test also gives a good idea of the effective-

ness, or otherwise, of any heat treatment given to the material.

The weight, or pendulum (fig. 23.2), is lifted through a known angle (i.e. to a known height), and then released. The kinetic energy it possesses at the moment of impact with the specimen can be calculated. If the specimen is broken, and the pendulum continues its swing and rises to some observed height, its final potential energy can be calculated. The difference between these two values will be the energy absorbed in breaking the specimen. In an actual machine, the energy absorbed can be read off directly. The specimen is made up in the form of a "notched bar"; in the case of the Izod test-piece, the dimensions are $75 \times 10 \times 10$ mm. with the notch at one-third of the length, the remaining two-thirds being gripped in the vice.

7. Efficiency of energy conversion

The efficiency of energy conversion may be written as the ratio

$$\frac{\text{output of energy in the required form}}{\text{input of energy in the available form}}$$

When converting from electrical to mechanical and from mechanical to electrical energy, quite high efficiency figures (over 90 per cent) can be obtained provided that the design of the apparatus is good. Mechanical and electrical energy are readily convertible into heat energy, all the energy taking part being so convertible. When, however, we start to convert heat energy into mechanical and then electrical energy (i.e. the reverse process), we find that the efficiency of conversion is very low, even when using plant of the best possible design. This subject is further considered in Chap. XXVII.

8. Conservation of energy *

It is important to realize that, however many transformations take place, no loss or gain of energy occurs. It is true that we do not always convert all of one form of energy into some other *required* form, but this so-called lost energy reappears in some recognizable form.

For instance, in a good electric power station, of the original energy stored in the coal only about 25 per cent is converted

* This paragraph must, however, be read in conjunction with Chap. IV, § 6 (Conservation of matter and energy), as, under certain conditions, it is possible for matter to be converted into energy.

into electrical energy. The remaining 75 per cent is accounted for by heat energy thrown up the chimney (about 17 per cent), heat energy given to the cooling water used to condense the steam in the condenser (about 50 per cent), and the rest by frictional losses in the machines, electrical resistance losses, etc., in the generator.

9. Perpetual motion

Whenever motion occurs, friction is set up, and a "loss" of energy in the form of heat takes place. The many attempts made in the past to build a self-contained machine which, once set going, would continue indefinitely, were therefore doomed to failure. Owing to the principle of conservation of energy, the energy possessed by such a self-contained system was constant, but the energy "going over" to heat was "lost" as regards maintaining the motion. Consequently, no machine will operate perpetually unless energy is supplied from some external source.

EXERCISES ON CHAPTER XXIII

1. At a certain instant a truck weighing 5 tons is moving along a level road at 6 m.p.h. Find the number of ft.-lb. of kinetic energy stored in the truck. Assuming all power shut off, how far would the truck move before coming to rest if the resistances were 12 lb. per ton. (U.L.C.I.)

2. If a hammer weighing 2 lb. and moving with a velocity of 25 ft. per sec. strikes a stationary block weighing 5 lb. and the hammer and block are free to move together after the impact, with what velocity will they move?

Find the kinetic energy of (a) the hammer before impact, (b) the hammer and block after impact. (U.L.C.I.)

3. The hammer of an Izod impact testing machine weighs 57.6 lb. and swings in an arc of a vertical circle. It is released from a point 25 in. higher than the lowest point of the arc, strikes a test-piece at this lowest point, breaks it, and rises to a vertical height of 1 ft. on the other side.

Find the kinetic energy and speed of the hammer on striking the test-piece, and the kinetic energy absorbed in breaking the test-piece.

4. (a) State Newton's three Laws of Motion.

(b) A projectile weighs 6 lb. and travels at a speed of 2000 ft. per sec. Find its kinetic energy in foot-tons. If it is stopped in a distance of 4 in. by armour plating, find the average resistance in tons.

(U.E.I.)

5. (a) Compare the functions of the governor and flywheel of an engine.

(b) The rim of a flywheel weighs 2000 lb. and has a mean diameter of 6 ft. Calculate the additional energy stored in ft.-lb. if, during the working stroke, the speed increases from 120 to 124 r.p.m. (U.E.I.)

6. The hammer of a pile driver weighs 5 cwt. and strikes the head of a pile after falling freely 20 ft. Calculate: (a) the velocity of the hammer at impact, (b) the common velocity of the hammer and pile immediately after impact if the pile weighs $1\frac{1}{2}$ tons, (c) the kinetic energy of the system immediately before and after impact, and obtain from this the energy lost in making the blow. (U.E.I.)

7. A flywheel weighing 8 tons and having a mean rim radius of 4.5 ft. runs at a speed of 120 r.p.m. How many foot-pounds of energy are stored in the wheel?

The wheel is fitted to a shaft which revolves in bearings of 6 in. diameter. Assuming the driving power suddenly cut off and the energy of the wheel absorbed by the friction of the shaft bearings, find how many revolutions the flywheel would make before coming to rest. Coefficient of bearing friction = 0.07. (U.L.C.I.)

8. A rigid framework consisting of bars OA, OB, and OC rotates about a vertical axis through O at 120 r.p.m. OA, OB, and OC are 2, $1\frac{1}{2}$, and 3 ft. long respectively, and are at right angles to the axis of the shaft. Masses of 4, 6, and 2 lb. are attached to the bars at A, B, and C respectively. Neglecting the weight of the bars, determine the kinetic energy of the system.

If the three masses be concentrated at a radius R feet, find the value of R if it be so chosen that the kinetic energy at the same speed as before remains unchanged. What name is given to this radius R ? (U.E.I.)

9. (i) During shunting operations a railway wagon weighing 20 tons and moving at 15 ft. per sec. overtakes another wagon weighing 15 tons moving at 10 ft. per sec. If the two wagons are automatically coupled, find their common velocity after the impact.

(ii) If after the two trucks move on together the resistance to motion is 20 lb. per ton, find how far they will travel before coming to rest. (S.W.E.T.C.)

10. A bullet with a velocity of 200 ft. per sec. is fired into a fixed trunk of wood into which it penetrates 10 in. If a similar bullet is fired with the same velocity into a piece of similar wood, find what thickness this should be if the bullet is to emerge with a velocity of 100 ft. per sec. (C.P.E.)

11. A body weighing 50 lb. has a velocity of 10 ft./sec. along a straight path; a second body weighing 30 lb. has a velocity of 5 ft./sec. in the opposite direction. After colliding, the two bodies separate

with a relative velocity of 3 ft./sec. Find the final velocity of each and determine the loss of kinetic energy. (C.P.E.)

12. A bullet weighing one ounce is fired from a rifle with a muzzle velocity of 1500 ft./sec. Calculate its kinetic energy.

If the bullet passes into and remains embedded in a block of wood weighing $1\frac{1}{2}$ lb. which can move freely, find how much kinetic energy is lost by the penetration of the bullet into the block. (C.P.E.)

13. Explain the terms (i) moment of inertia of a body about an axis, (ii) radius of gyration.

A flywheel weighing 6 tons has a radius of gyration of 4 ft. and a speed of 250 r.p.m. Find its kinetic energy. To what speed will the flywheel fall if 100,000 ft.-lb. of energy are taken from it?

(S.W.E.T.C.)

14. Explain the difference between Potential and Kinetic Energy. What units are they measured in?

The flywheel of a shearing machine weighs 1 ton and has a radius of gyration of 1.5 ft. At the beginning of the cutting stroke its speed is 100 r.p.m. and at the end its speed is 80 r.p.m. How much energy has been given out and, if 80 per cent of this is usefully employed on a 3-in. stroke, what average cutting force is exerted?

(S.W.E.T.C.)

15. A string passes over a frictionless pulley. On one end of the string is a weight of 12 lb. and on the other a weight of 17 lb. The 17-lb. weight is held at a height of 5 ft. above the floor and then released. What is the velocity of the string when the 17-lb. weight strikes the floor? How much higher will the 12-lb. weight rise, after the 17-lb. weight has struck the floor?

(S.W.E.T.C.)

16. A projectile weighing 80 lb. and having a velocity of 900 ft. per sec. crashes through a target and emerges with a velocity of 200 ft. per sec. If the target, which is initially at rest and weighs 3000 lb., is free to move in the direction of the projectile, find:

(i) The kinetic energy lost by the projectile.

(ii) The kinetic energy gained by the target. (C.P.E.)

17. Describe a method of measuring the acceleration due to gravity. A ball is projected vertically upwards with a velocity of 96 ft. per sec. Find the maximum height and the time the ball takes to reach it, also the time taken to reach the ground again. If the mass of the ball is 5 oz., what will be its kinetic energy after falling for two seconds? ($g = 32$ ft./sec.²)

(C.P.E.)

18. Two masses weighing 6 lb. and 8 lb. respectively are connected by a string passing over a light pulley. If the masses are initially at rest, find the total kinetic energy of the masses when the heavier mass has fallen through a distance of 6 ft.; hence find their speed and acceleration.

(C.P.E.)

CHAPTER XXIV

Circular Motion and Centrifugal Force

1. Circular motion

Consider a small body (a particle) moving at uniform speed in a circular path of radius r (fig. 24.1). It moves from A to B in t seconds, passing through an angle θ radians. Its angular velocity (ω) in radians per second will be

$$\omega = \frac{\theta}{t}.$$

By definition, the value of θ in radians = $\frac{\text{the arc AB}}{r}$, hence angular

$$\begin{aligned} \text{velocity } (\omega) &= \frac{\text{angle traversed}}{t} = \frac{\theta}{t} \\ &= \frac{AB}{r} \div t = \frac{AB}{r} \times \frac{1}{t} = \frac{AB}{t} \times \frac{1}{r}. \end{aligned}$$

As $\frac{AB}{t}$ = linear velocity of body (v), then

$$\text{angular velocity of body } (\omega) = \frac{v}{r}.$$

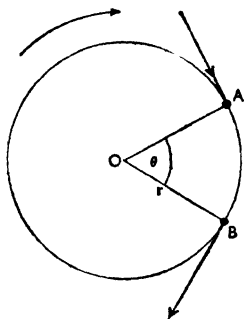


Fig. 24.1

2. Centripetal and centrifugal force

In accordance with Newton's First Law, a force applied to a body, uninfluenced by other external forces, would cause movement to take place in a straight line.

If, however, the body is attached to a string which is fixed to a point at its other end (see fig. 24.2), movement will take place in a circular path around this point.

To cause this change in motion, some force (other than the original) is required (*vide* Newton's First Law). This other force is obviously provided by the pull of the string on the body, and it is called **centripetal force**, because it acts *towards* the centre. Further, in accordance with Newton's Third Law, the body must exert an equal and opposite reacting force on the string. This force is called **centrifugal force**, because it acts *away* from the centre.

This reacting force only exists because of the pull of the string (centripetal force). Consequently, if the string should

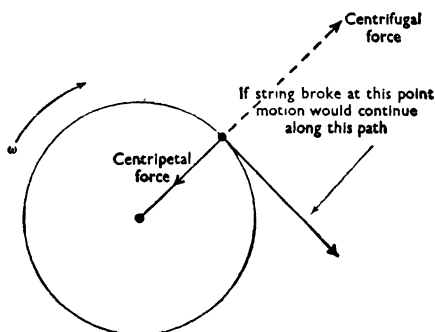


Fig. 24.2.—Circular motion

break, both forces would cease, and the body, because of its inertia, would continue its movement by travelling in a straight line, tangential to the circle (see fig. 24.2).

The greater the mass of the body, or the greater its speed of rotation, the greater will be the magnitude of the centripetal (and centrifugal) force produced, and it is quite possible that this force may rise to a value sufficient to cause the body to break away from its rotary path. Thus, a part may fly off a revolving body if it is not fixed securely enough to withstand the effect of centrifugal force. Similarly, a car rounding a curve will be subjected to this centrifugal force, and will tend to continue its movement in a straight line. If the friction between its tyres and the road is insufficient to stand up to this pull, skidding will result.

Many other instances of the effects of centrifugal force could

be quoted. In some cases, as with vehicles rounding curves, or revolving machinery, the effects have to be guarded against; in other cases, as with centrifugal dryers, pumps, or governors, the effects are put to good use.

We will now proceed to investigate the value of these forces.

3. Centripetal acceleration

When a body moves in a circular path, even with uniform angular speed, it is continually changing its direction. As velocity is a vector quantity, involving both the magnitude of speed and its direction, it will be evident that the velocity of the body is continually changing; and, as a change of velocity represents acceleration, it will be seen that the body is continually undergoing acceleration.

This acceleration is termed **centripetal acceleration**.

4. Value of centripetal acceleration

Consider again the body moving in a circular path of radius r (fig. 24.1). Its velocity changes (in direction) from u at point A to v at point B.

These changes can be represented vectorially (see fig. 24.3). In order to reach the final velocity v , the initial velocity u

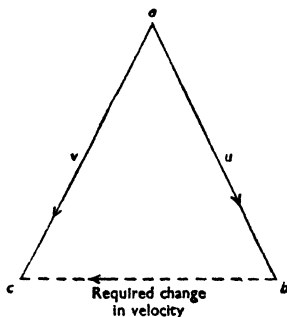


Fig. 24.3

must have added to it a velocity represented by the vector bc , i.e. vector ac is the resultant of vector ab plus vector bc .

Notice that vector bc , representing the change in velocity required in order that uniform circular motion may continue,

is directed from b to c , i.e. it is directed towards the centre of the circle.

We will now consider the magnitude of this change of velocity and of the resulting acceleration.

If θ is made small, the triangles OAB (fig. 24.1) and abc (fig. 24.3) become similar triangles.

$$\therefore \angle AOB = \angle cab = \theta.$$

From fig. 24.4,

$$\sin \frac{1}{2}\theta = \frac{\frac{1}{2}bc}{v(\text{or } u)},$$

$$\therefore bc = 2v \sin \frac{1}{2}\theta.$$

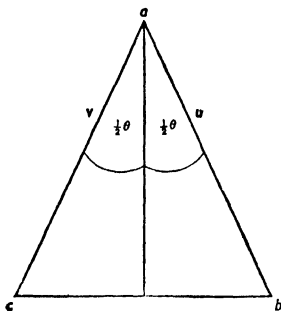


Fig. 24.4

But angle θ is very small. Under these conditions, $\sin \theta = \theta$ (in radians) approximately (refer to trigonometrical tables),

$$\therefore \text{change of velocity } (bc) = 2v \cdot \frac{1}{2}\theta = v\theta.$$

Let t = time taken in travelling from A to B, then

$$\text{acceleration} = \frac{\text{change of velocity}}{t} = \frac{v\theta}{t} = v\omega.$$

$$\text{But } \omega = \frac{v}{r}, \therefore \text{centripetal acceleration} = v \times \frac{v}{r} = \frac{v^2}{r},$$

directed along the radius towards the centre.

Alternative expression for centrifugal acceleration

Since $v = \omega r$ we may also write

$$\text{centripetal acceleration} = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r.$$

5. Value of centripetal force

Having found the value of centripetal acceleration, we will now determine the value of the centripetal force.

Consider a body of weight W moving uniformly in a circular path on the end of a string of radius r . The velocity of this body will be subjected to a centripetal acceleration v^2/r towards the centre. To cause this acceleration a centripetal force or pull must act on the mass of the body along the string, so making it describe a circular path instead of continuing its motion in a straight line.

In accordance with Newton's Second Law, the magnitude of this centripetal force will be proportional to mass times acceleration.

$$\therefore \text{centripetal force} = m \times a = \frac{W}{g} \times \frac{v^2}{r}.$$

The centrifugal force, or reaction of centripetal force, will also have the same magnitude, but will be opposite in direction.

Alternatively, if it is desired to use angular velocity,

$$\text{centrifugal (or centripetal) force} = \frac{Wv^2}{gr} = \frac{W\omega^2 r}{g}.$$

Example 1.—A truck of 10 tons weight, moving at 30 m.p.h., negotiates a curve of radius 1100 ft. Find the outward pressure it will exert against the rail due to centrifugal force.

$$\text{Centrifugal force} = \frac{Wv^2}{gr} = \frac{10 \times 44^2}{32 \times 1100} = \frac{19360}{35200} = 0.55 \text{ ton.}$$

This force exerted by the wheel flanges against the rail is quite considerable, and it will increase as the square of the speed. At 60 m.p.h. (88 ft./sec.) it will be

$$\frac{10 \times 88^2}{32 \times 1100} = \frac{77440}{35200} = 2.2 \text{ tons.}$$

6. Banking of tracks

Owing to the effect of centrifugal force, it is possible for a derailment to occur if a curve is negotiated at too high a speed.

This can happen when the resultant of the weight of the vehicle and the centrifugal force is at such an angle that it passes outside the wheelbase at track level, so causing in-

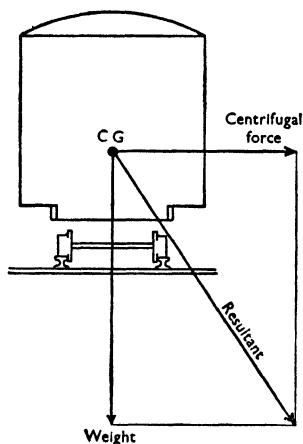


Fig. 24.5

stability (see fig. 24.5 and Chap. X, § 6). Note that both the weight and the centrifugal force act through the centre of gravity of the vehicle.

In order to prevent such derailments on railways, or the overturning of road vehicles, it is customary to bank the track at a curve, by raising the outer side above the level of the inner side.

Under these conditions, the resultant of the weight of the vehicle and the centrifugal force is designed to be approximately perpendicular to the ground surface at the normal operating speed, and to lie well within the track of the vehicle at any other speed contemplated.

The diagram (fig. 24.6) illustrates the method of finding the angle at which the track should be banked for a vehicle of weight W to travel round a curve of radius r with perfect safety at its normal operating speed v . The position of the centre of gravity and of the wheels, etc., have, of course, to be drawn to scale. Some increase in speed over the normal

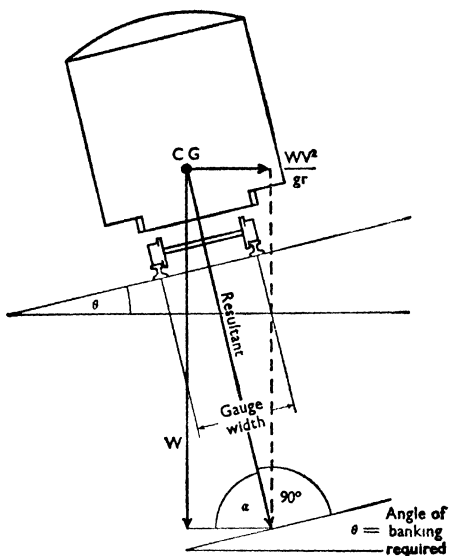


Fig. 24.6

operating value is also permissible, provided that the line of action of the resultant force set up lies within the gauge width.

In solving problems dealing with centrifugal force, the correct values of v and r to be used are the velocity of the centre of gravity of the body, and the radius from the centre of gravity to the axis of rotation.

Example 2.—The radius of curvature of a bend in a road may be taken as 50 ft. Determine the angle at which the road must be banked to eliminate all tendency to side slip for a vehicle travelling round the bend at 30 m.p.h. (I.E.E. Grad.)

The banking must be such that the resultant of the weight of the vehicle and the centrifugal force exerted is perpendicular to the track (see fig. 24.6).

Let $W = 1$, then

$$\text{centrifugal force} = \frac{Wv^2}{gr} = \frac{1 \times 44^2}{g \times 50} = \frac{1936}{1610} = 1.2.$$

$$\tan \alpha = \frac{W}{\text{C.F.}} = \frac{1}{1.2} = 0.833,$$

$$\therefore \alpha = 39^\circ 45'.$$

$$\begin{aligned} \text{Angle of banking required } (\theta) &= 180^\circ - (90^\circ + \alpha) \\ &= 180^\circ - 129^\circ 45' = 50^\circ 15'. \end{aligned}$$

N.B. By construction, in fig. 24.6 the angle between W and the resultant must also be θ .

$$\therefore \theta = \tan^{-1} \frac{Wv^2}{gr} \div W = \frac{v^2}{gr}.$$

7. The conical pendulum

A particle of weight W suspended by a thin string AB from a fixed point B, and arranged to move in a horizontal circle

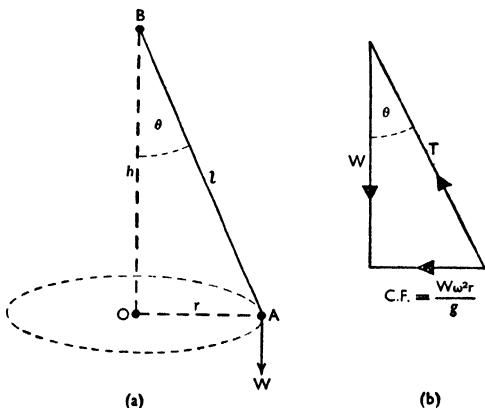


Fig. 24.7.—Conical pendulum

(see fig. 24.7a), constitutes what is termed a *conical pendulum*, the string describing a cone about the vertical axis OB.

Let the string AB be of length l (ft.),
 the radius (OA) of the horizontal circle = r (ft.),
 the height (OB) of the conical pendulum = h (ft.),
 and let tension in string = T (lb.),
 angular velocity = ω (radn./sec.),
 angle of inclination of string to vertical = θ .

Then, when rotating, the forces acting on the particle are W and T , and they have a resultant C.F. (see fig. 24.7b) which acts horizontally and is directed inwards, its value being that of the centripetal force $\frac{W\omega^2 r}{g}$.

The triangles in fig. 24.7a and b are similar,

$$\therefore \frac{T}{W} = \frac{l}{h} \quad \text{and} \quad \frac{W}{W\omega^2 r/g} = \frac{h}{r}.$$

$$\therefore h = \frac{Wr}{W\omega^2 r/g} = \frac{g}{\omega^2}.$$

Thus h is independent of l for a given value of ω .

Also $h = l \cos \theta$, $T = W \div \cos \theta$, $r = l \sin \theta$, etc.

The conical pendulum forms the basis of operation of the centrifugal governor, the variation in h with ω being used to operate some form of control mechanism.

Example 3.—A mass of 5 lb. weight is tied at the end of a thin cord 4 ft. long and revolves as a conical pendulum; the cord is always inclined at an angle of 45 degrees with the vertical. Find the tension in the cord and the velocity of the mass. (C.P.E.)

Tension (T) in cord = $W \div \cos 45^\circ = 5 \div 0.7071 = 7.07$ lb.

As the angle of inclination is 45° , then C.F. = $W = 5$ lb.

The radius (r) = $l \sin 45^\circ = 4 \times 0.7071 = 2.828$ ft.

As
$$\frac{W\omega^2 r}{g} = 5 \text{ lb.},$$

then
$$\omega^2 = \frac{5 \times 32.2}{5 \times 2.828} = 11.38,$$

and
$$\omega = \sqrt{11.38} = 3.37 \text{ radn./sec.}$$

$$\therefore \text{velocity of mass} = 3.37 \times 2.828 = 9.55 \text{ ft./sec.}$$

8. Motion in a vertical circle

When a particle runs around a smooth curve lying in a vertical plane, its speed at any point can be determined with the aid of the principle of conservation of energy. The kinetic energy gained at any point will be equal to the potential energy lost.

If m is the mass of the particle, u and v are its initial and final velocities, and h is the vertical height descended, then (using absolute units)

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgh,$$

$$\therefore v^2 - u^2 = 2gh$$

and

$$v^2 = u^2 + 2gh.$$

(The sign of g will be positive when descending, and negative when ascending.)

Example 4.—A car weighing 200 lb. runs round a vertical circular track of 20 ft. diameter. If its speed at the top of the circle is 25 ft. per sec., find the reaction between the car and the track. Find the speed at the lowest point of the track and the corresponding reaction. (C.P.E.)

At top of track:

$$\text{Centrifugal force} = \frac{W}{g} \times \frac{v^2}{r} = \frac{200}{g} \times \frac{25^2}{10} = \frac{200 \times 625}{32.2 \times 10} = 388.4 \text{ lb.}$$

The weight of the car will be acting downwards, but here the centrifugal force exerted will be acting upwards, and will hold the car to the track. Hence

$$\text{Reaction between car and track} = 388.4 - 200 = 188.4 \text{ lb.}$$

At lowest point of track:

Speed.—Let u = speed at top, v = speed at lowest point, h = diameter of track,

$$\text{then } v^2 = u^2 + 2gh = 25^2 + 2g \times 20 = 1913,$$

$$v = \sqrt{1913} = 43.75 \text{ ft./sec.}$$

$$\text{Centrifugal force} = \frac{W}{g} \times \frac{v^2}{r} = \frac{200}{g} \times \frac{43.75^2}{10} = 1188 \text{ lb.}$$

Reaction.—The weight of the car and the centrifugal force will now both act downwards. Hence

$$\text{Reaction between car and track} = 200 + 1188 = 1388 \text{ lb.}$$

9. Balancing

All the particles in a rotating body will exert centrifugal force and will thus pull on the axis of rotation. The resultant pull will be the vector sum of all these individual pulls. By careful attention to the design of a rotating body, the weight can be so distributed that this resultant action is reduced to zero. Under such conditions the rotating mass is said to be **balanced**. Note that, although such a body may be balanced, its structure must be such as to withstand the individual pulls. For example, in the case of a salient-pole type of alternator

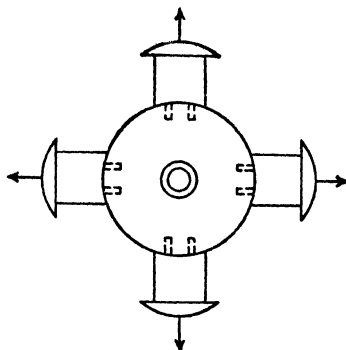


Fig. 24.8

(see fig. 24.8), the rotating poles may be balanced, but their fixing bolts, etc., must be of sufficient strength to withstand the centrifugal force produced.

If a rotating body is designed so that its centre of gravity coincides with its axis of rotation, then that body will be balanced.

An unbalanced rotating body will set up a pressure on its axis of rotation, and on the road surface if it is a vehicle wheel, and this will cause undue bearing friction and surface wear (in the case of the wheel). Also dangerous vibrations will possibly be set up.

Example 5.—What tensile stress will be set up in each of two similar bolts if they are used radially to attach a pole-piece of an

alternator to its rotor. The weight of the pole-piece is 64 lb., and its centre of gravity is 18 in. from the centre of the rotor shaft. Speed of alternator 480 r.p.m. Effective cross-sectional area of each bolt $1\frac{1}{2}$ sq. in.

$$\begin{aligned}\text{Centrifugal force set up by pole-piece} &= \frac{W}{g} \omega^2 r \\ &= \frac{64}{g} \times \left(2\pi \times \frac{480}{60}\right)^2 \times \frac{3}{2} = 7570 \text{ lb.}\end{aligned}$$

This force is resisted by a bolt cross-sectional area of $2 \times 1\frac{1}{2} = 2\frac{1}{2}$ sq. in.

$$\therefore \text{Stress} = \frac{7570}{2\frac{1}{2}} = 3028 \text{ lb./in.}^2$$

(N.B. This stress will be increased by $\frac{64}{2\frac{1}{2}}$ lb./in.² when the pole-piece hangs vertically down.)

An unbalanced rotating mass can be balanced by one or more weights attached to it in such positions that they cancel out the effect of its centrifugal force.

The centrifugal force due to each rotating mass will be $W\omega^2 r/g$. But the angular velocity (ω) will be the same for all the rotating masses, and, as g is common, balance will therefore be obtained if the moments of the weights (Wr) are balanced, *provided that* all these weights lie in the same plane.

A body balanced so that the moments about its geometric centre are balanced (i.e. $\Sigma M = 0$) is said to be *statically balanced*, and it will remain stationary in any position.

It will also be balanced at any speed, or *dynamically balanced*, if the weights of its component parts all lie in the same plane.

Example 6.—Three weights A, B, and C are attached to a disc which revolves about an axis passing through its centre O at right angles to the plane of the disc. A = 4 lb., B = 6 lb., and C = 7 lb. AO = 10 in., BO = 9 in., CO = 9 in.; the angles measured clockwise from A are AOB = 90°, AOC = 210°. Find the unbalanced force acting when the disc rotates 120 times a minute.

Find what weight placed at a radius of 8 in. from O will establish balance, and the direction in which it must lie.

Method.—First find the weight required to produce balance. This will be independent of speed.

$$\text{Centrifugal force of A} \propto Wr = 4 \times 10 = 40.$$

$$,, \quad ,, \quad B \propto Wr = 6 \times 9 = 54.$$

$$,, \quad ,, \quad C \propto Wr = 7 \times 9 = 63.$$

By drawing a force polygon the equilibrant required to produce balance can be obtained (fig. 24.9).

By measurement.—The equilibrant (balance weight) must lie at an angle of 57° , measured anticlockwise from OA, and must act outwards.

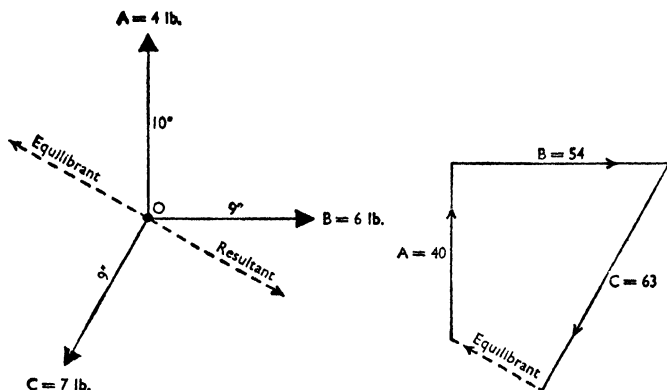


Fig. 24.9

[The unbalanced force (or resultant) will be opposite in direction to this, i.e. at 123° measured clockwise from OA.] The product Wr of the required balance weight = 26.8. Therefore, at a radius of 8 in.,

$$W = \frac{26.8}{8} = 3.35 \text{ lb.}$$

The unbalanced centrifugal force acting at 120 r.p.m. is

$$\frac{W}{g} \omega^2 r = \frac{3.35}{g} \times \left(2\pi \times \frac{120}{60} \right)^2 \times \frac{2}{3} = \frac{3.35}{g} \times 16\pi^2 \times \frac{2}{3} = 10.95 \text{ lb.}$$

10. Dynamic balance

If the balance weights and the rotating masses do not all lie in the same plane, static balance may be secured about its centre line; but this does not necessarily mean that the body will be balanced when rotating at any speed.

Consider the crankshaft of an inside-cylinder type locomotive

(see fig. 24.10a). It is statically balanced ($\Sigma Wr = 0$), but when in motion, the centrifugal forces set up by the two cranks would not balance each other as they are not in the same plane. In order to secure dynamic balance, the sum of the moments of all of the centrifugal forces taken about any point in the crankshaft (i.e. at right angles to the planes of rotation) must also be zero. These conditions are illustrated diagrammatically in fig. 24.10b.

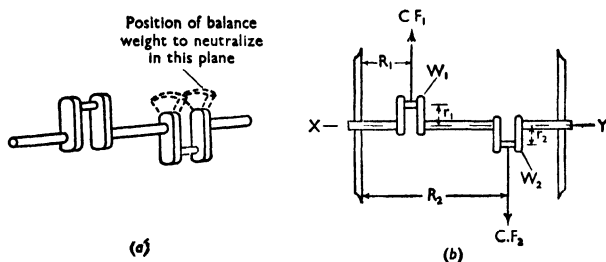


Fig. 24.10

To secure dynamic balance:

ΣWr must equal zero, and

$\Sigma (\text{centrifugal force}) \times R \quad ,, \quad ,, \quad \text{zero.}$

$\therefore W_1 r_1 \quad ,, \quad ,, \quad W_2 r_2$, and

$(\text{centrifugal force}) \times R_1 \quad ,, \quad ,, \quad (\text{centrifugal force}) \times R_2.$

N.B. The moments for each set of conditions are taken about planes mutually at right angles.

To obtain the above conditions for dynamic balance, the webs of each crank could have weights added to them to neutralize the unbalanced forces in the planes in which they are set up.

To neutralize the unbalanced forces set up by the various rotating and reciprocating masses in the more complex case of a locomotive with both inside and outside cylinders, balance weights are often fitted to the driving wheels. The presence of any unbalanced force in the driving wheels sets up a highly undesirable hammering action on the track; this must be eliminated as far as possible.

Example 7.—The revolving parts of a crankshaft of a gas engine are equivalent to a mass of 257·6 lb. at 1 ft. radius. The crankshaft is 3 ft. long between bearings, and the crank centre-line is 1 ft. from the right-hand bearings (and situated between the bearings). If the crankshaft is unbalanced, find the forces on each bearing when it is rotating at 240 r.p.m.

$$\begin{aligned}\text{Centrifugal force} &= \frac{W}{g} \omega^2 r = \frac{257\cdot6}{32\cdot2} \times \left(\frac{240 \times 2\pi}{60} \right)^2 \times 1 \\ &= 8 \times (8\pi)^2 \times 1 = 5050 \text{ lb.}\end{aligned}$$

The moment of this centrifugal force must be balanced by the bearings (see fig. 24.11).

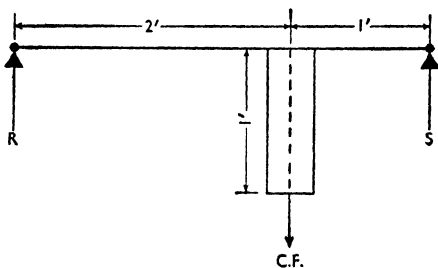


Fig. 24.11

Taking moments about *R*,

$$(\text{C.F.}) \times 2 = 3S.$$

$$\therefore S = \frac{5050 \times 2}{3} = 3366\cdot6 \text{ lb.,}$$

and

$$R = 5050 - 3366\cdot6 = 1683\cdot3 \text{ lb.}$$

EXERCISES ON CHAPTER XXIV

1. A pole-piece weighing 48 lb. is attached to the rotor of an alternator, and has its centre of gravity 16 in. from the axis of the shaft. Calculate the pull in the attachment to the rotor when the speed of the alternator is 420 r.p.m. (U.E.I.)

2. A light rod which can rotate in a vertical plane about a horizontal axis has a mass of 5 lb. weight attached to the free end at a distance of 3 ft. from the axis of rotation. If the greatest tension in the rod is not to exceed 60 lb. weight, what is the maximum angular speed of the rod in revolutions per minute?

What are the greatest and least tensions in the rod when running at one-half of its maximum speed? (C.P.E.)

3. A point moves along the circumference of a circle of radius r with a constant speed v . Find the change of velocity between points A and B on the circle 60° apart, and find the mean acceleration in magnitude and direction in the time taken to traverse the arc AB. (C.P.E.)

4. A body weighing 40 lb. is bolted to a disc which rotates about an axis passing through its centre and at right angles to its face. The centre of gravity of the body is at a distance of 12 in. from the axis of rotation. What radial force will act on the bolt connecting the body to the disc when the latter is rotating at 120 r.p.m.? Also, what weight placed diametrically opposite the body and 20 in. from the disc centre would give perfect balance? (U.L.C.I.)

5. A wheel of a railway van is 4 ft. diameter, weighs 750 lb., and has its centre of gravity $\frac{1}{10}$ in. from its geometrical axis. Find the unbalanced force due to centrifugal action when the van is travelling at 40 m.p.h. If the vertical load on the wheel is 3 tons, find the greatest and least pressures on the rail at the above speed. (U.L.C.I.)

6. Determine the height at which the outer rail of a curve on a railway should be banked above the inner rail, for safe running, given the following details:

Gauge of railway, 4 ft. $8\frac{1}{2}$ in.

Radius of curve, 40 chains.

Normal operating speed, 45 m.p.h.

7. A locomotive weighing 60 tons traverses a curve of radius 800 ft. at a speed of 60 m.p.h. Find the force exerted towards the centre of the circle by the rails on the wheel flanges.

8. A motor-car weighing 20 cwt. travelling at a speed of 30 m.p.h. traverses a dip in the road, the dip being part of a circle of radius 21 yd. Find the reaction between the car and the road at the lowest point of this dip.

9. Three masses, X , Y , and Z , in the same plane revolve about an axis which pierces the plane of rotation at A . X weighs 20 lb. and is situated at 25 in. from A . Y weighs 40 lb. and is situated at 16 in. from A . Z weighs 30 lb. and the angle XAY is 135° . Find where Z must be placed if these three revolving masses are to be in perfect balance. (U.L.C.I.)

10. Two masses A and B are attached to a disc, centre O , which revolves at a speed of 120 r.p.m., the plane of the disc being at right angles to the axis of rotation which passes through O . The mass A weighs 8 lb. and is 10 in. from O ; and B weighs 10 lb. and is 8 in. from O . The angle AOB is 120° . Determine the resultant effect of the centrifugal forces of A and B . (U.L.C.I.)

11. Two masses, $A = 18$ lb. and $B = 24$ lb., are attached to a rotating disc at distances of 9 in. and 10 in. respectively from O , the axis of rotation. The angle between the lines joining O to the centres of gravity of A and B is 120° . Find the resultant force on the axis when the speed of rotation is 240 r.p.m.

If balance is obtained by a third mass C , placed at a distance of 13 in. from O , find the magnitude and angular position of C . (U.L.C.I.)

12. A body weighing 20 lb. is constrained to move in a vertical circular path. It falls from rest through a vertical height of 30 in. before it reaches the bottom of the circle. What is the velocity of the body when in its lowest position? If, in this position, 20 ft.-lb. of energy are withdrawn from the body, through what vertical height would the body rise on the other side? Neglect all frictional resistances. (U.L.C.I.)

CHAPTER XXV

Thermometry

1. Temperature

The sensations of heat and of cold are familiar to all. We may say that one body is hotter than another, after making use of our sense of touch. This, however, is not a true guide to the degree of hotness possessed by the bodies. For example, if a block of iron and a block of wood are touched, both having stood together in a room for some time, the iron will feel colder than the wood. After a brisk walk out of doors on a cold day, our senses on going indoors will indicate, perhaps, a considerable degree of warmth. A person who has been sitting indoors may, however, feel quite cold. Again, it feels much colder outdoors on a cold wet day than on a cold dry day. The relative degree of hotness (or intensity of heat) of a body is termed its **temperature**. To determine it we make use of an instrument called a **thermometer**.

2. Thermometry and thermometers

Thermometry is the name given to the science of temperature measurement. In order to specify the temperature of a body, some fixed standard, or standards, must be used for comparison. Ice is cold and steam is hot, so these states, under carefully defined conditions, may be used as our standards.

Thermometers.—A change of temperature may be indicated by some such physical change in a body as an alteration of its length, volume, pressure, or state. The common glass type of thermometer, containing mercury or alcohol, makes use of the change in volume to indicate temperature change, an increase in temperature causing the volume of the liquid to increase to a greater extent than the volume of its containing glass tube.

The **mercury thermometer** consists of a glass tube of small bore, known as a **capillary tube**, fitted with a bulb at one end (see fig. 25.1, p. 360).

The walls of this bulb are thin compared with the walls of the tube, so enabling heat to reach the mercury quickly, thus making the thermometer more sensitive. This bulb, and a part of the tube, are filled with mercury, and the tube is then "sealed off" by a process which ensures that no air is left above the mercury. As the mercury column is readily seen, and as it increases considerably in volume for a small temperature rise, quickly reaching the temperature of the body in contact with the bulb, it is very suitable for thermometric use.

3. Calibrating the thermometer

The position of the mercury in the tube is marked when the thermometer is being subjected to the specified standard conditions. These conditions give what are known as the **fixed points**, and are:

1. For the **lower fixed point**.—The temperature of pure melting ice under normal atmospheric pressure.

2. For the **upper fixed point**.—The temperature of steam at normal atmospheric pressure.

Therefore, by placing the thermometer in melting ice, and then in steam at atmospheric pressure (usually this is done by means of an instrument termed a **hypsometer**), these fixed points (freezing-point and boiling-point) may be marked on the scale.

4. Thermometer scales

Two different thermometer scales are in common use.

- (a) The **Fahrenheit scale** (F.), introduced in 1714 and used for general work, especially in this country.

- (b) The **Centigrade** (C.) **scale**, introduced around 1742 and used particularly in scientific work.*

Both scales make use of the two fixed points mentioned above.

On the Fahrenheit scale the lower fixed point is marked 32° and the upper 212° , whereas on the Centigrade scale the lower fixed point is marked 0° and the upper 100° .

The space between these two fixed points on the thermometer tube is divided up uniformly, consequently the number

* Now sometimes called the Celsius scale, after the first user of a scale divided into a hundred degrees.

of divisions (or degrees) between these points will be $212^{\circ} - 32^{\circ} = 180^{\circ}$ on the Fahrenheit scale, and 100° on the Centigrade scale (see fig. 25.1).

Readings below zero on either scale are referred to as "minus" temperatures (e.g. -40° C.). Readings above boiling-point can be obtained with a normal mercury thermometer to a temperature approaching the boiling-point of mercury

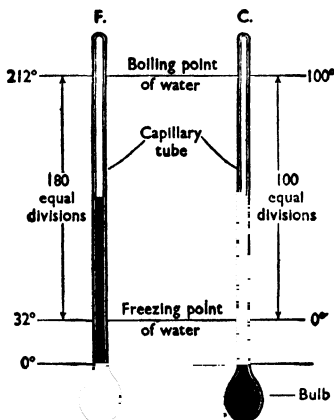


Fig. 25.1.—Thermometer scales

(674° F.). Above this a special thermometer, or a **pyrometer**, must be used. As mercury freezes at -37.8° F., it is usual to employ an alcohol thermometer for recording low temperatures. (Alcohol can be used down to -173° F.) Alcohol cannot, however, be used for measuring high temperatures, its boiling-point being just over 172° F.

5. Conversion of temperatures

It will be noticed that 180° rise on the Fahrenheit scale corresponds to a 100° rise on the Centigrade scale. Or, putting it another way,

A 100° interval on the C. scale equals, on the F. scale,

$$\frac{9}{5} (\text{C. interval}) = \frac{9}{5} \times 100 = 180^{\circ}.$$

Similarly, a 180° interval on the F. scale equals, on the C. scale,

$$\frac{5}{9} (\text{F. interval}) = \frac{5}{9} \times 180 = 100^\circ$$

As freezing-point is not zero, but 32° on the F. scale, this must be taken into account when converting temperatures.

Consequently, if F and C represent Fahrenheit and Centigrade temperatures respectively, we obtain

$$F = \frac{9}{5}C + 32,$$

and

$$C = \frac{5}{9}(F - 32).$$

This may be expressed by the rules:

- (1) When converting from Centigrade to Fahrenheit, multiply by $\frac{9}{5}$ and *then* add 32.
- (2) When converting from Fahrenheit to Centigrade, subtract 32, and *then* multiply by $\frac{5}{9}$.

Example 1.—Convert: (a) 100° C. to degrees F.; (b) 15° C. to degrees F.; (c) -40° C. to degrees F.

$$(a) \left(\frac{9}{5} \times 100\right) + 32 = 180 + 32 = 212^\circ \text{ F.}$$

$$(b) \left(\frac{9}{5} \times 15\right) + 32 = 27 + 32 = 59^\circ \text{ F.}$$

$$(c) \left\{\frac{9}{5} \times (-40)\right\} + 32 = (-72) + 32 = -40^\circ \text{ F.}$$

Note particularly the method when dealing with temperatures below zero. Also that the temperature scales coincide at -40° .

Example 2.—Convert: (a) 212° F. to degrees C.; (b) 77° F. to degrees C.; (c) -22° F. to degrees C.

$$(a) \frac{5}{9}(212 - 32) = \frac{5}{9} \times 180 = 100^\circ \text{ C.}$$

$$(b) \frac{5}{9}(77 - 32) = \frac{5}{9} \times 45 = 25^\circ \text{ C.}$$

$$(c) \frac{5}{9}(-22 - 32) = \frac{5}{9} \times (-54) = -30^\circ \text{ C.}$$

6. Maximum and minimum thermometers

The maximum and minimum temperatures reached over a period can be recorded by a thermometer such as Six's self-

registering type shown in fig. 25.2. The working substance used is alcohol, which is contained in the tube AB, as well as in C. The connecting U-tube is filled with mercury. An increase in temperature will cause the alcohol in AB to expand; this expansion will depress the mercury column at B, and raise it at C. The maximum index will be raised by this column,

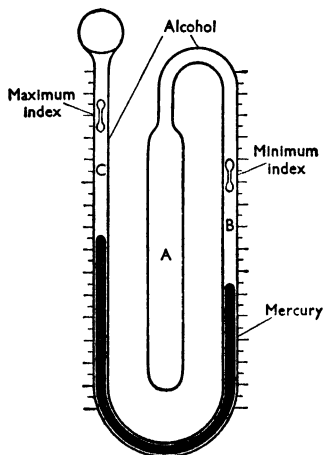


Fig. 25.2.—Six's maximum and minimum thermometer

and left at the highest position reached, so indicating the maximum temperature. A decrease in temperature will cause the alcohol in AB to contract, and the mercury will push the minimum index up tube B to indicate the minimum temperature reached. Resetting an index can be carried out by means of a magnet.

7. Pyrometers

As the liquid-in-glass type of thermometer is fragile and has certain limitations, it is often replaced by various other types for industrial purposes. When used for measuring high temperatures these may be called **pyrometers**.

One most useful instrument makes use of the **thermo-electric effect**. If the junction of two dissimilar metals is heated an **electromotive force** (e.m.f.) is developed across the cold ends of these metals, its magnitude being proportional to the difference of temperature of the **hot junction** and the cold ends. This e.m.f., although small, is readily measurable by a **voltmeter**, which can be calibrated to read temperature directly. An arrangement suitable for measuring furnace temperatures is shown in fig. 25.3. The fact that the measuring voltmeter can be installed remote from the furnace

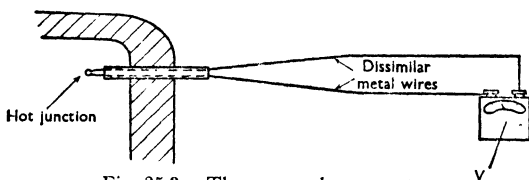


Fig. 25.3.—Thermocouple pyrometer

V is a voltmeter connected across cold junction and calibrated to read temperature (cold junction temperature must be kept constant for accurate results).

is often advantageous. The dissimilar metal wires of the couple need not necessarily be taken right up to the voltmeter, but can be connected to it by a pair of leads.

The dissimilar metals used as the couple may be **base metals** (as in the copper and constantan couple), or **rare metals** (as in the platinum and platinum-iridium couple).

A further type of instrument makes use of the variation of the electrical resistance of a wire (such as platinum) when exposed to temperature change.

8. Heat and expansion

The effect of heat on most substances is to cause an increase of dimensions. This is referred to as **expansion**. Cooling, similarly, causes **contraction**. All materials, however, do not expand, or contract, at the same rate. It is possible to make various alloys which do not undergo dimensional changes to any appreciable extent when subjected to a change of temperature. One such alloy is termed **Invar**.

Expansion must be allowed for in many practical cases.

Gaps must be left between rails, steel bridges are not fixed rigidly at both ends, steam and hot-water pipes are fitted with expansion joints, etc.

The property of expansion is used in fixing steel tyres to locomotive wheels. The tyre is made of slightly less diameter than the rim to which it is to be fitted. The tyre is then heated and expands. It is pushed over the rim and allowed to cool. In doing so it contracts to its original size and thus binds itself tightly on to the rim. Such a method of jointing is known as a **shrink fit**.

9. Thermostats

The expansion of a metal rod with increase of temperature can be utilized to operate a switch or control, thereby cutting off the supply of some heating medium at a prearranged temperature. An instrument which performs this function is called a **thermostat**.

For instance, in an electric water-heater, a thermostat may be arranged to maintain the water at 180°F . When the water reaches this temperature, the expansion of the rod, acting through some suitable multiplying system, causes a switch to open, so shutting off the electricity. When the temperature of the water falls, the contraction of the rod causes the switch to close, and brings the heater into operation again.

A somewhat similar arrangement is often fitted to a gas cooker. When the oven reaches some preset temperature, the expansion of the control rod causes the closure of a diaphragm, which restricts the flow of gas to the burners.

The **bimetal-pattern thermostat** makes use of the different rates of expansion of two different metals. A strip of each of these materials is fastened together to form what is known as a **compound bar** or **bimetal strip**. A change of temperature causes this strip to buckle, and consequently if one end is fixed the other end will move by an amount depending upon this temperature change. By arranging a pair of electrical contacts so that they are opened or closed by the bimetal strip, it is possible for the temperature of electric heaters, rooms, furnaces, etc., to be maintained at some preset value. Fig. 25.4 shows the construction of a **thermostat** for a domestic electric temperature-controlled iron. The temperature of the sole plate may be maintained at values suitable for ironing

such materials as linen (which needs a comparatively high temperature), wool, cotton, and silk (needing a lower tem-

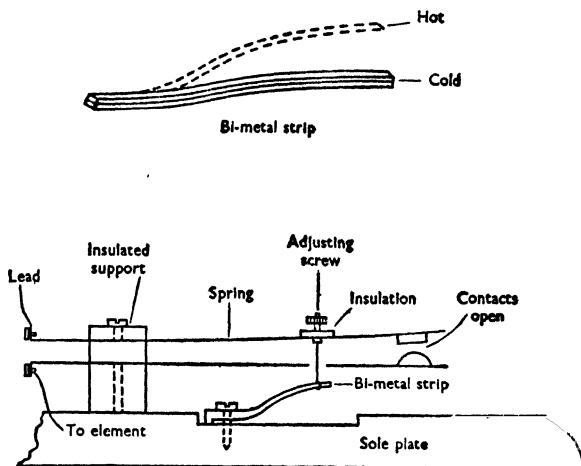


Fig. 25.4.—Thermostat on a temperature-controlled iron
(When the iron is cold the contacts are held closed against the action of the spring)

perature), by setting the control knob to the appropriate position.

10. Coefficient of expansion

When heat is applied to a block of material, it will expand in length, breadth, and depth, i.e. it expands cubically. All materials will expand in this manner, but, in the case of objects which are long in comparison with their other dimensions, it is often convenient to consider their expansion in a linear direction only.

In general a rise of temperature of five degrees causes five times as much expansion as a rise of one degree, i.e. expansion is uniform, and is proportional to temperature rise, for a given material.

The amount of linear expansion per degree of temperature

rise per unit of the original length is known as the **coefficient of linear expansion** of that material.

If L = final length, L_0 = original length, then

$$\text{expansion} = L - L_0,$$

and the coefficient of linear expansion (α)

$$= \frac{L - L_0}{L_0 t}, \text{ where } t = \text{temperature rise.}$$

From this, the final length

$$L = L_0(1 + \alpha t).$$

It will be evident that two values of α will exist, according to whether the temperature rise (t) is expressed in degrees F. or C. The latter value will be $\frac{9}{5}$ of the former.

A few representative values of α are given in Table 5.

TABLE 5
COEFFICIENTS OF LINEAR EXPANSION

Material	Per degree C.	Per degree F.
Lead	·000 0280	·000 01555
Copper	·000 01719	·000 00955
Cast Iron	·000 01113	·000 00618
Steel	·000 012	·000 0066
Water	·000 1586	·000 088
Ice	·000 0510	·000 0283
Invar (36% nickel steel)	·000 0009	·000 0005

11. Coefficient of superficial expansion

The length and breadth of a sheet of material will increase with temperature rise. Knowing the coefficient of linear expansion of the material in question, the **coefficient of superficial (or area) expansion** is, for all practical purposes, twice that of its coefficient of linear expansion.

This can be shown by considering a square sheet of metal of

unit side heated through one degree. Each side will become of length $(1 + \alpha)$, and its area will become

$$(1 + \alpha)^2 = 1 + 2\alpha + \alpha^2.$$

As α is a very small quantity (see Table 5), α^2 will be very much smaller in comparison, and can safely be neglected. Consequently we may write

$$\text{final area } (A) = \text{original area } (A_0) \times (1 + 2\alpha t),$$

$$\text{i.e.} \quad A = A_0(1 + 2\alpha t).$$

12. Coefficient of cubical or volume expansion

Similarly, for all practical purposes, the coefficient of cubical expansion of a material is three times its linear coefficient.

This follows when considering a cube. Each side, when heated through one degree, will become of length $(1 + \alpha)$, and its volume will become

$$(1 + \alpha)^3 = 1 + 3\alpha + 3\alpha^2 + \alpha^3.$$

The quantities $3\alpha^2$ and α^3 , being small compared with α , can be neglected for all practical purposes.

$$\therefore \text{Final volume } (V) = \text{original volume } (V_0) \times (1 + 3\alpha t),$$

$$\text{i.e.} \quad V = V_0(1 + 3\alpha t).$$

Example 3.—A steel railway line is 60 ft. long when measured at 60° F. Calculate the difference in its length caused by a change from a summer temperature of 95° F. to a winter temperature of 10° F.

The coefficient of linear expansion of steel = 0.000 0066 per deg. F.,

$$\begin{aligned} \therefore \text{summer length} &= L_0(1 + \alpha t) = 60(1 + 0.000\,0066 \times 35) \\ &= 60 \times 1.000\,231 = 60.013\,86 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{Winter length} &= L_0(1 + \alpha t) \text{ (where } t \text{ is a fall of } 50^\circ \text{ F., i.e. } t = -50) \\ &= 60(1 - 0.000\,0066 \times 50) = 60 \times 0.999\,67 = 59.9802 \text{ ft.} \end{aligned}$$

Difference between summer and winter length

$$\begin{aligned} &= 60.013\,86 - 59.9802 = 0.033\,66 \text{ ft.} \\ &= 0.033\,66 \times 12 = 0.403\,9 \text{ in.} \end{aligned}$$

Example 4.—A steel tyre is to be shrunk on to a locomotive wheel 6.02 ft. in external diameter. If the tyre is 6 ft. in internal diameter, find the minimum temperature rise it requires before it can be fitted. (Take $\alpha = 0.000\ 0066$ per $^{\circ}\text{F.}$)

The tyre must be heated until its internal diameter (L) is at least 6.02 ft.

$$L = L_0(1 + \alpha t).$$

$$6.02 = 6(1 + 0.000\ 0066t) = 6 + 0.000\ 0396t.$$

$$t = \frac{6.02 - 6}{0.000\ 0396} = 505^{\circ}\text{F.}$$

Example 5.—A steel rod 10 in. long is heated from 60°F. to 180°F. and is then secured by fastening in an appliance so that it cannot contract when cooling. Calculate the tensile stress set up in it when the appliance cools to 60°F. ($\alpha = 0.000\ 0066$ per $^{\circ}\text{F.}$ and Young's modulus for the steel is 13,000 tons/in.²)

$$\text{Extension produced} = 10 \times 0.000\ 0066 \times 120.$$

$$\begin{aligned}\text{Strain} = \text{extension per unit length} &= \frac{10 \times 0.000\ 0066 \times 120}{10} \\ &= 0.000\ 792.\end{aligned}$$

$$\text{As Young's modulus } (E) = \frac{\text{stress}}{\text{strain}} = 13,000 \text{ tons/in.}^2 \text{ (see Chap. XI),}$$

$$\text{then stress} = \text{strain} \times E = 0.000\ 792 \times 13,000 = 10.29 \text{ tons/in.}^2$$

13. Expansion of liquids

Only the cubical expansion of liquids need be considered. The coefficient of cubical expansion of liquids is not appreciably constant, as with solids. It increases as the temperature rises. Consequently, when stating the coefficient, the temperature range applicable should be mentioned.

The expansion of water must be considered separately, owing to its peculiar behaviour. Starting with water at freezing-point (0°C.), increases of temperature cause *contraction* until 4°C. is reached. Further increases of temperature beyond 4°C. cause expansion to occur. Water, therefore, has a point of maximum density at 4°C.

These facts are of major importance, as they affect the mechanism of freezing. As the water cools down on the surface, it becomes denser, and sinks to the bottom, being replaced by

the warmer water from the bottom. This process continues until all of the water has cooled to its point of maximum density (4°C.). Further surface cooling causes the water to become less dense, and consequently it stays on the surface and eventually freezes, forming a thin layer of ice. This ice formation continues to increase in thickness, but the ice as formed, being less dense, remains on the surface. Consequently water freezes from the surface downwards, a fact of great importance. Deep water will not freeze solidly.

14. Boyle's law

A gas may be readily compressed. Therefore, if pressure is applied to a gas enclosed in some containing vessel, the volume of this gas will decrease. It can be shown by experiment that, to decrease the volume by one-half, the applied pressure must be doubled; to decrease the volume by one-third, the applied pressure must be trebled, and so on in similar fashion. The temperature of the gas must, however, be constant throughout. This relationship between pressure and volume was first observed by Robert Boyle in 1662, and consequently is called "Boyle's Law". This law states that:

"The volume of a given mass of a gas varies inversely as the applied pressure, provided that its temperature remains constant during the change".

The law is usually expressed symbolically, using P for pressure, V for volume, and C for a constant.

Thus $V \propto \frac{1}{P}$ when temperature is constant (N.B. $\propto$ means "is proportional to"),

and
$$V = C \times \frac{1}{P},$$

from which we get

$$PV = C,$$

this being the usually remembered form.

Actual gases depart slightly from this law, but for all practical purposes it can be regarded as true. A gas which would obey the law perfectly is called a **perfect gas**.

If a graph of pressure and volume is plotted for a perfect gas, the resulting curve is known as a **rectangular hyperbola**

(fig. 25.5). The expansion or compression of this gas is said to be "hyperbolic", and, as it is assumed that the temperature remains constant throughout, it is also said to be **isothermal** (i.e. equal temperature) expansion or contraction. The pressure is usually stated in terms of its absolute value, i.e. its pressure above that of an absolute vacuum. **Gauge pressure** means pressure above that of the atmosphere (approx. 15 lb. per sq. in.).

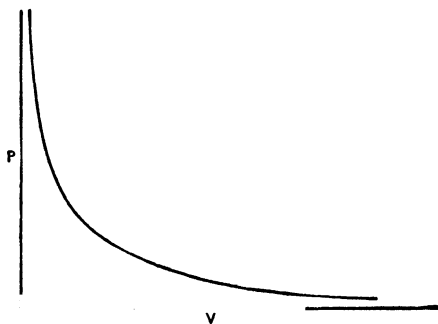


Fig. 25.5

To make use of Boyle's law in order to find the pressure or volume in one state, knowing the pressure and volume in some other state, it is usual to write it as follows:

$$P_1 V_1 = C \quad P_2 V_2,$$

where the suffixes (1) and (2) denote the two states.

This can be rearranged if desired as

$$\frac{P_1}{P_2} = \frac{V_2}{V_1},$$

Example 6.—If the volume of a gas at a pressure of 200 lb. per sq. in. (absolute) is 3 cu. ft., find its volume when the pressure is reduced to 15 lb. per sq. in. (the temperature remaining constant).

$$P_1 V_1 = C = P_2 V_2.$$

$$200 \times 3 = 600 = 15 \times V_2.$$

$$V_2 = \frac{600}{15} = 40 \text{ cu. ft.}$$

15. Charles' law. Expansion of gases

The Frenchman Charles discovered in 1787 that the volume of a given mass of gas increased with temperature by a constant fraction of its volume at 0°C . for each degree of temperature rise, provided that the pressure remained constant.

This fractional increase in volume per degree C was the same for all gases, and careful experiment gave its value as $\frac{1}{273}$.

Thus the volume of a gas at temperature t° (V_t) can be found from the expression

$$V_t = V_0(1 + \frac{1}{273}t),$$

where V_0 is the volume at 0°C .

It has also been determined that the pressure exerted by a gas increases by the same fractional amount per degree when the temperature is increased, provided that the volume is kept constant, i.e.

$$P_t = P_0(1 + \frac{1}{273}t).$$

16. Absolute temperature

From the above formulæ, as the volume of a gas at 0°C . decreases by $\frac{1}{273}$ of its volume for a decrease of 1°C ., it would seem that its volume would become zero at -273°C . Also its pressure would become zero at -273°C . This temperature is termed **absolute zero**. In actual fact the gas does not entirely vanish at this temperature, as Charles' law does not hold good at such low temperatures, because by then all gases would have solidified. A perfect gas, however, is assumed to obey Charles' law.

N.B. Temperatures within a small fraction of a degree of absolute zero have been obtained experimentally.

The *absolute temperature scale* makes use of this absolute zero as its starting-point. Thus:

0° absolute (A) is equal to -273°C .			
273°A .	„	„	0°C .
373°A .	„	„	100°C .

Any reading on the Centigrade scale therefore becomes {reading ($^\circ \text{C}$.) + 273} on the Centigrade absolute scale,

e.g. $15^\circ \text{C} = 15 + 273 = 288^\circ \text{A}$.

The Centigrade absolute scale is now often called the Kelvin scale (e.g. 288° K.).

The Fahrenheit absolute scale.—In this, the decrease in volume at freezing-point per 1° F. decrease in temperature is

$$\frac{1}{273} \times \frac{5}{9} = \frac{1}{491.4}.$$

As 0° C. corresponds to 32° F., then the volume would become zero at $(-491.4 + 32) = -459.4^{\circ}$ F. (often taken as -460° F.). This, then, is absolute zero on the Fahrenheit scale.

$\therefore$ Fahrenheit absolute temperature = {reading ($^{\circ}$ F.) + 459.4}.

17. Charles' law

Charles' law can now be restated as follows:

"The volume of a given mass of gas varies directly as the absolute temperature, provided that the pressure is kept constant throughout."

Thus $V \propto T$ (P being constant),

or $V = CT$, where C is a constant,

from which $\frac{V}{T} = C$.

As with Boyle's law, this can be rewritten as

$$\frac{V_1}{T_1} = C = \frac{V_2}{T_2}, \quad \text{or} \quad \frac{T_1}{T_2} = \frac{V_1}{V_2},$$

where the suffixes (1) and (2) refer to different states.

Notice that, if the volume is kept constant, the law can be written:

"The absolute pressure of a given mass of a gas varies directly as its absolute temperature, provided that the volume is kept constant throughout."

Thus $P \propto T$ (V being constant),

from which $\frac{P}{T} = \text{a constant},$

and $\frac{P_1}{T_1} = C = \frac{P_2}{T_2}.$

18. The gas law. Combination of Boyle's and Charles' laws

The pressure, volume, and temperature of a gas may all change at the same time. So far we have only considered the physical conditions resulting when one of these quantities remains fixed. A combination of Boyle's and Charles' laws, known as the Gas Law, enables us to determine the physical conditions of a gas resulting from changes of pressure, volume, and temperature when the conditions prior to the changes are known.

Boyle's law states that $V \propto \frac{1}{P}$.

Charles' law states that $V \propto T$.

$$\therefore V \propto \text{both } \frac{1}{P} \text{ and } T, \text{ or } V \propto \frac{T}{P}.$$

We can now write $V = C \frac{T}{P}$, where C is some constant. Cross-multiplying, we get

$$PV = CT,$$

and

$$\frac{PV}{T} = C.$$

This result may be written in the form of a law, as follows:

"The product of the absolute pressure and the volume divided by the absolute temperature of a quantity of gas is constant."

Consequently, the ratio PV/T for a gas under one set of conditions will be the same as that for some other set of conditions, i.e.

$$\frac{P_1 V_1}{T_1} = C = \frac{P_2 V_2}{T_2},$$

the suffixes (1) and (2) refer to these states, or conditions. If we mean of this law, then, if we know the conditions exist in one state, we can readily find them for some other state, set of conditions. This is useful when reducing the conditions of a gas to those which would exist under **normal temperature and pressure** (abbreviated to N.T.P.). These conditions are usually taken to mean a temperature of 0°C. , and an atmospheric pressure of 14.7 lb. per sq. in.

Example 7.—A quantity of gas occupies a volume of 15 cu. in. at a pressure of 60 lb. per sq. in. (absolute), and a temperature of 27° C. Find its volume at N.T.P.

$$\frac{P_1 V_1}{T_1} = C = \frac{P_2 V_2}{T_2}.$$

$$\frac{60 \times 15}{273 + 27} = \frac{900}{300} = 3 = \frac{14.7 V_2}{273}.$$

$$14.7 V_2 = 819.$$

$$V_2 = \frac{819}{14.7} = 55.6 \text{ cu. in.}$$

Example 8.—At the end of its suction stroke an internal-combustion engine cylinder is filled with air at 12 lb. per sq. in. (absolute), and at a temperature of 60° F. When the compression stroke has reduced the cylinder volume to one-fifth of its original value, the pressure is 72 lb. per sq. in. (absolute). Find the temperature of the air.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

It is required to find T_2 .

$$T_2 = \frac{P_2 V_2 T_1}{P_1 V_1} = \frac{72 \times 1 \times (460 + 60)}{12 \times 5} = 624^\circ \text{ F. absolute,}$$

or $624 - 460 = 164^\circ \text{ F.}$

EXERCISES ON CHAPTER XXV

Section A

1. Plot a graph which can be used for converting °F. to °C. Read off from your graph the Centigrade temperature equivalent to 55° F., and the Fahrenheit temperature equivalent to 89° C.

(S.W.E.T.C.)

2. Change the following temperatures in degrees Fahrenheit to degrees Centigrade: (i) 60° F., (ii) 0° F., (iii) 113° F., (iv) -4° F.; and (b) the following to degrees Fahrenheit: (i) 37° C., (ii) 1600° C., (iii) -38.9° C., (iv) 4° C.

3. Give a definition of the coefficient of linear expansion of a solid. What amount of expansion will take place in a steam pipe 120 ft.

long, the temperature of the steam being 350°F. ? Take the atmospheric temperature to be 50°F. and the coefficient of linear expansion $\cdot 000\ 00656$. (S.W.E.T.C.)

4. If steam be admitted into the cylinder of a steam engine at a pressure of 75 lb. per sq. in. above that of the atmosphere (take as 15 lb. per sq. in.), and the cut-off is one-third of the stroke, calculate the pressures at half, three-quarters, and the end of the stroke. Draw to scale a diagram which shows how the pressure varies during the stroke. (U.L.C.I.)

5. What do you understand by Charles' law? 10 cubic feet of air are heated from 10° to 16°C. , the pressure remaining constant. Find the final volume. (U.E.I.)

6. In a gas-engine trial it is found that 450 cu. ft. of gas are used per hour at a pressure of 4 in. of water. What would be the gas consumption at atmospheric pressure, i.e. at 15 lb./sq. in. absolute, without variation of temperature? You may assume that atmospheric pressure will support a column of water 34 ft. high.

If this gas consumption at atmospheric pressure was at a temperature of 18°C. , what would be the gas consumption at atmospheric pressure and at a temperature of 0°C. ? (U.E.I.)

Section B

7. Describe a method of measuring the coefficient of linear expansion of a metal rod. Give details of the calculations involved. Draw a diagram of a thermostat and explain its principles. (C.P.E.)

8. Describe some method for determining the relation between the pressure and the temperature of a given mass of gas at constant volume. A gas cylinder contains a gas at a pressure of 21 atmospheres when the temperature is 10°C. Calculate the pressure of the gas if the temperature were raised to 55°C. (C.P.E.)

9. Explain the meaning of the term "coefficient of expansion of a gas at constant pressure". Give an account of an experimental method by means of which its value for air can be determined. A balloon contains a gas at 13°C. , its volume being 910 cubic metres. What volume will it occupy at 35°C. , assuming the balloon to be expansible and the pressure to remain unchanged? (C.P.E.)

10. A centigrade thermometer is calibrated and it is found that at freezing-point the thermometer reads $2\cdot 5^{\circ}$, and at boiling-point it reads 96° . (a) What will the thermometer read when the true temperature is 65°C. ? (b) What is the true temperature when the thermometer reads 92°C. ? (S.W.E.T.C.)

11. (a) Show why the linear, superficial, and volumetric coefficients of expansion of a substance are in the ratio 1 : 2 : 3.

(b) A copper sphere (linear coefficient of expansion 0.000017 per °C.) of 10-cm. radius is heated at the rate of 2.3° C. temperature rise per sec. At what rate is it (i) increasing in surface, (ii) in volume? (S.W.E.T.C.)

12. An air compressor has a cylinder 12 in. diameter and 36 in. long. At the beginning of the compression stroke the cylinder is filled with air at a temperature of 18° C. and pressure 13.6 lb./sq. in. abs. After 2 ft. of the compression stroke the pressure is 60 lb./sq. in. abs. What is the temperature? (S.W.E.T.C.)

13. State Boyle's law. Describe briefly how you would verify it. A bubble of gas occupies 20 cu. mm. at the bottom of a pond of water 3.4 metres deep. A barometer at the surface of the pond stands at 71 cm. of mercury. Find the volume of the bubble when it reaches the surface of the pond. (S.G. of mercury 13.6.) (C.P.E.)

14. Describe in detail the construction of a mercury centigrade thermometer including the graduation. Give reasons for the suitability of mercury as the thermometric liquid. (C.P.E.)

15. Describe with the aid of a suitable diagram a maximum and minimum thermometer. Compare the properties of alcohol and mercury as thermometric liquids. (C.P.E.)

16. In the compression stroke of a diesel engine one-tenth of a cubic foot of air at a pressure of 13.5 lb./in.² absolute and temperature 100° F. is compressed to a pressure of 420 lb./in.² absolute and temperature 1200° F. Assuming the compression to follow the combined laws of Boyle and Charles, calculate the volume of the air at the end of compression. (Poly.)

17. Give an account of three uses whereby the expansion of solids can be utilized. Clearly labelled diagrams of these applications should be drawn and the functions of the parts should be carefully explained. (C.P.E.)

CHAPTER XXVI

Calorimetry

1. Measurement of heat

It should be appreciated that there is a definite distinction between "temperature" and "heat". Temperature is a measure of the **degree** or **intensity** of heat, but does not take into account the total **quantity** of heat possessed by a body. For instance, a block of red-hot iron may have a temperature of 900°C. ; if it is dropped into a tank of water at room temperature, the iron will be cooled, and the temperature of the water raised by an amount depending upon the quantity of water present. The total quantity of heat present in the iron and the water together will, however, be the same after quenching as before. Only the distribution of the heat will have been altered—the iron losing heat and the water gaining heat.

The **quantity of heat** possessed by a body will depend upon:

- (1) its temperature;
- (2) its mass;
- (3) the material of which it is composed.

The measurement of quantity of heat is called **calorimetry**, and the units employed depend upon the effect heat has of increasing the temperature of bodies.

2. Units of heat

In scientific work, the unit quantity of heat is the **calorie**. It may be defined as the quantity of heat required to raise the temperature of one gram of water at 15°C. by one degree Centigrade. It is sometimes called the **gram calorie**, or **small calorie**, to distinguish it from the **large, great** or **kilocalorie**, which is equal to 1000 calories.

In engineering work, the unit quantity of heat commonly employed is the **British Thermal Unit (B.Th.U.)**. It is defined

as the quantity of heat required to raise the temperature of one pound of water from 60° F. to 61° F. For all practical purposes this can be taken to mean the quantity of heat required to raise the temperature of one pound of water by 1° F.

The **Centigrade heat unit (C.H.U.)**, or pound centigrade heat unit (lb. C.H.U.), is sometimes employed. It may be defined as the quantity of heat required to raise the temperature of one pound of water through one degree Centigrade.

A unit of heat used particularly by gas engineers for the sale of gas is the **therm**. A therm is equal to 100,000 B.Th.U.

Conversion of heat units

$$1 \text{ C.H.U.} = \frac{9}{5} \text{ B.Th.U.} = 453.6 \text{ gram calories.}$$

$$1 \text{ B.Th.U.} = \frac{5}{9} \text{ C.H.U.} = 252 \text{ gram calories.}$$

3. Specific heat

The quantity of heat possessed by a body depends upon the material of that body. If equal weights of two different materials are heated to the same temperature under identical conditions, it will take longer to heat one than the other. That is to say, different materials have, weight for weight, the property of absorbing differing quantities of heat for the same temperature rise. This property is referred to as the **specific heat** of the material.

Specific heat may be defined as **the amount of heat required to raise unit mass of the substance through one degree temperature rise.**

As the various heat units are based on raising unit quantity of water through 1° temperature rise, we can also define the specific heat of a substance as **the ratio between the quantity of heat required to raise its temperature 1° to the quantity of heat required to raise the temperature of the same weight of water through 1°.**

Specific heat

$$= \frac{\text{heat required to raise unit weight of a substance through } 1^{\circ}}{\text{heat required to raise unit weight of water through } 1^{\circ}}.$$

Note that it does not matter what units are used for weight and temperature, provided that the same are used in both

numerator and denominator; also that, defined in this manner, specific heat is a number and needs no qualifying units.

If we say that the specific heat of copper is 0.100, it means that 0.100 B.Th.U. will raise the temperature of 1 lb. of copper 1° F., or that 0.100 calorie will raise the temperature of 1 gm. of copper 1° C.

The quantity of heat (H) given out, or received, by a quantity of a substance depends upon

- (a) the weight of the substance (W);
- (b) the specific heat of the substance (s);
- (c) the change in temperature ($t_1 - t_2$).

Thus
$$H = W \times s(t_1 - t_2).$$

If H is in B.Th.U., W must be in lb., and $(t_1 - t_2)$ in °F.

If H is in calories, W must be in gm., and $(t_1 - t_2)$ in °C.

Example 1.—Taking the specific heat of copper to be 0.1, find how many B.Th.U. are needed to raise the temperature of 60 lb. of copper from 60° F. to 120° F.

$$\begin{aligned}\text{Heat required (B.Th.U.)} &= 60 \times 0.1(120 - 60) = 60 \times 6 \\ &= 360 \text{ B.Th.U.}\end{aligned}$$

4. Mixing bodies of unequal temperatures

If we mix equal quantities of water at, say, 50° C. and 80° C. respectively, the temperature of the mixture will be $\frac{1}{2}(50 + 80) = 65^\circ \text{C.}$

But the temperature of a mixture of quantities of substances having different specific heats cannot be found so readily. We must take into account their specific heats. The resulting temperature of the mixture can be found by means of the formula

$$\text{Final temperature} = \frac{W_1s_1t_1 + W_2s_2t_2 + W_3s_3t_3 + \text{etc.}}{W_1s_1 + W_2s_2 + W_3s_3 + \text{etc.}},$$

in which

W_1, s_1, t_1	=	weight,	specific heat,	temperature of one substance,	
W_2, s_2, t_2	=	"	"	"	second "
W_3, s_3, t_3	=	"	"	"	third " .

This formula only holds true provided that no change of state occurs when mixing, such as ice melting or water vaporizing.

Example 2.—Find the temperature of a bath of 60 lb. of water initially at 60° F., after 10 lb. of copper at 120° F. has been dropped into it. (Take specific heat of copper as 0.1.)

$$\begin{aligned}\text{Final temperature} &= \frac{60 \times 1 \times 60 + 10 \times 0.1 \times 120}{60 \times 1 + 10 \times 0.1} = \frac{3600 + 120}{60 + 1} \\ &= \frac{3720}{61} = 61^\circ \text{ F. approx.}\end{aligned}$$

5. Water equivalent

The water equivalent of a body is the product of its weight and its specific heat.

$$\text{Water equivalent} = Ws,$$

i.e. it is that weight of water which would take up the same quantity of heat as the body when subjected to the same temperature rise.

It is sometimes called the body's heat or thermal capacity.

Example 3.—Find the water equivalent of a tank of cast iron weighing 100 tons and containing 60 tons of water. (S.H. of cast iron = 0.129.)

$$\begin{aligned}\text{Water equivalent} &= 100 \times 0.129 + 60 \times 1 \\ &= 129 + 60 = 189 \text{ tons.}\end{aligned}$$

6. Change of state. Sensible and latent heat

The state of a substance, solid, liquid, or gaseous, is determined by the speed and behaviour of its molecules, i.e. it largely depends upon temperature (see Chap. XXVII, § 2).

Starting with a solid body, the application of heat causes its temperature to rise until it begins to liquefy. Its temperature then remains *constant* until melting is completed. Further application of heat causes the temperature of the liquid to rise, until boiling begins; then the temperature again remains constant until all of the liquid has been turned into a gaseous vapour.

That heat which is used in causing temperature rise is referred to as **sensible heat**, as it is evident to the senses.

That heat which is used in changing the state of a substance is referred to as **latent heat**, as it lies hidden from the senses.

Latent heat is expressed in terms of the number of heat units required to change the state of unit mass of a substance, e.g. the latent heat of ice is 80 C.H.U. per lb. or 144 B.Th.U. per lb.

The latent heat required to melt a given substance is different from that required to evaporate it. The former value is referred to as the **latent heat of fusion**, and the latter as the **latent heat of evaporation** (or **vaporization**).

The value of latent heat for a given material also varies with temperature and pressure; therefore these should be specified when making a complete statement.

7. Conversion of ice to steam

The facts stated in the previous paragraph are illustrated in fig. 26.1, which shows the results of heating one gram of



Fig. 26.1

ice at -40°C ., and eventually converting it into steam at normal atmospheric pressure.

It will be seen that 40 cal. are needed to bring the ice to freezing-point (0°C .). Then, as the latent heat of fusion of ice is 80 cal. per gm., 80 cal. are needed to melt the ice, without the temperature changing. Then 100 cal. are used to bring the

water to boiling-point (100°C.), and no less than 539.22 cal. are then needed to convert the water into steam at this temperature. A further application of heat after evaporation * (the steam remaining at atmospheric pressure) will increase its temperature (i.e. it will superheat the steam). Taking the specific heat of steam as 0.5 , then 10 cal. will be needed to bring the steam to 120°C. Therefore the amount of heat required during the process will be:

$$40 + 80 + 100 + 539.22 + 10 = 769.22\text{ cal./gm.}$$

$$\begin{array}{ccccccccc} \left(\begin{array}{c} \text{sensible heat} \\ \text{of solid} \end{array}\right) & + & \left(\begin{array}{c} \text{latent heat of} \\ \text{fusion of ice} \end{array}\right) & + & \left(\begin{array}{c} \text{sensible heat} \\ \text{of liquid} \end{array}\right) & + & \left(\begin{array}{c} \text{latent heat of} \\ \text{vaporization} \end{array}\right) & + & \left(\begin{array}{c} \text{superheat} \\ \text{of steam} \end{array}\right) \end{array}$$

8. Total heat of steam

The value given in the previous paragraph is the total heat of the steam per gram from -40°C. The total heat could obviously be stated from any particular temperature, but in practice it is given from the freezing-point of water.

9. Dryness fraction

If the full amount of latent heat of evaporation has been supplied to a quantity of steam it is termed **dry steam**, i.e. no unevaporated water will be present in it. Sometimes steam contains a proportion of such unevaporated water. It is then said to be **wet**, meaning that the full amount of latent heat required for complete evaporation has not been supplied. The ratio

$$\frac{\text{latent heat actually supplied}}{\text{latent heat required for complete evaporation}}$$

is known as the **dryness fraction** (q) of the steam.

Example 4.—In the case of the conversion of ice to steam already considered above, find the total heat of the steam per gram from 0°C. if its dryness fraction is 0.9 .

* This heat must be applied to the steam when it is *not* in contact with water.

Total heat of steam from 0° C.

$$\begin{aligned} &= \text{sensible heat of liquid} + (\text{latent heat of steam} \times 0.9) \\ &= 100 + (539.22 \times 0.9) = 585.3 \text{ cal./gm.} \end{aligned}$$

N.B. The steam must be 100 per cent dry before it can receive any superheat.

10. Condensation

If a vapour is cooled sufficiently, it will change into its liquid state. The process consists of the removal of the latent heat of evaporation of the vapour by means of the cooling medium, and is termed **condensation**.

11. Effect of pressure on boiling-point

We have seen that under normal atmospheric pressure (14.7 lb. per sq. in.), the boiling-point of water is 100° C. (212° F.). If this pressure is increased, the boiling-point is raised; if the pressure is decreased, the boiling-point is lowered.

It can be shown that, for any given pressure, there is only one temperature for the steam whilst it is in contact with the water from which it has been generated.

This boiling-point is usually referred to as the **saturation or evaporation temperature** of the water at the given pressure.

12. Steam Tables.

Values of saturation temperature, sensible heat of the liquid (liquid heat), latent heat of the vapour, and total heat for various pressures have been determined with great accuracy, and are published as Steam Tables. These figures are of considerable use when making calculations dealing with thermodynamics, heat engines, etc. Tables with values in either Centigrade or Fahrenheit units are available, the latter being generally used in British engineering practice.

N.B. In these tables heat values are given in heat units per pound from the freezing-point of water.

Use of Steam Tables

An extract from the Steam Tables prepared by Professor Callendar is given in Table 6, pp. 452-3.

The column headed h gives the **total heat of the water**, i.e. the number of heat units required to raise the temperature of one pound of water from freezing-point to boiling-point at the stated absolute pressure.

The total heat of water in B.Th.U./lb. can be found approximately, without the table, by subtracting 32° from its particular temperature. This does not give the exact value, because amongst other things the specific heat of water varies slightly from unity at different temperatures. However, this method is accurate enough for most practical purposes.

The **latent heat of vaporization**, given in the column headed L , is the number of heat units required to change one pound of water, at the boiling-point corresponding to the given pressure, into dry saturated steam at the same temperature.

It will be noticed that the amount of latent heat diminishes as the pressure (and hence the boiling-point) is increased. Eventually a **critical state** is reached where the latent heat is zero, the liquid going straight into the vapour state at this **critical temperature and pressure** without further addition of heat.*

It should be realized that, below the critical state, latent heat is being supplied whenever steam is being generated. The full quantity of latent heat given in the Steam Table must be supplied in order to produce **dry saturated steam**. If it is not, then the steam is **wet**. Any further heat supplied after all the water present has been converted into steam goes to **superheat** the steam.

The **total heat of dry saturated steam** given in the column headed H_s is the total number of heat units required to raise the temperature of one pound of water from freezing-point to boiling-point at the stated absolute pressure, and then to convert it all into steam at that pressure. Consequently,

$$H_s = h + L.$$

The total heat of wet steam of dryness fraction q is given by

$$H = h + qL.$$

Notice that

$$q = \frac{H - h}{L}.$$

* This state, however, is not reached in normal usage, as it does not occur until the pressure reaches $3206.2 \text{ lb./in.}^2 \text{ abs.}$, the corresponding saturation temperature being 705.4° F.

If it is required to find the amount of heat necessary to raise one pound of water to the boiling-point from a temperature higher than freezing-point, and then to convert it into steam, we must deduct the liquid heat initially in the water, i.e. for all practical purposes

$$\text{heat necessary} = H_s - (t - 32) \text{ B.Th.U./lb.,}$$

or

$$H_s - t \text{ C.H.U./lb.}$$

where t is the initial temperature of the water.

The column headed V_s gives the volume of 1 lb. of dry saturated steam at the stated pressure, i.e. its **specific volume**,

The specific volume of wet steam will be qV_s .

The weight of steam in a given vessel will be:

$$w(\text{lb}) = \frac{\text{vol. of vessel (cu. ft.)}}{\text{specific vol. of steam (cu. ft./lb.)}}.$$

13. Total heat of superheated steam

Although it has been mentioned earlier that the specific heat of steam is about 0.5, this value should not be used for calculating the **total heat of superheated steam**, except for very rough calculations, as the actual specific heat varies from about 0.47 to 0.9. Steam Tables for superheated steam are available and should be used.

Examples on the use of the Steam Tables (p. 452).

Example 5.—Determine from the Steam Tables the total heat in 1 lb. of dry saturated steam at a pressure of 120 lb./in.² (absolute). Give also the saturation temperature of this steam.

Referring to the table, it will be seen that the total heat (H_s) is 1191.4 B.Th.U. per lb., whilst the saturation temperature (t) is 341.3° F.

Example 6.—If the dryness fraction of the steam in Example 5 is 0.9, what will the total heat be now?

A dryness fraction of 0.9 implies that only 0.9 of the full amount of latent heat has been supplied.

$$\begin{aligned} \therefore \text{Total heat} &= h + 0.9(L) = 312.5 + 0.9(878.9) \\ &= 1103.51 \text{ B.Th.U. per lb.} \end{aligned}$$

Example 7.—Find the amount of heat required to produce one pound of steam in the condition given in Example 6, if the feed water entering the boiler is at a temperature of 72° F.

Heat required = total heat — $(t - 32)$

$$= 1103.51 - (72 - 32) = 1063.51 \text{ B.Th.U. per lb.}$$

Example 8.—2 lb. of steam at atmospheric pressure and of dryness fraction 0.9 is blown into 4 gal. of water at 60° F. Assuming no losses, find the temperature to which this water will rise.

Heat lost by steam = heat gained by water.

Let t be the final temperature, and q the dryness fraction. Then

$$\text{Wt. of steam} \times ((h - t) + qL) = \text{wt. of water} \times (t - 60).$$

From Steam Tables, $h = 180.1$, and $L = 970.6$,

$$\therefore 2((180.1 - t) + 0.9 \times 970.6) = 4 \times 10(t - 60),$$

$$(360.2 - 2t) + 1747 = 40t - 2400,$$

$$42t = 360.2 + 1747 + 2400,$$

$$t = 107.3^\circ \text{ F.}$$

EXERCISES ON CHAPTER XXVI

Section A

1. (a) Differentiate between heat and temperature. State the units which are commonly used for their measurement.

(b) A steel steam pipe is 300 ft. 6 in. long at a temperature of 15° C. What is its increase in length when wet steam at 200 lb. per sq. in. absolute flows through it. The coefficient of linear expansion of steel = 0.000 006 per °F. (Use steam tables for temperature of steam.) (S.W.E.T.C.)

2. (a) What is latent heat?

(b) The latent heat of steam at atmospheric pressure is 539 C.H.U./lb. One pound of such steam is condensed by letting it flow into 20 lb. of water at a temperature of 10° C. To what temperature is the water raised?

3. Define specific heat.

If a piece of metal weighing 2 lb. is heated to 120° C. and dropped into 6 lb. of water at 15° C., determine the final temperature of the water, neglecting losses. Specific heat of metal to be taken as 0.011.

4. What is meant by (a) liquid heat, (b) latent heat of steam? Find the total heat of 1 lb. of steam of dryness 0.9, the pressure being 130 lb. per sq. in. absolute, given liquid heat = 177.07, latent heat 487.76 C.H.U. per lb.

5. A piece of metal of weight 400 gm. and temperature 80°C ., the specific heat of which is 0.095, is placed in a vessel containing 900 c.c. water at a temperature of 10°C . To what temperature will the water rise if no heat is absorbed by the vessel, and if the water is well stirred? What is the water equivalent of the piece of metal?

6. How much heat is required to raise the temperature of 0.2 lb. of copper through 1050°C .? Assume the specific heat of copper to be 0.1. If this heated copper is plunged into 2.2 lb. of water at 15°C ., what will be the final temperature of the water, assuming no losses by radiation? (U.E.I.)

7. What is the specific heat of a substance?

If 100 gm. of water at 100°C . are poured into 120 gm. of turpentine at 9°C ., and the resulting temperature is found to be 70°C ., what is the specific heat of the turpentine? What reason have you for supposing that the result you give may not be correct? (U.L.C.I.)

8. Define sensible heat, latent heat, and total heat.

The following figures have been extracted from a steam table. Complete the table and calculate the latent heat contained in 8 lb. dry steam at a pressure of 190 lb. per sq. in. absolute.

Pressure lb. per sq. in. absolute	Sensible heat, C.H.U.	Latent heat, C H U.	Total heat, C H.U.
160		480.61	667.22
190	194.94		669.13
220	202.32	468.38	

(U.L.C.I.)

Section B

9. Distinguish between boiling and evaporation. What effect has a change of atmospheric pressure on the boiling-point of a liquid? Illustrate by suitable experiments, and give examples of any practical uses of the effects. (C.P.E.)

10. Define the term specific heat. Describe an experiment to find the specific heat of a solid, giving full details of the experimental procedure and of the calculations involved. 600 grams of a liquid at 19°C . are contained in a copper calorimeter of mass 300 gm., and

a piece of material of mass 100 gm. is dropped into it from a furnace at a temperature of 600°C ., the resulting temperature being 69°C . Find the specific heat of the substance. (S.H. of the liquid = 0.5. S.H. of copper = 0.1.) (C.P.E.)

11. Explain what is meant by the terms thermal capacity and latent heat of vaporization of a liquid. A refrigerator produces ice by the evaporation of ammonia. How much liquid ammonia must be evaporated to convert 800 gm. of water at 20°C . into ice at -4°C .? Assume no loss of heat. (Latent heat of ammonia = 340 gm. cal.; latent heat of ice = 80 gm. cal.; specific heat of ice = 0.5.) (C.P.E.)

12. Define latent heat of fusion of a solid. Describe some method for the determination of the latent heat of fusion of ice. What mass of ice at -10°C . must be mixed with 600 gm. of water at 20°C ., contained in a vessel of water equivalent 27 gm., to lower its temperature to 10°C .? (Latent heat of ice = 80 gm. cal. Specific heat of ice = 0.5.) (C.P.E.)

13. Steam is generated in a boiler at 250 lb. per sq. in., its condition being 93 per cent dry.

If the heat supplied per lb. of steam was 918 B.Th.U., what was the temperature of the feed water?

The steam passes from the boiler to a superheater, where its temperature is raised to 501°F . How much heat is supplied per lb. of steam in the superheater? (Use steam table below.) (U.L.C.I.)

Abs. pressure, lb. per sq. in.	Saturated steam Tempera- ture $t^{\circ}\text{F}$.	Heat in B.Th.U. h	H_s	Total heat of superheated steam, degrees of superheat		
				80	100	120
	257.6	226.3				
250	401.0	376.1	1202.1	1251.7	1263.4	1274.9

14. Explain the terms "specific heat" and "thermal capacity". Describe the experimental determination of the specific heat of a liquid.

If the price of gas is 1s. 4d. per therm, how much will it cost to heat 2000 gal. of water through 150°F . (1 gal. of water weighs 10 lb.) (1 therm = 100,000 B.Th.U.) (C.P.E.)

15. (a) Explain the terms, sensible, latent, and total heat of 1 lb. of superheated steam.

(b) Find the heat required to produce 1 lb. of steam at 150 lb. per sq. in. absolute and dryness fraction 0.9 from feed water at 140°F .

(c) If this steam then passes through a superheater until the temperature is 400°F ., find the additional heat required per lb.

(For superheated steam assume specific heat to be 0.5.) (U.E.I.)

16. (a) Fill in the blanks of the following steam table:

Pressure	t	h	Fahrenheit Units L	H_g
10	193.25	—	981.90	1143.02
14.689	212	180.00	—	1150.74
15	213.05	181.04	970.12	—
120	341.15	312.34	882.72	1195.06

(b) What heat is required to change 20 lb. of water at 62° F. into steam of dryness fraction 0.9, the pressure being 120 lb. per sq. in. absolute?
(S.W.E.T.C.)

17. (a) Define the terms (i) specific heat, (ii) water equivalent.

(b) A piece of copper of weight 1.25 lb. is placed in a current of hot gas and then immersed in 5 lb. of water at temperature 60° F. If the temperature of the water is raised thereby to 65° F., calculate the temperature of the copper. (Assume the water to receive all heat lost by the copper. Sp. ht. of copper = 0.094.) (S.W.E.T.C.)

CHAPTER XXVII

Heat Energy and its Transmission

1. Supply of heat

Heat can be derived from the following sources:

(1) physical, (2) chemical, (3) mechanical.

Physical sources of heat include those derived directly from natural sources, of which the most important is the sun. In fact, it seems not unlikely that all heat in this world emanated originally from the sun. This heat reaches the world by radiation from the sun and produces the well-known evaporation of water, warming of substances, and generally the storing of energy. The earth itself possesses natural heat, as is evident if one goes down a mine. The deeper one goes, the higher becomes the temperature. This is thought to be because the centre of the earth has not cooled down from the state which existed when the earth was flung from off the sun as a nebulous mass of molten matter in the dim ages of the past. Owing to the earth being such a poor conductor of heat, this interior natural heat has little effect in warming the surface, not raising the surface temperature by more than about $.05^{\circ}$ F.

N.B. Physical heat also includes that derived from atomic disintegration and other intra-atomic actions. Such actions are considered to be the source of the sun's heat (see Chap. IV).

Chemical sources of heat are derived from the combination of substances that act chemically on one another, heat being evolved. When this heat is due to the combination of oxygen with carbon, hydrogen, or certain other substances, and it is produced in a sufficient quantity to maintain the action, the process is called **combustion**, or more generally, **burning**. Note that oxygen *must* be present for combustion to take place. A chemical source of heat which once ignited maintains the process of combustion is called a **fuel**.

Mechanical sources of heat are derived from processes such as friction, hammering, and compression which involve the

expenditure of work. The production of heat by friction, as for instance the heating of a bearing, has been considered in Chap. XIV. Hammering a piece of metal will make it hot. The compression of a gas causes a rise in temperature, as is evident when a bicycle pump becomes hot after vigorous use.

2. The kinetic theory of heat

Up to the nineteenth century heat was regarded as an intangible fluid, called "caloric", present in hot bodies.

About the middle of the nineteenth century, however, a new theory began to be evolved. A hot body was regarded as one in which the molecules, or atoms, were in a rapid state of movement or vibration. Cooling the body resulted in a slowing down of this movement, whilst heating it up further increased the rate of movement. This theory is now generally accepted.

If heat is continually applied to a solid, its molecular vibrations increase in intensity, i.e. its temperature rises, and eventually the movement becomes so great that the solid becomes soft and eventually changes into liquid form; the molecules move with great violence and collide with each other frequently, so that they spread out over any plane surface. Further application of heat still further increases the rate of molecular movement and causes the molecules to leave the boundaries of the material in all directions, so forming a gas.

If this gas is enclosed in a vessel, it will bombard the walls, i.e. it will exert pressure on them. The greater the temperature to which the gas is raised, the greater becomes the force with which its molecules hit the containing walls.

Whilst there appears to be no upper limit to the speed of molecular movement, and hence of temperature, it will be apparent that there is a lower limiting temperature, namely that temperature at which all molecular movement ceases. This temperature is referred to as **absolute zero**.

3. Heat as a form of energy

It follows from the kinetic theory of heat that heat is a form of energy, being due to the kinetic energy of moving or vibrating atoms. In accordance with the Law of Conservation of Energy, it will therefore be convertible to other forms. Also, other forms of energy can be used to produce heat. The action, it will be seen, is reversible. Many instances of this could be given.

For example, consider a piston and cylinder (fig. 27.1*a*). Doing work on the piston by forcing it down compresses the air, causes more frequent molecular collisions, and thereby produces heat. This production of heat will be evident to anyone who has pumped up a tyre. On the other hand, if heat is applied to the cylinder (fig. *b*), the air in it will tend to expand owing to its increased molecular activity; the pressure set up will force the piston upwards, thereby performing work.

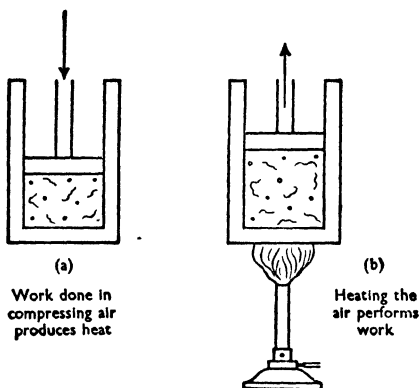


Fig. 27.1.—Heat and work

Hammering a piece of iron causes it to become hot, i.e. work is converted into heat. Similarly, rubbing produces friction, and heat is evolved. Actually, if two pieces of ice are rubbed together, sufficient heat can be produced to melt them.

The heat of a burning fire can be used to boil water, and the steam evolved can be used to operate a steam engine, i.e. heat can be converted into work. Similarly, the heat produced by burning a petrol/air mixture in the cylinder of an internal-combustion engine is made to do work. The methods of carrying out the heat to work conversion form the subject of Heat Engines.

4. The mechanical equivalent of heat

Experimental work on the conversion of mechanical energy into heat energy was carried out by various people early in the

nineteenth century. Dr. Joule of Manchester around 1843 measured the amount of work which had to be performed in order to produce a given quantity of heat. By means of falling weights he caused a paddle wheel to rotate in a box of water (fig. 27.2). The work done by the falling weights was readily calculated. The resulting rise in temperature given to the

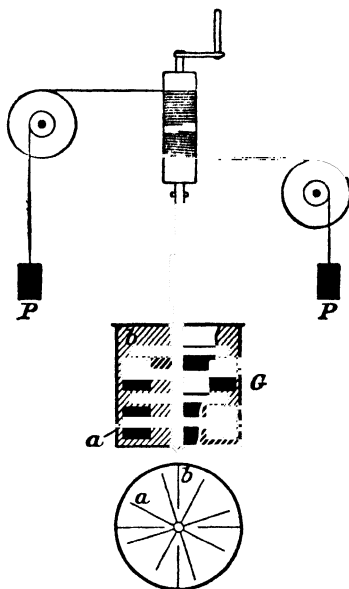


Fig. 27.2

mass of the water was determined, and from this the quantity of heat given out was calculated (taking into account the heat absorbed by the box, and in friction at the bearings, etc.).

By this method Joule determined that 772 ft.-lb. of work had to be done to raise 1 lb. of water through 1° F. Later and more accurate experiments gave 778.3 ft.-lb. of work as the figure, 778 being the usual value taken. Thus 778 ft.-lb. is equivalent to 1 B.Th.U.

This **mechanical equivalent of heat** can be expressed in other units if required, as follows:

$$1400 \text{ ft.-lb.} = 1 \text{ C.H.U.}$$

$$3.1 \text{ ft.-lb.} = 1 \text{ gram calorie.}$$

$$4.2 \text{ joules} = 1 \text{ gram calorie.}$$

$$1055 \text{ joules} = 1 \text{ B.Th.U.}$$

The relationship may be expressed in the form of the equation

$$W = JH,$$

where W = work done, H = heat units generated, J = Joule's equivalent.

The value of Joule's equivalent used will, of course, depend upon the units employed.

5. First and Second Laws of Thermodynamics

It is always possible to convert all the mechanical energy used in doing work into its equivalent in heat, in accordance with the expression $W = JH$. This is called the **First Law of Thermodynamics**.

It is not in general possible, however, to convert the whole of a quantity of heat into its equivalent in mechanical work. Only a fraction is convertible. This constitutes the **Second Law of Thermodynamics**.

The work $\rightarrow$ heat conversion occurs readily at the slightest opportunity, as in cases of friction.

The heat $\rightarrow$ work conversion is more difficult, and special machines, known as "heat engines", have to be employed to carry it out. In accordance with the Second Law, the overall efficiency (or thermal efficiency) of these engines will be a fraction of 100 per cent.

$$\text{Overall thermal efficiency} = \frac{\text{work output}}{\text{equivalent heat input}} = \frac{W}{JH}.$$

6. Heat energy of fuels

The heat energy input to an engine is usually obtained by the combustion of a fuel. The amount of such energy stored in a fuel can be determined experimentally, and is known as the **calorific value of the fuel**. It can be expressed in C.H.U. per

lb., or B.Th.U. per lb., the latter being normally used in engineering practice. The calorific value of good coal may be up to the order of 14,000 B.Th.U. per lb. (see Chap. XXVIII, § 9).

Example 1.—A steam engine burns 1 ton of coal per hour (of calorific value 14,000 B.Th.U./lb.) when developing 1000 h.p. Find its coal consumption per horse-power-hour, and its thermal efficiency.

$$\text{Coal consumption} = \frac{2240}{1000} = 2.24 \text{ lb. per h.p.hr.}$$

$$\begin{aligned} \text{Thermal efficiency} &= \frac{W}{JH} = \frac{1 \text{ h.p.hr.}}{\text{heat input per h.p.hr.}} \\ &= \frac{33,000 \times 60}{778 \times 2.24 \times 14,000} = 0.0812 \text{ or } 8.12 \text{ per cent.} \end{aligned}$$

Example 2.—If the mechanical efficiency of the above engine is 90 per cent, determine the heat lost per hour in friction.

$$\text{Mechanical efficiency} = \frac{\text{output}}{\text{input}} = \frac{1000}{\text{input}} = 0.9.$$

$$\begin{aligned} \text{Input} &= \frac{1000}{0.9} = 1111.1 \text{ h.p.} & \text{Friction losses} &= \text{input} - \text{output} \\ & & &= 1111.1 - 1000 = 111.1 \text{ h.p.} \end{aligned}$$

$$\text{Work converted into heat per hour} = 111.1 \times 33,000 \times 60 \text{ ft.-lb.}$$

$$\begin{aligned} \text{Heat lost per hour (H)} &= \frac{W}{J} = \frac{111.1 \times 33,000 \times 60}{778} \\ &= 283,000 \text{ B.Th.U.} \end{aligned}$$

The thermal efficiency calculated on a b.h.p. basis may be termed **brake thermal efficiency**.

The thermal efficiency calculated on an i.h.p. basis may be termed **indicated thermal efficiency**.

The thermal efficiency of the most efficient steam locomotive engine does not exceed about 8 per cent. A good steam turbine power station, fitted with efficient condensing plant, may have a thermal efficiency approaching 30 per cent. The thermal efficiency of a large Diesel engine may be of the order of 35 per cent.

Example 3.—During an engine trial the circulating cooling water entered the condenser at a temperature of 12° C. and left at 36° C. The quantity of water flowing was 500 lb. in 35 min. Calculate the heat carried away per minute by the cooling water, and give its equivalent in ft.-lb. (U.L.C.I.)

$$\begin{aligned}\text{Heat given to cooling water} &= \text{wt.} \times \text{sp. ht.} \times \text{temp. rise} \\ &= 500 \times 1 \times 24 = 12,000 \text{ C.H.U.}\end{aligned}$$

$$\text{Heat carried away per minute} = \frac{12,000}{35} = 342\frac{2}{7} \text{ C.H.U.}$$

$$\begin{aligned}\text{Energy equivalent of heat carried away per minute} \\ &= 342\frac{2}{7} \times 1400 = 480,000 \text{ ft.-lb.}\end{aligned}$$

7. Thermal efficiency of boilers

In a steam power-plant the heat energy of the fuel has first to be imparted to the water/steam **working fluid** by combustion in a boiler furnace before it can be used by the engine. Consequently, a high boiler thermal efficiency is an essential factor in securing the maximum possible overall thermal efficiency from a steam power-plant. The generally higher thermal efficiency of an internal-combustion engine is largely because a boiler is not required, combustion of the fuel taking place directly *inside* the cylinder.

The thermal efficiency of a boiler may be represented as follows:

Boiler thermal efficiency

$$= \frac{\text{actual heat required to produce } W \text{ lb. of steam}}{\text{calorific value of fuel per lb.}},$$

where W lb. is the quantity of steam produced per pound of fuel.

Example 4.—Find the efficiency of a boiler which generates 8.75 lb. of steam per lb. of coal consumed, from the following data:

Feed water temperature, °F., 100.

Boiler pressure, lb. sq./in. absolute, 170.

Dryness fraction of steam, 0.98.

Calorific value of coal (B.Th.U./lb.), 13,500.

(Use the Steam Table, p. 452.)

(Poly.)

Heat required to produce 1 lb. of steam under the stated conditions

$$= h + gL - (t - 32) = 341.2 + 0.98 \times 855.9 - (100 - 32)$$

$$= 341.2 + 838.78 - 68 = 1112 \text{ B.Th.U.}$$

$$\text{Boiler thermal efficiency} = \frac{1112 \times 8.75}{13,500} = 72.07 \text{ per cent.}$$

8. Electrical equivalent of heat

Joule investigated the heating effect of an electric current by passing it through a conductor. He found that the heat produced was proportional to the square of the current, the resistance of the conductor and the time the current flowed. Further, the equivalent between heat energy generated and electrical energy expended was the same as the mechanical equivalent of heat.

The joule is a unit of energy applicable to both mechanical and electrical problems. Expressed in electrical terms, the joule is the energy expended when a power of one watt is maintained for one second, i.e.

$$1 \text{ joule} = 1 \text{ watt-sec.} = 1 \text{ volt } (E) \times 1 \text{ ampere } (I) \times 1 \text{ sec. } (t)$$

and $\text{joules} = EIt.$

In accordance with Ohm's law, $E = IR$,

$$\therefore \text{energy expended (joules)} = I^2Rt.$$

But Joule's equivalent states that 4.2 joules = 1 gram calorie,

$$\therefore \text{heating effect of an electric current} = \frac{I^2Rt}{4.2} \text{ gram calories.}$$

Further, as 1 kilowatt-hour (kWh) = $60 \times 60 \times 1000 = 3,600,000$ watt-sec. or joules, and

$$1 \text{ B.Th.U.} = 1055 \text{ joules,}$$

$$\text{then } 1 \text{ kWh} = \frac{3,600,000}{1055} = 3412 \text{ B.Th.U.}$$

Example 5.—An electric kettle used on a 210-volt circuit will raise the temperature of a litre of water from 20° C. to boiling-point in 8 min. What is its current consumption, assuming no loss of heat? (4.2 joules = 1 calorie.) (C.P.E.)

1 litre of water weighs 1000 gm.

Temperature rise required = $(100 - 20) = 80^\circ \text{C.}$

Heat required = $1000 \times 80 = 80,000 \text{ gm. cal.}$

But heating effect of a current $= \frac{EIt}{4.2}$ gm. cal.

$$\therefore \frac{EIt}{4.2} = 80,000$$

and $EI = \frac{4.2 \times 80,000}{8 \times 60} = 700 \text{ watts.}$

$$\therefore \text{current consumption (I)} = \frac{700}{210} = 3\frac{1}{3} \text{ amperes.}$$

9. Transmission of heat

Heat may be transmitted from one body to another, and from one part of a body to another part, by three different methods—**conduction**, **convection**, and **radiation**.

(1) Conduction is the means by which heat is transferred by direct contact from one body or part of a body to another. It occurs only when there is a difference of temperature between the two parts, and the heat flows *only from the region of higher temperature to that of lower temperature*, the tendency being to equalize the temperature of the whole body. The rapidity of the heat flow depends upon the temperature difference, and the nature of the materials present.

The action of heat conduction can be appreciated with reference to the kinetic theory of heat. The molecules of the heated substance are in rapid vibratory motion and hence they collide readily with adjacent but cooler molecules, setting these dancing in rapid motion also (or, rather, increasing their rate of vibration). Consequently, heat spreads outwards from the region of high temperature (or of rapid molecular movement).

All materials conduct heat to some extent, but metals have the highest **thermal conductivity**. Such materials as glass, wool, wood, granulated cork, and paper are poor conductors, and can be referred to relatively as **non-conductors**.

The **miners' safety lamp** makes use of the high thermal conductivity of metal gauze. The lamp flame is surrounded by the metal gauze, which conducts the heat away rapidly and prevents the outside atmosphere from reaching a temperature likely to ignite any gas (e.g. fire-damp) present. This gas may ignite inside the lamp (when it burns with a blue flame), but

it cannot cause the temperature of the outside of the gauze to reach the "flash-point or ignition-point" of the gas. The action is illustrated in fig. 27.3. If the gauze is placed over a bunsen flame, only the gas below it will burn (fig. a). If, however, the gas is first lit above the gauze, the gas below it will not burn (fig. b).

The poor thermal conductivity of clay, asbestos, felt, and the other materials previously mentioned is utilized by lagging pipes, tanks, etc., with them. Heat losses can thus be reduced in steam pipes, etc., and cold-water pipes can also be prevented from losing their existing heat, and so are protected to some extent from the effects of frost.

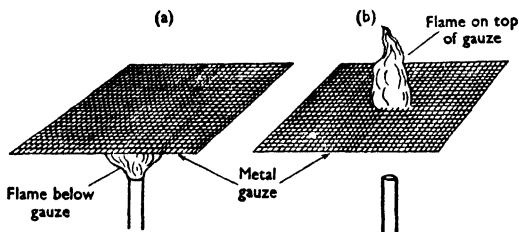


Fig. 27.3.—Illustrating thermal conductivity

(2) **Convection** is the means by which heat is conveyed from one place to another by heated portions of matter, acting as "carriers". These carriers collect heat from the hotter portion of the substance and rise in consequence, setting up what is known as "convection currents". The carriers surrender their heat when they reach the colder portion of the substance.

Since convection currents require the substance to be capable of being set in motion, convection can only take place in liquids and gases.

Convection in Liquids.—Domestic hot-water circulating systems, boilers (particularly the water-tube type), and motor-car cooling systems make use of convection. The process of convection may be either "forced" or "free", depending upon whether the fluid is moved by some external agency, or is moved by "natural" differences in density caused by the heating process. Diagrams of a natural-circulation domestic

hot-water system and of a forced-circulation motor-car cooling system are given in figs. 27.4 and 27.5.

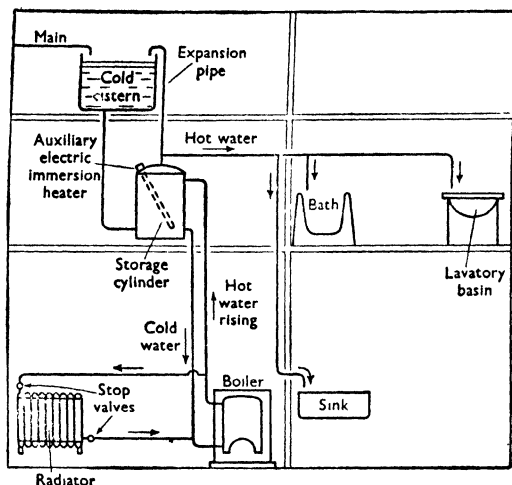


Fig. 27.4.—Domestic hot-water system (natural circulation)

In the hot-water system, the heated water in the back boiler expands, so becoming of lesser density than the cold water.

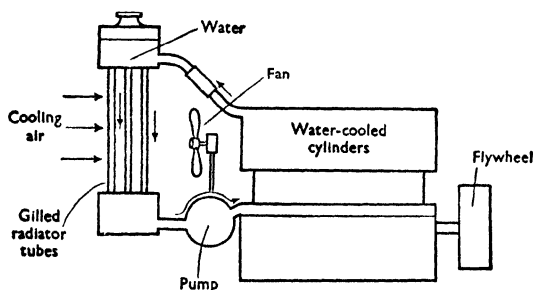


Fig. 27.5.—Water-cooled internal-combustion engine (circulation assisted by pump)

In consequence, it rises and passes up into the top of the storage cylinder. Here it displaces colder water, which falls down to

the bottom of the back boiler. Consequently, a continuous circulation is set up, and the temperature of the water in the storage cylinder rises. Hot water can be drawn off as required at the various outlets, cold water from the cistern replacing it, and becoming heated in its turn. Should the water reach boiling-point, the steam generated will force the water in the expansion pipe above its normal level, so permitting it to escape into the cistern and relieve the pressure. A radiator is shown connected in the circulating system and can be used for central heating when required. In order to give a supply of hot water in the summer, when the fire is not alight, an electric immersion heater can be fitted into the storage cylinder.

(3) **Radiation** is the means by which energy is transmitted from one place to another by waves in the *ether*.* Radiant heat is transmitted from one body to another without contact between them. Heating can be carried out at a considerable distance from the source of heat by this means, as all hot bodies emit radiations. An incandescent furnace, an electric reflector fire, and the sun are all radiators. They set up a wave motion which travels outwards and heats any body upon which it impinges. It should be realized that the waves themselves are *not* hot, but that, when impinging upon a body, they cause its molecules to increase their rate of vibration. The moving molecules may be forced to speed up to such an extent that they vibrate in sympathy, or *resonate*, with the incoming waves.

10. Reflection of radiated heat

Radiant heat energy travels in straight lines, in a manner similar to light. In fact, heat waves and light waves are both electromagnetic waves and differ only in wavelength. They can both be screened off, or they can be reflected (especially by polished surfaces), or they can be refracted (bent).

An electric fire is frequently fitted with a polished-metal parabolic reflector, which serves to direct the heat in the required direction. The beam of heat due to such a fire can readily be detected.

* The ether is an imaginary medium which pervades all space, from the vastness of interplanetary and interstellar space down to the space inside an atom.

11. Absorption of heat waves

When a wave of heat impinges on a substance, it may be reflected; it may pass right through; or it may be absorbed, depending upon the nature of the substance. A substance whose molecules readily resonate with the waves will readily absorb heat. If, however, the molecules do not resonate, then the heat waves will pass through without heating the substance. Water readily absorbs radiated heat, as is evident when one considers the effect of the sun on water. Rock salt does not absorb radiated heat.

The behaviour of glass in this respect is peculiar. It absorbs very little heat from the sun, but lets most of it pass right through. Heat from a low-temperature body, however, does not readily pass through it. These facts are made use of in greenhouses, window-panes, etc., which permit the sun's heat to pass through, but which do not readily permit internal warmth to escape.

12. Examples of heat transmission

Examples of the three methods of transmitting heat are furnished by considering some common electrical appliances.

In the electric iron (fig. 27.6) the heat from the electric

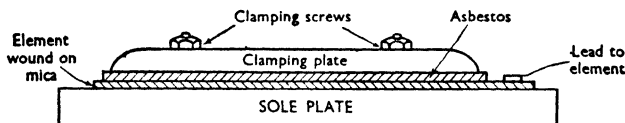


Fig. 27.6.—Electric iron illustrating heat conduction

element wire is transmitted by conduction to the iron sole-plate. Intimate contact is secured by fixing the element tightly between the sole-plate and an iron clamping plate. As heat is not required in this clamping plate, a piece of asbestos is placed between it and the element.

In the electric convector heater (fig. 27.7), the heating elements, usually operated at "black heat" (i.e. non-luminous), warm the air, causing it to rise and pass out into the room. Cold air, being heavier, then enters at the bottom to take its place,

becomes heated itself and passes out of the top of the appliance into the room. Thus a convection current of warm air is continually circulated from the appliance around the room, and then back to the appliance (after its heat has been given up).

In the electric reflector fire, or radiator (fig. 27.8), the heat from the element is directed in the required direction and

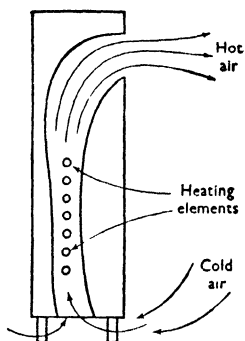


Fig. 27.7.—Electric convector

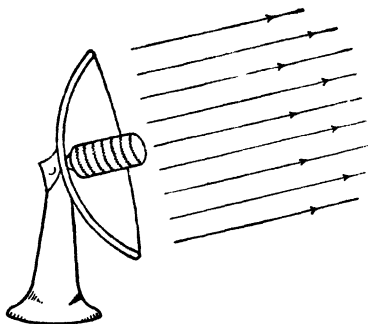


Fig. 27.8—Electric bowl fire (showing heat radiation)

warms objects in this direction by means of radiated heat waves.

These waves themselves are not hot, but if, for instance, the hand is placed in their path, the waves impinging on it cause its molecules to be stirred up into increased movement, thereby producing the sensation of heat.

13. Boilers

All three methods of heat transmission are made use of in a boiler. Boilers may be of two types:

- (i) fire-tube,
- (ii) water-tube.

In the fire-tube type, the products of combustion, on their way to the chimney, pass through tubes surrounded by water. In the water-tube type, water circulates through tubes surrounded by the products of combustion.

An outline diagram of a Yarrow three-drum water-tube

boiler is given in fig. 27.9. The heat from the furnace is transmitted to the water tubes by direct contact (conduction) with the flames and hot gases. Since these gases are in motion, convection currents will cause fresh hot gas to replace the gas cooled by contact. Also, and most important in modern boiler design, a large quantity of heat will be transmitted to the water by radiation.

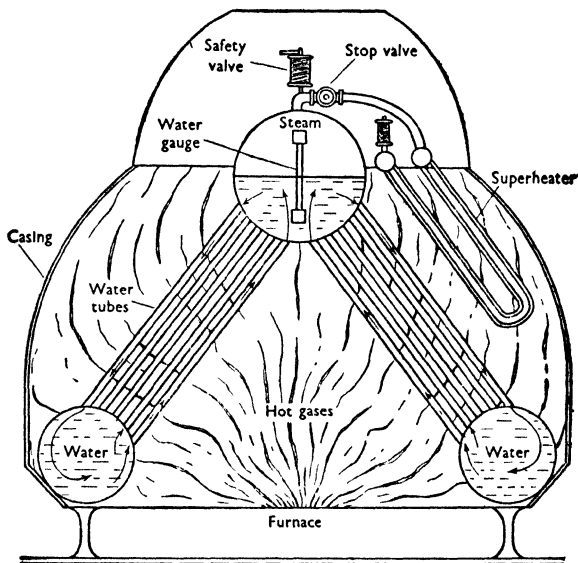


Fig. 27.9.—Yarrow-type water-tube boiler

The transmission of heat through the metal tubes and plates will be by conduction, whilst inside the boiler the heat is transmitted throughout the water by conduction (water is a bad conductor), but mostly by convection. The path of these convection currents in the water is shown in the diagram. The rapid steaming properties possessed by water-tube boilers are attained largely by a design which permits rapid circulation (convection) in the water, the heated water rising up the inner tubes and being replaced by cooler water descending the outer tubes.

14. Heat transmission through simple materials

A knowledge of the quantity of heat which will flow in a given time under specified conditions is of importance in many engineering problems.

The rate at which heat flows through a simple plate of a material when there is a temperature difference of one degree between its *faces* is a measure of the thermal conductance of that material. The thermal conductance per unit thickness of the plate is the **thermal conductivity** of the material. It is often expressed in terms of B.Th.U. transferred per square foot per hour per degree F. of temperature difference per inch of thickness.

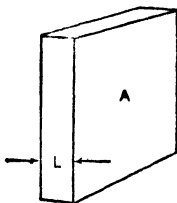


Fig. 27.10.—Thermal conductivity

In the plate of material shown in fig. 27.10, let

area of face = A sq. ft.,

thickness of plate = L in.,

temperature of hot face = t_1° F.,

,, ,, cold face = t_2° F.,

thermal conductivity = k ,

rate of heat flow = q B.Th.U. per hour;

then
$$q = \frac{kA(t_1 - t_2)}{L}.$$

N.B. The units used in the above equation must be carefully watched, as they are often varied. Sometimes k may be given per foot cube of material.

Example 6.—Find the heat loss per square foot per hour from the surface of an iron plate $\frac{1}{2}$ in. thick when the temperatures of the inner and outer surfaces are 250° and 200° F. respectively. Take the thermal conductivity of iron as being 26 B.Th.U. per hr. per sq. ft. per ft. thickness per $^{\circ}$ F.

The thickness of the plate in feet is $1/24$.

$$\text{Then } q = \frac{26 \times 1(250 - 200)}{1/24} = 31,200 \text{ B.Th.U. per hr.}$$

15. Heat transmission between media through slabs

In many practical problems heat is transmitted from one medium to another through a slab of material, and it is the temperature of these two media on either side of the slab which is known, e.g. the temperature of the flue gases *outside* a boiler water-tube and the temperature of the water *inside* may be known; or the temperature of the air inside and outside a building may be known.

In such cases heat is transmitted by convection through the first medium to the slab, then by conduction through the slab, and then by convection through the second medium. A considerable drop in temperature occurs at both surfaces of the slab due to:

(i) The presence of any deposit of soot, scale, etc., on these surfaces.

(ii) The presence of a thin stationary film of the fluid in close contact with the slab, through which the heat must pass by conduction and not by convection (fluids are poor conductors of heat).

These effects may be taken care of by using the **heat transfer coefficient** or **thermal transmittance** of the material instead of k .

Thermal transmittance is defined as the thermal transmission through unit area divided by the temperature difference between the fluids on either side.

EXERCISES ON CHAPTER XXVII

Section A

1. What is meant by the *mechanical equivalent of heat*? During an engine trial the circulating cooling water entered at a temperature of 12°C . and left at 36°C . The quantity of water flowing was 500 lb. in 35 min. Calculate the heat carried away per minute by the cooling water, and give its equivalent in ft.-lb.

1 centigrade heat unit is equivalent to 1400 ft.-lb. of work.
(U.L.C.I.)

2. Define the British Thermal Unit.

If all the heat used in raising the temperature of 0.5 gal. of water at 72°F . to boiling-point could be converted into mechanical work, through what vertical height could a weight of 1 ton be raised by the mechanical equivalent of the heat used?

1 B.Th.U. is equivalent to 778 ft.-lb. 1 gal. of water weighs 10 lb.
(U.L.C.I.)

3. The boiler of an oil-fired hot-water heating installation uses 0.4 lb. of oil per min. The oil used gives out 18,000 B.Th.U. per lb. when burned. If the temperature of the water entering the boiler is 60°F . and on leaving is 150°F ., calculate how many gallons of water pass through the boiler per minute if 60 per cent of the heat produced by the fuel is transferred to the water.
(U.L.C.I.)

4. Calculate the number of British Thermal Units in a kilowatt-hour.

The average daily domestic load for all purposes of a town of 20,000 dwellings is 5 units of electrical energy per dwelling. If every consumer by the exercise of moderate economy were to reduce his daily consumption by 5 per cent, calculate the annual saving in tons of coal.

Assume that 2 lb. coal are required to produce one electrical unit.
(746 W. = 1 h.p.; 1 B.Th.U. is equivalent to 778 ft.-lb.)
(U.L.C.I.)

5. Define: (a) sensible heat, (b) latent heat, (c) total heat.

Calculate the weight of coal required to melt one ton of cast iron if 50 per cent of the total heat of the coal is wasted.

Temperature of surrounding air, 60°F .; calorific value of the coal, 14,000 B.Th.U. per lb.; latent heat of fusion of iron, 50 B.Th.U. per lb.; specific heat of iron, 0.11; melting-point of iron, 2766°F .
(U.L.C.I.)

6. A tank contains 850 gal. of water at a temperature of 95°C . If the temperature of the water falls to 71.5°C . in 55 min., express the rate of loss of heat as horse-power.
(S.W.E.T.C.)

7. The weight of air in a room is 160 lb. Its specific heat is 0.24. What quantity of heat is required to raise the temperature of the room from 34°F. to 62°F. ? If the room cools to 34°F. again in 10 min., express the rate of loss of heat as horse-power.

(S.W.E.T.C.)

Section B

8. A boiler generates 7000 lb. of steam per hour at an absolute pressure of 160 lb. per sq in. from feed water at 80°C. If the efficiency of the boiler is 75 per cent, and the calorific value of the coal used is 8100 C.H.U. per lb., find the number of pounds of steam generated per pound of coal consumed. The total heat of steam at 160 lb. per sq. in. is 667.2 C.H.U. per lb.

(U.L.C.I.)

9. In a full-load test on a single-cylinder oil engine working on a four-stroke cycle the following results were obtained:

Fuel consumption 15.75 lb. per hour; r.p.m. 193; mean effective pressure 99.4 lb. per sq. in.; bore 11.8 in.; stroke 18.13 in.; calorific value of fuel 18,300 B.Th.U. per lb.; brake torque 959 lb.-ft.

Calculate the values of the indicated and brake horse-powers and indicated and brake thermal efficiencies.

(U.E.I.)

10. (a) The i.h.p. of an engine is 23. The mechanical efficiency is 85.5 per cent. What is the b.h.p.?

(b) If the engine burns fuel with a calorific value of 9000 C.H.U./lb., and the overall thermal efficiency of the engine is 24 per cent, what weight of fuel is used per hour?

(S.W.E.T.C.)

11. (a) Make a neatly proportioned sketch of any type of boiler with which you are acquainted, naming the important components.

(b) Describe clearly the means by which heat is transferred from the burning fuel to the water.

(U.E.I.)

12. Suggest explanations of the following facts: (a) When a bicycle pump is used for some time it gets hot. (b) Thin-walled glass vessels are less liable to crack than thick ones when hot water is poured into them. (c) Metal teapots are usually fitted with wooden handles.

(C.P.E.)

13. Give an account of the action of some type of engine with which you are familiar.

(C.P.E.)

14. Give at least two examples of (a) the conversion of work into heat energy, (b) the conversion of heat energy into work. How long should a car working at 10 h.p. be able to run on one gallon of petrol if one-quarter of the energy of the fuel is converted into useful work? (A gallon of petrol in burning gives 120,000 B.Th.U.)

(C.P.E.)

15. Describe how the radiating powers of various surfaces are compared. What are the results of such an experiment? How is the nature of the radiation affected by increasing the temperature of the source? Explain why glass is used for greenhouses.

(C.P.E.)

16. What is meant by the mechanical equivalent of heat? Describe an experiment by which its value can be obtained and give a full account of the calculations required.

(C.P.E.)

17. Draw a labelled diagram of a central heating system of a building. Explain the working of this system and give an account of the part played in it by the various modes of heat transference.

(C.P.E.)

18. What is meant by conduction of heat? Give an account of the principle of the Davy safety lamp. What experiments could be performed to compare the conductivities of several solid materials? Give a fully detailed account of these experiments and the interpretation of the results.

(C.P.E.)

CHAPTER XXVIII

Fuels and Combustion

1. Fuels: definition and classification

A fuel is a chemical source of heat which, once it has been raised to ignition temperature, maintains the process of combustion, provided that sufficient oxygen is available; i.e. it maintains a self-supporting burning action.

In practice, fuels may be classified according to their state:

(1) **Solid fuels** include coal and coke, wood, peat, and other substances containing mainly carbon, i.e. carbonaceous fuels.

(2) **Liquid fuels** include the many commercial fuels obtained from crude oil (petroleum), such as paraffin, petrol (gasolene as the Americans call it), and heavy fuel oil; also various forms of alcohol obtained from many sources, e.g. methylated spirit.

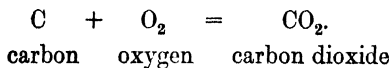
(3) **Gaseous fuels** include commercial "town or coal" gas, producer gas, water gas, and acetylene gas.

The principal constituents of any form of fuel are hydrogen and carbon, hence the term **hydrocarbon** fuel. Coal consists of about four-fifths by weight of carbon, about one-twentieth hydrogen, the remainder being mostly ash, oxygen, and sulphur.

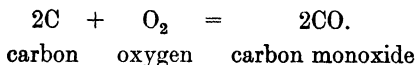
Petroleum fuels contain about 85 per cent carbon and 15 per cent hydrogen. "Gas" consists of a mixture of hydrogen, carbon monoxide (CO), and methane or marsh gas (CH₄).

2. Combustion equations

When carbon is burned completely in a plentiful supply of oxygen, the result can be represented by the chemical equation

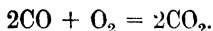


If insufficient oxygen is available, combustion will not be complete, and the chemical equation may be written:

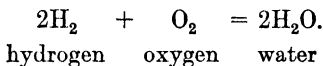


This gas carbon monoxide which is produced is itself a fuel; therefore, if it escapes to the atmosphere, it represents a loss. Furthermore, this gas is highly poisonous and has no smell, so its production during combustion and its subsequent escape must be avoided.

The combustion of carbon monoxide as a fuel may be represented by

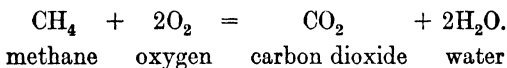


Similarly, the burning of hydrogen may be represented by



It will be seen that in this case the product of combustion is water, which, of course, will be in the form of steam.

The combustion of methane may be represented as follows:



The combustion of any type of fuel can be represented by chemical equations if the constituents of the fuel are known, an equation being written for the combustion of each constituent.

3. Importance of air supply

From the foregoing it will be appreciated that an adequate supply of oxygen is essential in order to secure complete combustion, and therefore the maximum amount of heat from the fuel.

This oxygen has to be obtained from the atmosphere (air), although in some cases a certain amount of oxygen is itself a constituent of the fuel.

The air itself is composed mainly of oxygen and nitrogen, with the addition of a small amount of carbon dioxide, and negligible amounts of the rare gases argon, neon, krypton, etc.

For all practical purposes, however, we can consider that the air is made up of 23 per cent by weight of oxygen, the remaining 77 per cent being nitrogen.*

Thus 1 lb. of air contains 0.23 lb. of oxygen, and 1 lb. of oxygen will be contained in

$$\frac{1}{0.23} = 4.35 \text{ lb. of air.}$$

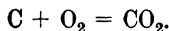
4. Air required for combustion

The exact amount of oxygen (and hence of air) required in order to ensure complete combustion can readily be calculated from the chemical equations and a knowledge of the atomic weights of the elements (see Chap. IV and Table 3, p. 33).

In these equations the letters represent elements (e.g. C and H), and a complete expression represents a molecule of either an element (e.g. O_2 represents a molecule of oxygen) or a compound (e.g. CH_4 represents a molecule of methane).

By referring to the table of atomic weights we can obtain the **molecular weight** of a substance.† Thus the weight of a molecule of oxygen is $16 \times 2 = 32$, and of a molecule of methane it is $12 + (1 \times 4) = 16$, i.e. a molecule of oxygen consists of two atoms of oxygen, and a molecule of methane consists of one atom of carbon and four of hydrogen.

Considering the equation for the complete combustion of carbon, we have



Substituting molecular weights, we have

$$12 + (16 \times 2) = 12 + (16 \times 2)$$

$$12 + 32 = 44.$$

Thus, to burn completely 12 lb. carbon, we need 32 lb. oxygen, and we thereby produce 44 lb. carbon dioxide. As 1 lb. oxygen is contained in 4.35 lb. air, we therefore need

$$32 \times 4.35 = 139.2 \text{ lb. air.}$$

* By volume the air consists of approximately 21 per cent oxygen and 79 per cent nitrogen.

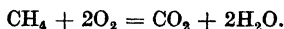
† The atomic weights are taken as whole numbers.

Therefore, in order to burn 1 lb. carbon, we need

$$139.2 \div 12 = 11.6 \text{ lb. air.}$$

The weight of air required in any other combustion equation can be calculated in a similar manner, as in the following examples.

Example 1.—Find the weight of air required for the complete combustion of 1 lb. of methane.



$$12 + (1 \times 4) + 2(16 \times 2) = 12 + (16 \times 2) + 2(1 \times 2 + 16).$$

$$16 + 64 = 44 + 36.$$

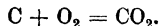
$\therefore$ 16 lb. methane need 64 lb. oxygen,

1 lb. methane needs $64 \div 16 = 4$ lb. oxygen,

1 lb. methane needs $4 \times 4.35 = 17.4$ lb. air.

Example 2.—Determine the theoretical weight of air required for the complete combustion of 1 gal. of petrol, given that the composition by weight of the petrol is 85 per cent carbon and 15 per cent oxygen. (Air contains 23 per cent by weight of oxygen, and the S.G. of the petrol is 0.8.) If 1 lb. of air at N.T.P. occupies 13.1 cu. ft., find also the volume of air required.

The complete combustion of carbon follows the equation



$$12 + 32 = 44.$$

Dividing by 12,

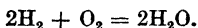
$$1 + \frac{32}{12} = \frac{44}{12},$$

or 1 lb. C needs $2\frac{2}{3}$ lb. O_2 to produce $3\frac{1}{3}$ lb. CO_2 .

$\therefore$ 0.85 lb. C needs $0.85 \times 2.666 = 2.27$ lb. O_2 ,

or $2.27 \times \frac{100}{23} = 9.88$ lb. air for complete combustion.

The complete combustion of hydrogen follows the equation



$$2 \times 2 + 32 = 36.$$

Dividing by 4,

$$1 + \frac{32}{4} = \frac{36}{4},$$

or 1 lb. H_2 needs 8 lb. O_2 to produce 9 lb. water.

$$\therefore 0.15 \text{ lb. } H_2 \text{ needs } 0.15 \times 8 = 1.2 \text{ lb. } O_2,$$

$$\text{or } 1.2 \times \frac{100}{23} = 5.22 \text{ lb. air for complete combustion.}$$

Therefore the theoretical minimum amount of air required for complete combustion of 1 lb. of petrol = $9.88 + 5.22 = 15.1$ lb.

1 gallon of petrol of S.G. 0.8 will weigh 8 lb. (1 gallon water weighs 10 lb.),

$$\therefore \text{Minimum amount of air required per gallon} = 8 \times 15.1 = 120.8 \text{ lb.}$$

$$\text{Volume of air required at N.T.P.} = 120.8 \times 13.1 = 1582 \text{ cu. ft.}$$

Example 3.—Determine the weight of the products of combustion formed by burning 1 gallon of the petrol in the last example.

The products of combustion will be CO_2 and H_2O .

1 lb. C needs $2\frac{3}{8}$ lb. O_2 to produce $3\frac{3}{8}$ lb. CO_2 . Therefore 0.85 lb. C will produce $0.85 \times 3\frac{3}{8} = 3.116$ lb. CO_2 .

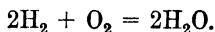
1 lb. H_2 needs 8 lb. O_2 to produce 9 lb. H_2O . Therefore 0.15 lb. H_2 will produce $0.15 \times 9 = 1.35$ lb. H_2O .

Therefore products of combustion of 1 gallon of petrol will be

$$8 \times 3.116 = 24.93 \text{ lb. } CO_2 \text{ and } 8 \times 1.35 = 10.8 \text{ lb. } H_2O.$$

5. Combustion of a fuel containing oxygen

A particular case occurs when a solid or liquid fuel contains oxygen. Usually it is assumed that this oxygen is combined with some of the hydrogen in the fuel in the form of water. This amount of hydrogen is therefore “not available” for combustion. Knowing the weight of oxygen in the fuel, the weight of hydrogen thus rendered “not available” can be determined from the equation



$$4 + 32 = 36.$$

Consequently, the weight of “fixed” hydrogen will be $4 \div 32 = \frac{1}{8} \times$ the weight of oxygen in the fuel. The hydrogen available for combustion will therefore be

total amount of hydrogen in fuel — “fixed” hydrogen.

Example 4.—Determine the weight of air required for the combustion of 1 lb. of coal of the following percentage analysis by weight. Carbon 76, hydrogen 6, oxygen 7, nitrogen 2, sulphur 1, ash 8.

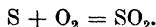
Oxygen required to burn 0.76 lb. carbon = $0.76 \times 2\frac{2}{3} = 2.026$ lb.

Weight of fixed hydrogen = $0.07 \div 8 = 0.0087$ lb.

Available hydrogen = $0.06 - 0.0087 = 0.0513$ lb.

Oxygen required to burn available hydrogen = $0.0513 \times 8 = 0.4104$ lb. (8 lb. of oxygen being required to burn 1 lb. of hydrogen.)

The only remaining combustible material is the sulphur. Complete combustion of sulphur takes place according to equation



$$32 + 32 = 64.$$

Hence oxygen required to burn 0.01 lb. sulphur = 0.01 lb.

Total amount of oxygen required = $2.026 + 0.410 + 0.01 = 2.446$ lb.

$$\therefore \text{weight of air required} = 2.446 \times \frac{100}{23} = 10.63 \text{ lb.}$$

In the case of a gaseous fuel containing oxygen there is no need to adjust its hydrogen content when making calculations, as is the case with solid and liquid fuels, because this oxygen is free to take part in combustion.

6. Combustion of gaseous fuels

Gaseous fuels are normally measured by volume (usually in cubic feet) rather than by weight.

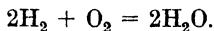
The combustion equations are the same, but are usually stated quantitatively in volume form, rather than by weight.

To do this requires a knowledge of **Avogadro's law**. This law states that

“The molecules of all gases occupy the same volume at the same temperature and pressure.”

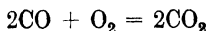
This means that the volume of one molecule of hydrogen is the same as that of one molecule of oxygen, or of any other gas (the temperature and pressure being constant).

Consider the combustion equation for hydrogen:



This means that two volumes (or molecules) of hydrogen

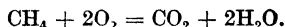
combine with one of oxygen and result in **two** of steam (at the same temperature and pressure). Again,



means that two volumes of carbon monoxide require one volume of oxygen for complete combustion, and the resulting product of combustion is **two** volumes of carbon dioxide.

Example 5.—Find the minimum volume of air required for the complete combustion of 1 cu. ft. of a gas containing 60 per cent methane (CH_4) and 40 per cent carbon monoxide (CO) by volume.

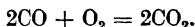
For the combustion of methane,



$$1 \text{ vol.} + 2 \text{ vol.} = 1 \text{ vol.} + 2 \text{ vol.}$$

$$\therefore 0.6 \text{ cu. ft. } \text{CH}_4 \text{ requires } 2 \times 0.6 = 1.2 \text{ cu. ft. oxygen.}$$

For the combustion of carbon monoxide,



$$2 \text{ vol.} + 1 \text{ vol.} = 2 \text{ vol.}$$

$$\therefore 0.4 \text{ cu. ft. } \text{CO} \text{ requires } \frac{1}{2} \times 0.4 = 0.2 \text{ cu. ft. oxygen.}$$

Therefore the minimum volume of oxygen required for combustion of 1 cu. ft. of the gas = $1.2 + 0.2 = 1.4$ cu. ft.

As the air consists of approximately 21 per cent oxygen by volume, the minimum volume of air required = $\frac{100}{21} \times 1.4 = 6\frac{2}{3}$ cu. ft.

7. Excess air

It should be realized that the amount of air necessary for complete combustion as calculated is the amount theoretically required. It assumes that all of this air comes into intimate contact with the particles of fuel. This condition is only approached in practice under ideal conditions, and, as incomplete combustion is most undesirable, it is consequently usual to arrange that a quantity of "excess air" is supplied. The total amount of air supplied may approach twice the theoretical amount, i.e. the amount of excess air may approach 100 per cent, but the modern tendency is to reduce the amount to from 25 to 50 per cent. A large amount of excess air has a cool-

ing effect on the process of combustion, and so causes heat losses.

In many modern furnace installations the air supply is sent through a "preheater" before entering the furnace, so as to avoid this cooling effect. This preheater raises the temperature of the air by making use of the hot flue gases on their way to the chimney, and, as these gases would otherwise be wasted, an increase in efficiency thereby results.

8. Forced and induced draught

In some installations, air is forced through the furnaces under the pressure created by "forced-draught" fans. These increase the output which can be obtained from a given size of boiler, by increasing the amount of air passed through the furnace, so increasing the **rate** of combustion. The quantity of air required per pound of fuel, however, remains unchanged. "Induced-draught" fans may also be fitted in the boiler uptake to the chimney, and serve a similar purpose.

9. Calorific value

When purchasing or considering the use of a fuel, it is often desirable to know not only its chemical constitution but the actual amount of heat which could be produced by, say, one pound of it when completely burned.

This theoretical amount of heat which can be produced by a unit quantity of fuel is known as its **calorific value**, and is usually expressed commercially in B.Th.U. per pound. It can also be expressed in C.H.U. per pound, calories per gram, B.Th.U. per gallon or per cubic foot as required.

The calorific value of a fuel is usually determined by completely burning a known quantity in oxygen, under laboratory conditions; the total quantity of heat given out is found by observing the rise in temperature of a known mass of water, etc., which is arranged to surround the burning fuel. The instrument used is called a **calorimeter**.

If the chemical analysis of a fuel is available, it is possible to estimate its calorific value if the calorific values of the constituent elements are known.

10. Measurement of calorific value

One form of fuel calorimeter which is used for the determination of the calorific values of fuels is shown in fig. 28.1. It is called the **bomb calorimeter**.

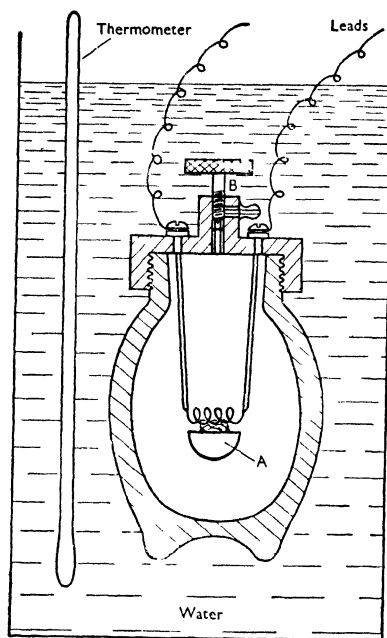


Fig 28.1.—Bomb calorimeter

A carefully weighed sample of the fuel is placed in the container (A). Oxygen under pressure is introduced through the valve (B), which is then screwed down. The bomb is then completely immersed in water. When this water (and the bomb and contents) has reached a steady temperature, this temperature is recorded, and the electric circuit through the fuse wire is completed. The fuel now ignites and continues to burn until it has all been consumed. The rise in temperature of the water in the calorimeter due to combustion is then recorded.

Then

Heat given up by the sample of fuel

$$= (\text{weight of water} + \text{water equivalent of calorimeter system}) \\ \times \text{temperature rise.}$$

Therefore, calorific value of fuel

$$= \frac{\text{heat given up by sample of fuel}}{\text{weight of fuel sample}}.$$

Example 6.—Determine the calorific value of a sample of coal from the following data: weight of coal burned $\frac{1}{10}$ oz.; weight of water in calorimeter 4 lb.; water equivalent of system $\frac{1}{2}$ lb.; initial temperature of water 60° F.; final temperature 66° F.

$$\begin{aligned} \text{Calorific value of fuel} &= \frac{(4 + \frac{1}{2}) \text{ lb.} \times (66 - 60)^{\circ} \text{ F.}}{\frac{1}{30} \times \frac{1}{16} \text{ lb.}} = \frac{4\frac{1}{2} \times 6}{\frac{1}{30} \times \frac{1}{16}} \\ &= 27 \times 16 \times 30 = 12,960 \text{ B.Th.U. per lb.} \end{aligned}$$

11. Higher and lower calorific values

When determining calorific values by the above method, a special case occurs if the fuel contains hydrogen. It will be remembered that, in this case, water is one of the products of combustion, and that it is evolved in the form of steam, owing to the temperature of combustion. When tested in the calorimeter, this steam will condense and give up its latent heat and some of its sensible heat in cooling down to the final temperature of the water in the calorimeter. Consequently the temperature rise of the water in the calorimeter, and hence the calorific value of the fuel obtained by the test, will have a higher value than the calorific value which can be obtained in practice, where, of course, the temperature of the waste gases will not permit the steam to condense.

This **higher or gross calorific value** (H.C.V.) obtained in a calorimeter thus represents the total heat which can be obtained from a fuel containing hydrogen. The quantity of heat which can be obtained from it commercially is known as the **lower or net calorific value** (L.C.V.) of that fuel, and this is the value usually quoted in practice, although heat-engine trials to the recommended requirements necessitate the gross value being used.*

* "Report on Tabulating the Results of Heat Engine Trials", the Institution of Civil Engineers.

If the higher calorific value is known, the lower value can be determined as follows:

Lower calorific value = higher C.V. — weight of steam formed
 $\times$ (latent heat of steam + sensible heat)

(all values being “per lb.”).

In practice, the amount subtracted from the H.C.V. is often taken as 1055 B.Th.U. per lb. of steam formed, this being the latent heat of steam at 60° F.

Example 7.—Calculate the higher calorific value of the coal the analysis of which was given in Example 4, given that the calorific value of carbon is 14,500 B.Th.U./lb., of hydrogen (H.C.V.) 62,000 B.Th.U./lb., and of sulphur 4000 B.Th.U./lb.

Heat due to combustion of carbon

$$= 0.76 \times 14,500 = 11,020 \text{ B.Th.U.}$$

Heat due to combustion of available hydrogen

$$= 0.0513 \times 62,000 = 3180 \text{ „}$$

Heat due to combustion of sulphur

$$= 0.01 \times 4000 = 40 \text{ „}$$

$$\therefore \text{Higher calorific value of the coal} = \underline{14,240 \text{ B.Th.U./lb.}}$$

EXERCISES ON CHAPTER XXVIII

1. A coal contains by weight 81 per cent C, 6 per cent H_2 , and the remainder ash. (a) What is the minimum weight of air required to burn 1 lb. coal? (b) How much steam and CO_2 are formed? Air contains 23 per cent oxygen by weight. (Poly.)

2. A sample of boiler coal yields the following analysis: carbon 80 per cent, hydrogen 5 per cent, sulphur 1.25 per cent, nitrogen 1.2 per cent, and ash 4.55 per cent. How much air would be theoretically required per lb. of coal to ensure complete combustion? How much might actually be required in practice? (U.E.I.)

3. The percentage composition of a sample of coal is 89 per cent carbon, 3.1 per cent hydrogen, and 3 per cent oxygen. Calculate the minimum weight of air required for the complete combustion of 1 lb. of this coal. If 50 per cent of excess air is supplied, find, assuming complete combustion, the weights of the respective flue gases per lb. of coal burned. (U.L.C.I.)

4. Determine from first principles the weight of air required to burn completely 1 lb. of carbon and 1 lb. of hydrogen respectively.

Estimate the weight of air required for the complete combustion of 1 lb. of oil consisting of 87 per cent carbon and 13 per cent hydrogen. What weights of steam and carbon dioxide are formed during combustion? (U.L.C.I.)

5. The composition per lb. of oil fuel supplied to a boiler consists of 0.85 lb. carbon, 0.13 lb. hydrogen, and 0.02 lb. oxygen. The air supply is 70 per cent in excess of that required for complete combustion. Calculate the weight of flue gases passing up the chimney per hour when 1400 lb. of oil are burned per hour. (U.L.C.I.)

6. The analysis by weight of a certain coal is carbon 82 per cent, hydrogen 5 per cent, non-combustibles 13 per cent. Determine: (a) The theoretical weight of air required for the complete combustion of 1 lb. of this coal. (b) The percentage composition, by weight, of the products of combustion, assuming no excess air was supplied.

Note that there is 1 lb. oxygen in 4.35 lb. of air. (U.E.I.)

7. An analysis of an oil fuel showed that it consisted of 85 per cent carbon and 15 per cent hydrogen. Determine, from first principles, the minimum weight of air necessary for the complete combustion of one pound of this fuel. Find also the higher calorific value of the fuel.

The calorific value of carbon is 14,500 B.Th.U. per pound and that for hydrogen 62,000 B.Th.U. per pound. Atomic weights: carbon 12; hydrogen 1; oxygen 16. Air contains 23 per cent by weight of oxygen. (Poly.)

8. (a) Define the term "calorific value".

(b) The following data were recorded during an experiment to find the C.V. of a sample of coal: weight of coal burned 1 gm.; weight of water in calorimeter 1020 gm.; water equivalent of the calorimeter 170 gm. Initial temperature of water 16.2°C .; final temp. of water 23.3°C . Determine the calorific value of one pound of the sample of coal tested in (i) C.H.U. and (ii) B.Th.U. (S.W.E.T.C.)

9. The calorific value of a fuel is to be determined experimentally. Sketch and describe the apparatus you would use. State the observations which you would make, and show how you would obtain the calorific value from these observations. The fuel may be taken as *either solid or gaseous*. (U.L.C.I.)

10. Calculate the calorific value of a sample of fuel in B.Th.U. per lb. and C.H.U. per lb., from the following experimental observations: Weight of fuel = 1.04 gm. Weight of water = 2338 gm. Initial temperature of water in calorimeter = 14.70°C . Final temperature of water in calorimeter = 17.65°C . Water equivalent of apparatus = 428 gm. (U.L.C.I.)

CHAPTER XXIX

Hydrostatics—Fluids at Rest

1. Fluids

A fluid may be defined as anything which will flow and which consequently can be poured from one vessel to another. This definition therefore includes both liquids and gases.

A **liquid** will take the shape of any vessel into which it is placed, as far as its volume will permit, i.e. it has a definite volume, and also an upper flat surface.

A **gas**, however, will **completely fill** any vessel into which it is placed, i.e. it has no definite volume. Consequently, when speaking of the volume of a gas, we mean its volume under some stated conditions of temperature and pressure. Very often we specify its volume at normal temperature and pressure (see Chap. XXV, § 18).

The molecules of a fluid permit internal movement of that fluid more or less readily, according to the nature of the fluid. The amount of such molecular frictional resistance to movement offered by a fluid is a measure of its **viscosity**. A gas has practically no viscosity; liquids such as water or alcohol possess very little, whilst syrup, lubricating oil, honey, and **wax** possess increasingly greater amounts. As the borderline between liquids and solids is approached, the amount of viscosity increases.

A gas possesses very little **cohesion** (or attraction between its molecules), and in consequence it can expand to fill any space.

Owing to cohesion, the molecules of a liquid, however, will cling together to some extent, and thus drops can be formed.

A gas can readily be compressed, but a liquid cannot be compressed to any appreciable extent.

If several gases are placed in the same vessel they will

diffuse, or mix amongst each other, until each completely fills the space. The total pressure of this mixture will be made up of the **partial pressures** of the individual gases. This property of **diffusion** is due to the wide spacing of the gas molecules, the molecules of one gas consequently finding room between the molecules of the other gases.

2. Fluid pressure

When speaking of fluid pressure, we usually mean intensity of pressure (in the same way that stress usually implies intensity of stress). Consequently **intensity of pressure** (loosely, pressure) may be defined as

the total force acting perpendicularly to any surface, divided by the area of that surface.

It can be expressed in such units as pounds per square inch, or tons per square foot. In symbols,

$$P = \frac{F}{A},$$

where P = intensity of pressure,

F = total force acting normal to the surface,

A = area of surface.

N.B. The expression “normal to” means “perpendicular to”, or “at right angles to”.

3. Laws of fluid pressure

The following laws can be applied to fluids at rest.

(1) Fluid pressure is always regarded as being normal to any surface on which it acts.

(2) The intensity of pressure is the force per unit area.

(3) The intensity of pressure at any point is the same in all directions.

(4) The pressure applied to a fluid is transmitted undiminished in all directions.

4. The hydraulic press

An illustration of the fourth law above is provided by the **hydraulic press** (fig. 29.1).

In it a pressure is set up by the action of a small piston (A) in a cylinder. This pressure is transmitted undiminished in all directions, and, in consequence, it acts on the area of the large piston (B), causing it to rise and exert pressure on any article

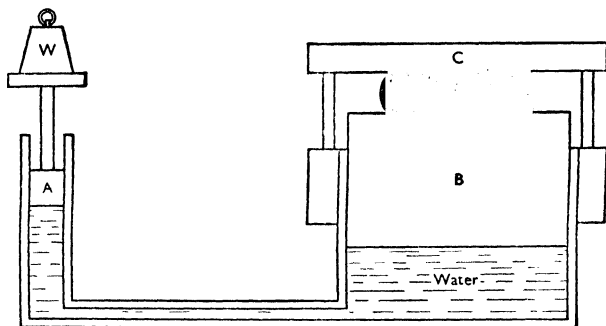


Fig. 29.1.—Hydraulic press

placed between this piston and the anvil (C). The total force exerted by the large piston, due to a given intensity of pressure set up by the small piston, will be much greater than the total force exerted on this small piston, owing to their great difference in areas.

Example 1.—A hydraulic press has a ram with an area of 800 sq. in., and is operated by means of a force pump with a piston of area $1\frac{1}{4}$ sq. in. The force exerted on this pump is 400 lb. Find the total force exerted by the ram.

Intensity of pressure created by force pump,

$$P = \frac{F}{A} = \frac{400}{1\frac{1}{4}} = 400 \times \frac{4}{5} = \frac{1600}{5} = 320 \text{ lb./sq. in.}$$

Total force exerted by ram = $800 \times 320 = 256,000 \text{ lb.}$

= $114\frac{1}{4}$ tons (approx.).

5. Fluid pressure due to gravity

As a fluid is acted upon by gravity (i.e. it possesses weight), it will therefore exert a force upon any body immersed in the fluid. The magnitude of this fluid pressure will depend upon the density of the fluid and the depth at which the body is located.

Take the case of a submarine whose deck is submerged to a depth of 150 ft. in sea-water of density 64 lb. per cu. ft. The intensity of pressure on 1 sq. ft. of the deck will be equal to the weight of the superimposed sea-water, or

$$\begin{aligned}\text{pressure} &= \text{area of surface} \times \text{depth} \times \text{density} \\ &= 1 \times 150 \times 64 \\ &= 9600 \text{ lb. per sq. ft. or } 4.285 \text{ tons per sq. ft.}\end{aligned}$$

This pressure, in accordance with the laws of fluid pressure, will always act normal to the surface under consideration. Thus it will act, not only on the deck of a submerged sub-

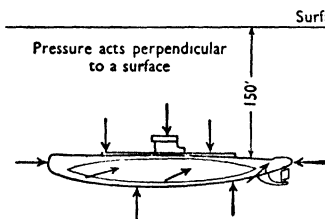


Fig. 29.2

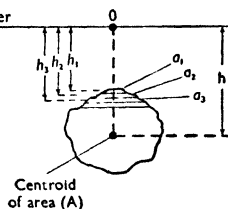


Fig. 29.3

marine, but also on its sides and bottom (see fig. 29.2). The pressure on the sides and on the bottom will, of course, be slightly greater, owing to their slightly greater depth.

If a submerged surface is not horizontal, the depth of its centre of area should be used when calculating the force on it. This can be proved by dividing the submerged surface into a large number of narrow strips each parallel to the surface of the liquid (see fig. 29.3).

Let the areas of these strips be a_1 , a_2 , a_3 , etc., and let their depths below the free surface be h_1 , h_2 , h_3 , etc.

Let w be the density of the liquid, and A the total submerged area. Then

$$\begin{aligned}\text{total force on } A &= a_1 h_1 w + a_2 h_2 w + a_3 h_3 w + \dots \text{etc.} \\ &= w(a_1 h_1 + a_2 h_2 + a_3 h_3 + \dots \text{etc.})\end{aligned}$$

But $a_1 h_1 + a_2 h_2 + \dots$ etc. is the sum of the moments of the individual areas about a point on the surface of the liquid, and can be replaced by a resultant moment Ah , where h is the depth of the centre of area (centroid) of the submerged surface (see Chap. X).

6. Head of water

Water pressures are often spoken of in terms of **head**, where head means "the length of the column of water producing the pressure".

Thus, if the water surface in a cistern on the roof of a building is 30 ft. above an outlet tap, then the head of water is 30 ft.

As fresh water weighs 62.4 lb. per cu. ft., this is equivalent to a pressure of 62.4×30 lb. per sq. ft. at the tap, or $62.4 \times 30 \div 144 = 13$ lb. per sq. in.

N.B. This pressure represents the pressure difference between the water surface in the cistern and the tap. It neglects any other pressure (such as atmospheric pressure) which may be acting on that water surface.

7. The hydrostatic paradox

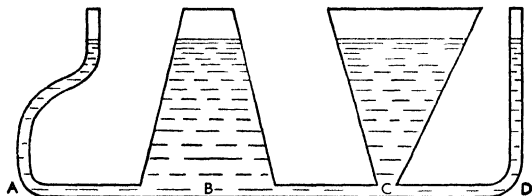


Fig. 29.4.—Hydrostatic paradox

It has been observed that the intensity of pressure due to a fluid is dependent on depth. If a liquid is poured into the arrangement shown in fig. 29.4, it will find its own level, this

level being the same in each vessel. Since the liquid is at rest, it is evident that the *intensity* of pressure at points A, B, C, and D must be the same. Consequently, this pressure must be independent of the shape of the containing vessel and of the weight of liquid it contains, and **must depend solely upon depth**. The pressure due to a small column of liquid (D) thus counter-balances that due to a much larger column (B or C) of much greater weight. This apparently contradictory situation is known as the "hydrostatic paradox".

As a result of this, the pressure in a water main, or against a dam at the end of a reservoir, is independent of the volume of water in the main, or in the reservoir, and depends only upon the head of water.

Example 2.—A rectangular lock gate is 30 ft. wide; the depth of water on one side is 20 ft., and on the other side 12 ft. Calculate the total load on each side. (Density of sea-water 64 lb./cu. ft.)

Depth of centroid (centre of area) of submerged surface on deep side = $20 \div 2 = 10$ ft.

Total force on deep side = area of submerged surface $\times$ depth of centroid of submerged surface $\times$ density

$$= 30 \times 20 \times 10 \times 64 = 384,000 \text{ lb.}$$

Depth of centroid of submerged surface on shallow side = $12 \div 2 = 6$ ft.

Total force on shallow side = $30 \times 12 \times 6 \times 64 = 138,240$ lb.

Example 3.—A sluice gate of triangular form, each side of length 4 ft., is inclined at an angle of 60° to the surface of fresh calm water, with one side level with the water. Calculate the total force exerted on it.

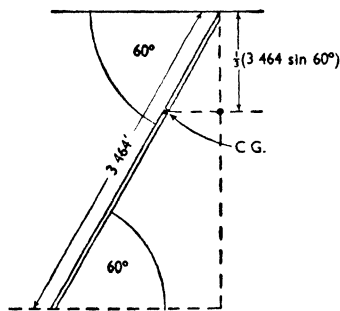


Fig. 29.5

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}B \times H = \frac{1}{2} \times 4 \times \sqrt{(4^2 - 2^2)} \\ &= 2 \times \sqrt{12} = 2 \times 3.464 = 6.928 \text{ sq. ft.} \end{aligned}$$

Depth of centre of area (centroid) (see fig. 29.5)

$$= \frac{1}{3}(3.464 \sin 60^\circ) = 1 \text{ ft.}$$

$$\text{Total force exerted} = 6.928 \times 1 \times 62.4 = 432.5 \text{ lb.}$$

Example 4.—A vertical rectangular dock gate is subjected to water pressure on one side. Assuming the width of the gate to be equal to one half of the depth of water, find the depth of water when the total pressure on the gate is 50 tons weight.

1 cu. ft. of water weighs 64 lb.

(C.P.E.)

Let depth of water = d ft.

then width of gate = $\frac{1}{2}d$ ft.

Total pressure (force) on gate = area $\times$ depth of centroid $\times$ density.

$$50 \times 2240 = (d \times \frac{1}{2}d) \times \frac{1}{2}d \times 64,$$

$$112,000 = \frac{1}{4}d^3 \times 64 = 16d^3,$$

$$d^3 = 7000.$$

$$\therefore \text{depth of water} = \sqrt[3]{7000} = 19.13 \text{ ft.}$$

8. Pressure of air due to gravity

The atmosphere exerts a fluid pressure on the earth because it is acted upon by gravity, i.e. air possesses weight. The

magnitude of this pressure due to the weight of air is not generally appreciated, perhaps because everybody is subjected to it, and therefore tends to forget it. One cubic foot of air weighs about 36.6 gm., or 1.3 oz. This weight acting on the surface of a liquid can be made to support a column of that liquid contained in a tube sealed at one end and inverted over the liquid, if the space between the top of the column and the sealed end of the tube is empty of air.

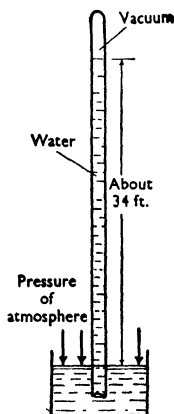


Fig. 29.6.—Water barometer

This is the principle of the **barometer**, an instrument used for measuring air pressure (see fig. 29.6). If water is used as the liquid, it will be found that atmospheric pressure will support the weight of a column of water approximately 34 ft. high.

We can, therefore, say that atmospheric pressure is equivalent to a head of water of 34 ft. (approx.).

To convert this pressure into pounds per square inch, assume the water column to be of 1 sq. ft. cross-sectional area. Its

volume will then be 34 cu. ft., and as fresh water weighs 62.4 lb. per cu. ft., its weight will be 34×62.4 lb.

$\therefore$ pressure of atmosphere

$$= 34 \times 62.4 \div 144 = 14.72 \text{ lb. per sq. in.}$$

9. The mercury barometer

An instrument containing a column of water 34 ft. high is not a very convenient device for measuring air pressure. As mercury has a density 13.6 times greater than water, we can use it in place of the water, and so considerably reduce the height of the barometer. Atmospheric pressure will support a column of mercury approximately 30 in. high.

One form of mercury barometer in common use is known as the **Fortin barometer** (see fig. 29.7). It consists of a glass tube with a scale placed alongside. To adjust the mercury surface to the zero-point on this scale, the mercury cistern is provided with a flexible leather bag which can be raised or lowered by means of a screw. This adjustment is correct when the ivory pointer (P) just touches the mercury. The height of the mercury is then read off by adjusting the slider, by means of the milled knob, until its bottom edge corresponds with the top of the mercury column. A vernier scale is provided to facilitate exact reading.

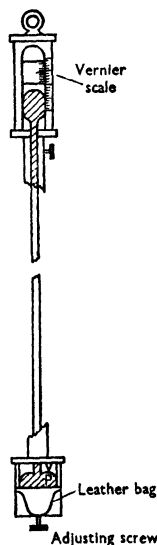


Fig. 29.7. — Fortin barometer

10. The aneroid barometer

This type of barometer does not contain mercury and thus can be made portable. It consists essentially of a corrugated metal container from which the air has been exhausted (fig. 29.8). An increase in atmospheric pressure will cause this container to collapse to some extent, whereas a decrease in atmospheric pressure will allow it to dilate slightly under the action of a spring. These small movements are magnified by a system of levers, which

finally produce rotation of a pointer by pulling a fine chain wound round a pulley fixed on the pointer. The pointer moves

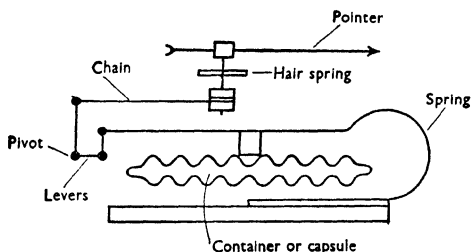


Fig. 29.8.—Aneroid barometer

round a dial calibrated in suitable units of pressure. A hair-spring is fitted in order to keep the chain taut and to take up any backlash.

11. Variation of atmospheric pressure with height

As the earth's atmosphere does not extend upwards for more than a hundred or so miles, it is evident that the pressure due to its weight will decrease as we ascend into it, i.e. the weight of air above us will become less. This reduction of pressure with altitude will not, however, follow a straight-line graph, because the air becomes less dense as we ascend.* The temperature of the air also decreases as we ascend, causing a slight increase in air density.

It will be seen, therefore, that barometric pressure can be used as an indication of height, and that the actual height can be measured if the true relationship between barometric pressure and altitude is known.

The altitude of an aircraft is normally measured by means of an **altimeter**, which is nothing more than a carefully designed and calibrated **aneroid barometer**.

* Above a height of about 5 miles the atmosphere becomes so rarified that it will not support human life.

12. Relationship between height and atmospheric pressure

It will be realized that the relationship between altitude and atmospheric pressure is difficult to lay down with great accuracy, depending as it does on such variables as air temperature and density at each particular height. Atmospheric pressure at sea-level is, of course, also variable, as, amongst other things, the moisture content of the air varies, possibly from hour to hour. As this moisture content increases, it causes a decrease in air density, and a corresponding drop in barometric pressure. Study of variations in barometric pressure forms part of the science of **meteorology**, or the study of the weather.

In order to calibrate an aneroid barometer so that it can be usefully employed as an altimeter, certain assumptions are therefore necessary. In practice two methods of relating altitude and barometric pressure are widely employed. These are known as

- (1) The isothermal convention.
- (2) The International Commission for Air Navigation (I.C.A.N.) convention.

The isothermal convention assumes that the temperature of the atmosphere remains constant at 10° C. irrespective of height. The actual height is obtained from the formula

$$H = 62.58 \log \frac{P_0}{P},$$

where

H = height in thousands of feet,

P = pressure at H ,

P_0 = ground pressure.

The I.C.A.N. convention assumes that the temperature of the atmosphere falls uniformly with increase of height at the rate of 1.98° C. per 1000 ft., from a value of 15° C. at ground level to -56.5° C. at 36,090 ft., after which it remains constant. This method is the more accurate of the two. The actual height (up to 36,090 ft.) can be calculated from the formula

$$\frac{P_0}{P} = \left(\frac{288}{288 - 1.98H} \right)^{5.256}$$

3. Absolute pressure and gauge pressure

As the pressure of the atmosphere is a variable quantity of average value around 14.7 lb. per sq. in., which is exerted upon everything around us, it will be realized that, to find the total intensity of pressure on something already being subjected to some other form of pressure, we must add the atmospheric pressure.

For example, in the case of the tap under a head of 30 ft. of water (a pressure 13 lb. per sq. in. — see § 6), the total pressure at the tap will be $13 + 14.7 = 27.7$ lb. per sq. in. This total pressure is termed **absolute pressure**, and it will be evident that the absolute zero of pressure will occur when there is no atmosphere present, i.e. in a vacuum.

Most pressure gauges used for steam, water, compressed air, etc., indicate the intensity of pressure above (and sometimes below) that of the atmosphere. Zero on such a gauge therefore means a pressure of 14.7 lb. per sq. in. absolute, and an indication of, say, 60 lb. per sq. in. means an absolute pressure of $60 + 14.7 = 74.7$ lb. per sq. in.

In order to distinguish between these pressures we call the actual gauge indication the **gauge pressure**. Then, we have

absolute pressure = gauge pressure + atmospheric pressure.

Example 5.—A point in a hydraulic system is under a 24-ft. head of fresh water. The surface of the water is also subjected to an air pressure of 10 lb./sq. in. gauge. Find the total absolute pressure at this point.

Water pressure = $62.4 \times 24 \div 144 = 10.4$ lb./sq. in.

Absolute air pressure = gauge pressure + atmospheric pressure
 $= 10 + 14.7 = 24.7$ lb./sq. in.

Total absolute pressure at point = $10.4 + 24.7 = 35.1$ lb./sq. in.

14. Centre of pressure on an immersed plane area

Any plane area immersed in a fluid will be subjected to an intensity of pressure which increases from point to point with depth.

The total force on this area will be

area $\times$ depth of centre of area $\times$ density of fluid.

As intensity of pressure increases with depth, it will be seen that the total force is not uniformly distributed over the area. Thus, in fig. 29.9, the intensity of pressure on the vertically suspended immersed area ABCD increases uniformly from zero at the surface to a maximum at CD. It is often convenient in calculations to regard the total force as being concentrated at a point within the area. This point at which the total force is considered to act is called the **centre of pressure**.

It would be incorrect to take the position of this point as being the point of average pressure (i.e. at half the vertical

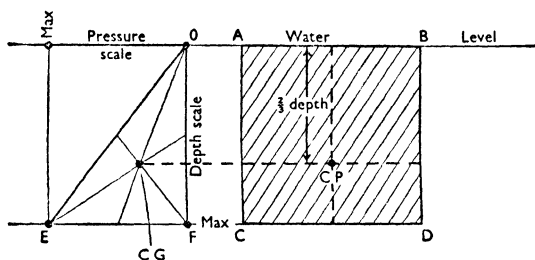


Fig. 29.9.—Centre of pressure

depth), because the pressure on the lower half of the plane is much greater than on the upper half. Thus, considering fig. 29.9, we can draw a graph showing how the intensity of pressure increases with depth, and it will be seen to enclose a triangle OEF. The centre of area of this triangle will correspond with the point about which the total force on the area ABCD is considered to act. The centre of area of this triangle can be determined (see Chap. X), and will be found to lie at a depth two-thirds of the distance between A and C. Therefore the centre of pressure of a rectangle or parallelogram with its upper edge on the surface of a fluid lies at $\frac{2}{3} \times$ (vertical depth).

It should be realized that the centre of pressure of any immersed plane area is not fixed, but depends upon the depth

at which it is submerged, its shape, and position (inclination, etc.). The matter will not be pursued any further here, but, for

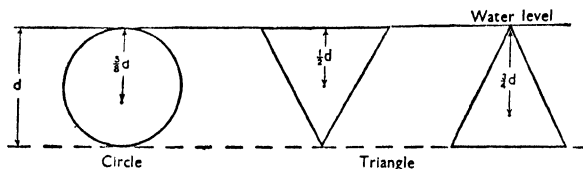


Fig. 29.10.—Position of centre of pressure on some immersed plane areas

reference, fig. 29.10 shows the position of the centre of pressure in some typical cases.

Example 6.—A dam 150 ft. long and 35 ft. deep is placed across a reservoir 27 ft. deep. Find (1) the total force exerted by the water on the dam, (2) the position of the centre of pressure, (3) the overturning moment acting on the dam.

$$\begin{aligned}\text{Total force} &= \text{wetted area} \times \text{depth of its centroid} \times \text{density} \\ &= (150 \times 27) \times 13\frac{1}{2} \times 62\cdot4 \\ &= 3,410,000 \text{ lb.} = 1520 \text{ tons (approx.)}\end{aligned}$$

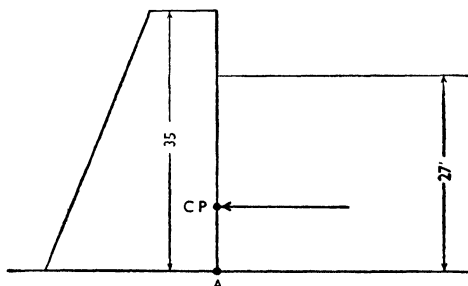


Fig. 29.11

The position of the centre of pressure will be on a vertical line 75 ft. from either end of the dam and situated at a distance of $\frac{2}{3} \times 27 = 18$ ft. below the surface, or 9 ft. from the bottom (see fig. 29.11).

The total overturning moment will act about A, and will be

$$1520 \times 9 = 13,680 \text{ ton-ft.},$$

or $\frac{1520}{150} \times 9 = 91.2 \text{ ton-ft. per ft. length of dam.}$

Example 7.—A hinged metal door is fitted to the end of a rectangular channel containing water. The channel is 14 in. deep and 12 in. wide. The door, 18 in. deep and 16 in. wide, is placed symmetrically about the opening of the channel and is hinged at its inner top edge (as shown in fig. 29.12). Calculate the thickness of the door

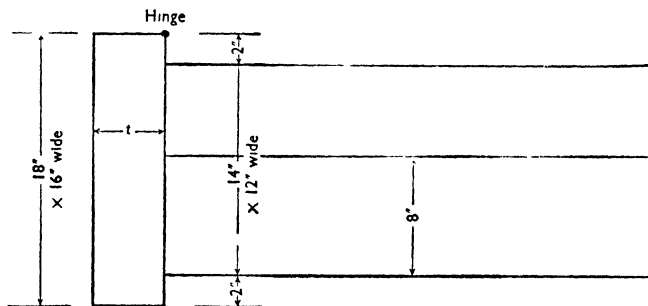


Fig. 29.12

if it just opens when the height of water in the channel reaches 8 in. Assume the weight of water to be 62.5 lb./ft.³ and the weight of metal to be 0.28 lb./in.³ (I.C.E. and I.E.E. Joint Section A.)

Total force exerted by water = area $\times$ depth of centroid $\times$ density

$$= \left(\frac{8 \times 12}{144} \right) \text{ sq. ft.} \times \frac{4}{12} \text{ ft.} \times 62.5$$

$$= 13.88 \text{ lb.}$$

This force can be considered as acting at the centre of pressure, which in this case is situated at $\frac{2}{3} \times 8 \text{ in.} = 5\frac{1}{3} \text{ in.}$ from surface of water, or at $2 + 6 + 5\frac{1}{3} = 13\frac{1}{3} \text{ in.}$ from hinge (fulcrum).

The weight of the door = $18 \times 16 \times t \times 0.28 \text{ lb.}$

This will act vertically downwards through the centre of gravity of the door, situated at a perpendicular distance of $\frac{1}{2}t$ from the hinge.

Take moments about hinge (see fig. 29.13).

$$\text{Clockwise moments} = 13.88 \times 13.33.$$

$$\begin{aligned}\text{Anticlockwise moments} &= 18 \times 16 \times t \times 0.28 \times \frac{1}{2}t \\ &= (9 \times 16 \times 0.28)t^2.\end{aligned}$$

For the door to be just about to open, the clockwise and anticlockwise moments must be equal,

$$\therefore t^2 = \frac{13.88 \times 13.33}{9 \times 16 \times 0.28} = 4.59,$$

$$\text{and} \quad t = \sqrt{4.59} = 2.142 \text{ in.}$$

15. Buoyancy

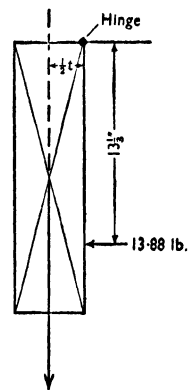
If a body is placed in a fluid of greater density, it will float in that fluid. If, however, the density of the body exceeds that of the fluid, it will sink. Thus, most woods will float in water; iron will float in mercury; and a balloon filled with hydrogen gas will float in air. Again, a steel shell, such as a ship's hull, will float in water because its density (mass $\div$ volume) is less than that of the water; but a steel block will sink.

If the floating body is forced down into the fluid, it will rise again when this force is removed, although gravity is acting downwards on it all the time. Consequently, the fluid must exert an upward thrust or force on the body.

This upward force is called **buoyancy**, and is due to the fluid pressure exerted on the surface of the body. **Archimedes' principle** may be written as stating that

this upward, buoyant force on a body submerged in, or floating on, a fluid is equal to the weight of the fluid displaced by the body.

Consider a submarine. Its volume is fixed, but its weight can be varied by pumping water into, or out of, its ballast tanks. Consequently, its density can be varied at will, and it will submerge or rise to the surface as required. If its density is adjusted so that it equals that of the water, it will float "awash". By going ahead with its engines and depressing



Wt. of door
Fig. 29.13

its hydroplanes whilst in this condition, a downward force is set up, which will cause the submarine to dive under the water. If the engines are stopped whilst in this condition, the vessel will rise to the surface.

Similarly, a fish can vary its density, and so rise or sink in the water at will. In this case, however, its weight is fixed, but it varies its volume, and hence the volume of water displaced, by varying the size of its swimming bladder.

16. Equilibrium of floating bodies

A body submerged in a fluid is acted upon by two forces:

(1) Its weight acting vertically downwards (through the centre of gravity of the body).

(2) The vertically upward thrust of the fluid (buoyancy), equal in value to the weight of the fluid displaced.

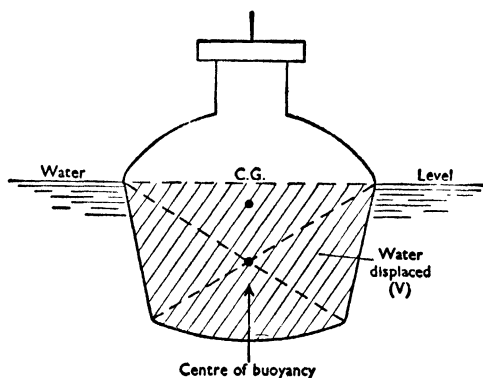


Fig. 29.14

Flotation will therefore occur when the weight of the body is equal to, or less than, this upward thrust due to the weight of the fluid displaced; and the body will rise in the fluid until forces (1) and (2) are equal and opposite.

In the case in which the weight of the body is less than the upward thrust on it when totally submerged, the body will rise and float partially immersed, so that the weight of the fluid *now* displaced (V in fig. 29.14) is equal to the weight of the body.

The centre of gravity of this displaced fluid is known as the **centre of buoyancy** of the body. Note that its position differs from that of the centre of gravity of the whole body. If this centre of buoyancy lies in the same vertical line as the centre of gravity of the whole body, then this body will be in a state of equilibrium.

17. Stability of ships

A floating body may be in a state of equilibrium, as just defined, but this equilibrium may be either stable, unstable, or neutral (see Chap. X, § 6). Obviously a ship must maintain a state of stable equilibrium, even when subjected to rolls of many degrees from the vertical, i.e. it must return to its original condition after a roll. Fig. 29.15 shows the cross-section of a

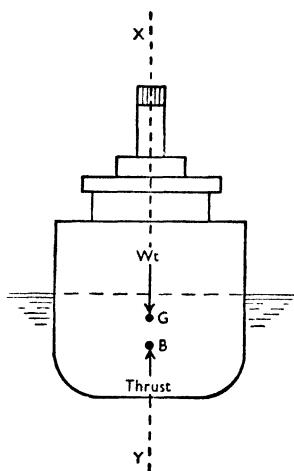


Fig. 29.15

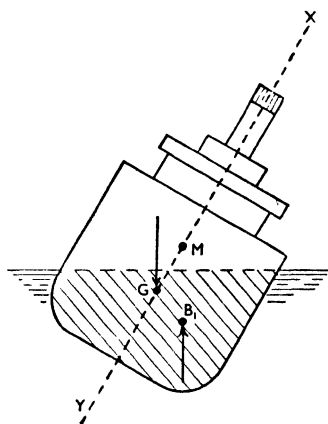


Fig. 29.16

ship. Its weight acts downwards through its centre of gravity (G), and the thrust of the water acts vertically upwards through the centre of buoyancy (B). Both of these forces are acting in the same vertical line, so the ship is in equilibrium. In fig. 29.16, the ship has rolled through several degrees. Its centre of gravity remains as before, but the centre of buoyancy, being the centre of gravity of the displaced water (the shaded portion), has now

shifted to B_1 . The vertical upward thrust through B_1 will now tend to turn the ship about its centre of gravity G in an anti-clockwise direction, i.e. it will create a turning moment tending to "right" the ship. Consequently, the ship is in a state of stable equilibrium.

In fig. 29.17, the ship is unladen and consequently its centre of gravity is higher than before.

The same amount of roll will now produce a clockwise turning moment, tending to overturn the ship. Consequently

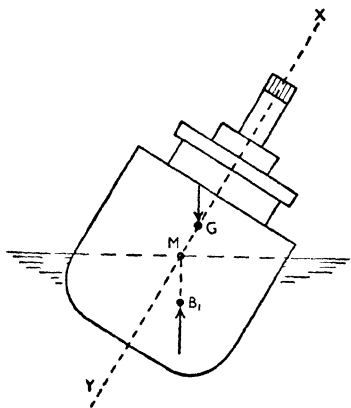


Fig 29.17

the ship is in a state of unstable equilibrium. Hence the necessity for "ballasting" an empty ship.

A vertical line drawn upwards from the centre of buoyancy (B_1) will cut the centre line XY at the point M . This point is termed the **metacentre**.

It will be noticed that in fig 29 16 the metacentre lies above the centre of gravity, whereas in fig 29 17 it is below. Hence, in order to obtain stable equilibrium in a floating body, its metacentre must be above its centre of gravity.

The distance between G and M is known as the **metacentric height**, and can be found experimentally by moving a heavy weight across the deck and measuring the resulting angle of heel.

EXERCISES ON CHAPTER XXIX

1. Describe any form of barometer and illustrate it by suitable diagrams. Explain the principles upon which the instrument depends and describe experiments to exemplify them. (C.P.E.)

2. Explain the principle of the hydraulic press. The pistons of such a press have diameters of 1 in. and 8 in. and it is worked by a lever whose effort arm is ten times as long as the resistance arm. A force of 10 lb. wt. is applied to the lever; what will be the force on the ram assuming 100 per cent efficiency? (C.P.E.)

3. State Archimedes' principle and describe an experimental verification of it. What weight of wire (S.G. = 9) must be wrapped round a cork weighing 8 g. (S.G. = 0.25), in order to just make it sink in water? (C.P.E.)

4. A straight wooden rod ABC of uniform section is floating with the part AB immersed in water. A vertical cord attached to C maintains the rod in an inclined position. Find the specific gravity of the rod when the immersed length $AB = 0.4 \times AC$, and determine the tension in the cord if the rod weighs 0.1 lb. Water weighs 62.4 lb./ft.³ (C.P.E.)

5. A U-tube with open ends has vertical limbs containing mercury. One limb A is connected to a vessel in which the air-pressure is maintained at 5 lb./in.² above atmospheric pressure while the other limb B remains open to atmosphere. Find by how much the mercury levels in the two tubes differ.

If water is now poured into limb B until the mercury level in A is 2 ft. below the free water surface in B, what is the difference in the mercury levels? Mercury weighs 848 lb./ft.³ and water 62.4 lb./ft.³ (C.P.E.)

6. A cube of 5 in. side and weighing $17\frac{1}{2}$ lb. is completely submerged in a vessel containing oil and mercury. The cube is partially immersed in each liquid with the upper face horizontal and below the free surface of the oil. Find how much of the cube is in each liquid. Specific gravity of oil 0.85, and of mercury 13.6. 1 cu. ft. of water weighs 62.4 lb. (C.P.E.)

7. A lock gate is subjected to water pressure on both sides. If the depths of water on the two sides are unequal, draw a diagram to show how the intensity of pressure on the gate varies with the depth.

If the gate is 12 ft. wide and the depth of water on one side is 8 ft., find the two depths possible on the other side of the gate when the resultant pressure on the gate is 10 tons. 1 cu. ft. of water weighs 62.4 lb. (C.P.E.)

8. A U-tube, with limbs vertical and of equal length and whose ends are open, contains mercury in the lower part; the level of the mercury is 28 in. below the open ends. The tube is of constant cross-section and has an internal diameter of $\frac{1}{4}$ in. Find the greatest volume of water that could be poured into one limb, and determine what would be the pressure at the common surface for a barometric pressure of 30 in. of mercury. Specific gravity of mercury = 13.6.

(C.P.E.)

9. Two identical vessels, one containing water and the other an equal weight of oil, are placed on the scale pans of a weighing machine so that the two pans are in balance. Two identical pieces of iron, each weighing 2 lb., are held in suspension in the liquids by fine wires, one piece of iron being in the water and the other in the oil. To which scale pan should an additional weight be added to make the pans balance and what is the magnitude of the weight?

Specific gravity of oil = 0.8, specific gravity of iron = 7.2, 1 cu. ft. of water weighs 62.4 lb.

(C.P.E.)

10. A vertical cylindrical water tank is 4 ft. in diameter and 8 ft. high. Find the total water pressure on the bottom and on the curved surface when the depth of water in the tank is 6 ft. (U.L.C.I.)

11. A water tank 10 ft. long and 8 ft. wide, with vertical sides and ends, has its base inclined at an angle of 30° to the horizontal in the direction of its length. The depth of the water in the tank at the shallow end is 4 ft. Find the total water pressure on the base of the tank, and state in what direction this pressure acts. (U.L.C.I.)

12. The centre of a circular door 3 ft. in diameter in the side of a tank is 40 ft. below the surface of the sea. Determine the intensity of pressure at this depth in pounds per sq. in. and the total pressure on the door. (1 cu. ft. of sea water weighs 64 lb.) (U.L.C.I.)

13. A hydraulic press has a hand-operated pump unit. The ram is 10 in. diameter and the plunger 1 in. diameter. The centre line of the plunger is 2 in. from the fulcrum of the hand lever, and an effort of 36 lb. is applied to this lever 30 in. from the fulcrum. Find the force on the ram, in tons, if the efficiency of the press is 85 per cent.

(U.E.I.)

14. A tank is of square section, base 4 ft. $\times$ 4 ft., and contains water to a depth of 8 ft. Calculate the total fluid pressure on each side of the tank and on the base of the tank. A vertical pipe leads downwards from the tank and is closed by a plug 1 in. in diameter. What is the pressure on the plug if it is placed 28 ft. below the base of the tank? (1 cu. ft. of water weighs 62.3 lb.) (U.E.I.)

15. The water in a dock is 24 ft. deep. Find the total force, in tons, on one foot-length of the wall. What will be the overturning moment on the wall about its base due to this pressure? (1 cu. ft. of sea water weighs 64 lb.) (U.E.I.)

16. The supply to a water turbine is controlled by a flat rectangular sluice gate 2 ft. wide, weighing 100 lb., and sliding in vertical guides. It is raised by a rope passing round a 5-in. diameter drum which is turned by a handle of effective radius 20 in. Water is on one side of the gate only to a depth of 3 ft. The coefficient of friction, gate to guides, is 0.20. Find the force required to be exerted at the handle to lift the gate. 1 cu. ft. of water weighs 62.5 lb. (U.E.I.)

17. A vertical rectangular surface is subjected to water pressure on one side. The rectangle is 6 ft. wide by 10 ft. deep and the top shorter edge is in the water surface. Find the total or resultant pressure on the rectangle. Determine the distance from the water surface of a line AB parallel to the water surface such that the resultant water pressures above and below the line are equal. 1 cu. ft. of water weighs 62.4 lb. (C.P.E.)

CHAPTER XXX

Hydrodynamics—Flow through an Orifice

1. Velocity of flow

The velocity with which a fluid will flow through any opening, nozzle, or orifice depends upon

- (a) the head, or height of the fluid above the orifice,
- (b) the shape of the orifice,
- (c) the frictional resistance to the flow.

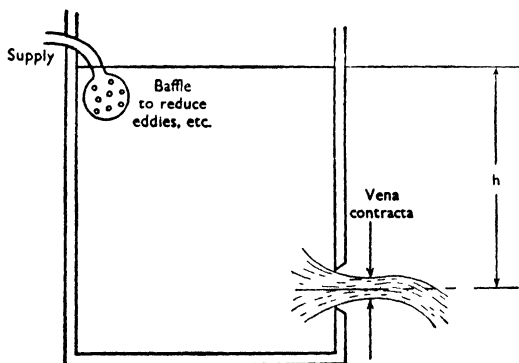


Fig. 30.1.—Flow through an orifice

The theoretical value of this velocity can be determined from considerations of kinetic energy.

Imagine a tank from which water flows through a sharp-edged opening, under the action of a head of h ft. (see fig. 30.1).

A quantity of W lb. of water at the surface level will possess a potential energy of Wh ft.-lb. This energy will be converted into kinetic energy in falling through h ft. to the orifice, the

amount of this kinetic energy being $Wv^2/2g$ ft.-lb. (see Chap. XXIII).

As potential energy = kinetic energy,

$$\text{then} \quad Wh = \frac{Wv^2}{2g},$$

$$\therefore v^2 = 2gh$$

$$\text{and} \quad v = \sqrt{2gh}.$$

If this water were directed vertically upwards through a suitable jet, whilst possessing this theoretical velocity, it would rise up to its original height of h ft.

The actual velocity of the water will, however, be less than the theoretical value, due to friction losses. The ratio

$$\frac{\text{actual velocity}}{\text{theoretical velocity}}$$

is called the **coefficient of velocity** (c_v), then

$$\text{actual velocity} = c_v \sqrt{2gh}.$$

2. Coefficient of contraction

It will be noticed from fig. 30.1 that a reduction in the cross-sectional area of the jet occurs on its leaving the orifice, the point of minimum cross-section being termed the **vena contracta**. The ratio

$$\frac{\text{cross-sectional area of jet}}{\text{cross-sectional area of orifice}}$$

is termed the **coefficient of contraction** (c_c).

3. Quantity of flow

The theoretical quantity of water which will flow through an orifice in unit time depends upon (1) the velocity (v), (2) the cross-sectional area of the orifice (A), i.e.

$$\text{quantity } (Q) \propto v \times A.$$

We have seen that the theoretical velocity is reduced by friction, so that actual velocity = $c_v \sqrt{2gh}$; also that the

cross-sectional area of the issuing jet may be less than that of the orifice, so that

actual cross-sectional area of jet = $c_c A$.

∴ actual quantity which will flow in unit time
 $= c_v \sqrt{(2gh)} \times c_c A$,

which is equal to $cA\sqrt{(2gh)}$, where c is the product of c_v and c_c , and is known as the **coefficient of discharge**.

N.B. It follows that the coefficient of discharge (c) is the ratio

$$\frac{\text{actual discharge}}{\text{theoretical discharge}}.$$

4. Coefficient of discharge

The value of c varies with the design of the orifice or nozzle, a few typical values being indicated in fig. 30.2.

	Circular sharp-edged orifice	External parallel mouthpiece	Internal (Borda's) mouthpiece	Streamlined mouthpiece
c_v	0.95	0.8	0.96	1.0
c_c	0.63	1.0	0.5	1.0
c	0.6	0.8	0.48	1.0

Fig. 30.2.—Orifices and nozzles

Notice the formation of a waist in the projecting pipe of the external parallel mouthpiece. The fluid jet, however, then expands and fills the pipe, as indicated by the value of c_c being unity.

Notice that in Borda's mouthpiece the area of the jet is half that of the full bore ($c_c = 0.5$), whereas, by careful design of the streamlined mouthpiece, maximum efficiency of discharge can be obtained, as indicated by its value of c approaching unity.

Example 1.—Water under a constant head of 10 ft. discharges through an orifice. Find its theoretical and actual velocity of discharge if the coefficient of velocity is 0.8.

Theoretical velocity $= v = \sqrt{2gh} = \sqrt{64.4 \times 10} = 25.4$ ft./sec.

Actual velocity $= c_v \sqrt{2gh} = 0.8 \times 25.4 = 20.32$ ft./sec.

Example 2.—If the orifice in Example 1 is of 2 in. diameter, and the coefficient of discharge is 0.65, find (i) the quantity discharged in cubic feet per second, and (ii) the kinetic energy expended per second.

(i) Volume discharged per second

$$= cA \sqrt{2gh} = 0.65 \times \frac{\pi}{144} \times \sqrt{644} = 0.36 \text{ cu.ft.}$$

(ii) K.E. expended per second

$$= \frac{Wv^2}{2g} = \frac{0.36 \times 62.3 \times 644}{64.4} = 224.3 \text{ ft.-lb.}$$

5. Energy due to pressure

We have seen that a fluid may possess potential and kinetic energy. It can also transmit energy due to pressure.

Consider one cubic foot of a fluid, of weight w lb. and under a pressure of P lb. per square foot, admitted to a cylinder and acting on a piston of area A sq. ft. Then

Force exerted on piston $= PA$ lb.

Distance moved by piston $= \frac{\text{volume}}{\text{area}} = \frac{1}{A}$ ft.

$\therefore$ Work done by 1 cu. ft. of fluid $= PA \times \frac{1}{A} = P$ ft.-lb.

But this work has been done by w lb. of fluid,

$\therefore$ Energy transmitted per lb. of fluid due to its pressure

$$= \frac{P}{w} \text{ ft.-lb.}$$

This pressure energy may be converted into kinetic energy by passing the fluid through a suitable nozzle (such as a fire hose), from which it issues with considerable velocity. Thus

pressure energy per lb. = kinetic energy per lb.

$$\frac{P}{w} = \frac{v^2}{2g}$$

and velocity (v) of jet = $\sqrt{\frac{2gP}{w}}$

6. Force exerted by a jet

If a jet of water hits an object and has its direction of motion changed, a force will be exerted on that object (Newton's First Law).

This force will be proportional to the rate of change of momentum and acts in the direction of the change of momentum (Newton's Second Law).

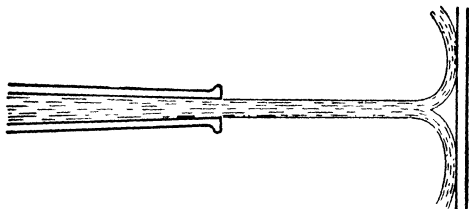


Fig. 30.3.—Impact of jet on fixed plate

The force exerted by the water on the object will set up an equal and opposite reacting force on the water (Newton's Third Law).

Imagine a jet to strike a fixed plate (fig. 30.3). The velocity and hence the momentum of the jet in the direction of the motion will become zero.

Thus, if v = original velocity (ft./sec.),

and $Q = \frac{W \text{ (lb.)}}{t \text{ (sec.)}} = \text{rate of flow (lb./sec.)},$

then the original momentum of the water issuing from the nozzle per second

$$= \frac{Q}{g} \times v,$$

and the change of momentum per second or the

$$\text{rate of change of momentum} = \frac{Qv}{g} - 0 = \frac{Qv}{g}.$$

This value will represent the force exerted on the plate in pounds (Newton's Second Law), and also the reacting force set up by the plate on the water (Newton's Third Law).

If the jet were to strike a fixed cup-shaped object (as shown in fig. 30.4), the direction of motion of the jet would be changed through 180° (i.e. from $+v$ to $-v$, or a change of $2v$).

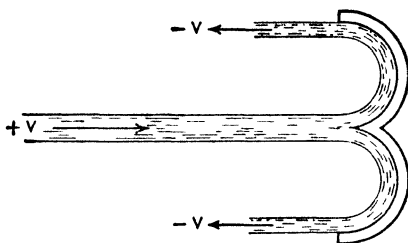


Fig 30.4.—Impact of jet on fixed bucket

If no losses occur, then the speed of the jet would remain unaltered, but its velocity would undergo a change of $2v$.

$\therefore$ Force exerted = rate of change of momentum

$$= \frac{W}{g} \times \text{rate of change of velocity} = \frac{Q}{g} \times 2v.$$

7. Water wheels and turbines

The force exerted by a fluid jet is utilized to develop power by operating various forms of water wheels, turbines, etc. In such cases the object (or objects) upon which the jet exerts a force are not fixed, but are mounted on the periphery of a wheel. The change of velocity on striking a series of flat vanes will be $(v - v_1)$, where v_1 is the velocity with which the wheel moves away. If all of Q reaches the vanes, then

$$\text{force exerted} = \frac{Q}{g} (v - v_1).$$

With cup-shaped vanes or buckets (like those in fig. 30.4) mounted on the wheel as in the **Pelton-wheel** type of water turbine, the force exerted (if all of the quantity of water Q reaches the vanes) will be

$$2 \frac{Q}{g} (v - v_1).$$

In most practical cases the jet does not either strike a vane squarely, or leave it after being turned through exactly 90° or 180° . It may approach and leave the vanes at any angle.

To cover such cases the general formula

$$\text{force exerted} = \frac{Q \times (\text{change of velocity})}{g}$$

may be used (if all the quantity Q reaches the vanes), the change of velocity being found vectorially.

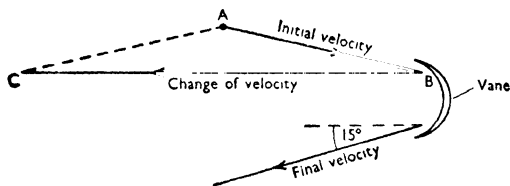


Fig. 30.5

Example 3.—A jet of water 1 in. in diameter strikes a smooth, fixed, cup-shaped vane at a velocity of 72 ft./sec. and at an angle of 15° to the normal from the centre of the vane. It is deflected and leaves at 15° to the normal in the opposite direction. Find the force exerted on the vane. (1 cu. ft. of water weighs 62.4 lb.)

$$Q = \text{area} \times 62.4 \times v = \frac{\pi \times 1 \times 1}{4 \times 144} \times 62.4 \times 72 = 24.5 \text{ lb./sec.}$$

The change of velocity can be found by direct measurement on the vector diagram (fig. 30.5).

AB represents the initial velocity of 72 ft./sec.

AC represents the final velocity after deflection by the vane.

As the vane is smooth this speed is still 72 ft./sec. BC then represents the change of velocity.

By measurement $BC = 140$ ft./sec.

By calculation $BC = 2(72 \cos 15^\circ) = 139.1$ ft./sec.

$$\therefore \text{force exerted} = \frac{24.5}{g} \times 139.1 = 105.9 \text{ lb.}$$

EXERCISES ON CHAPTER XXX

1. Explain the terms coefficient of velocity and coefficient of discharge, when applied to the flow of water through orifices and nozzles.

A certain water supply is measured by passing it through a tank with a constant head of 25 ft. and discharging it through a sharp-edged orifice 2.5 in. in diameter. Find the velocity of discharge and the quantity of water discharged in gallons per hour. Assume the coefficients of discharge and of velocity to be 0.62 and 0.95 respectively. (U.L.C.I.)

2. A jet of water 1 in. diameter issues from a nozzle under a head of 40 ft. Find (a) the velocity of the water leaving the nozzle, (b) the weight of water issuing per second, and (c) the amount of kinetic energy leaving the nozzle per second. (U.L.C.I.)

3. Water discharges from a sharp-edged circular orifice in the vertical side of a tank. The diameter of the orifice is 0.3 in., and the head of water above the centre of the orifice is kept constant at 4 ft. Determine the theoretical quantity of water discharged in cubic feet per minute. If the actual discharge is 0.28 cu. ft. per min., what is the coefficient of discharge for the orifice? (U.E.I.)

4. A sharp-edged opening in a tank is 1 sq. in. in area and its centre is 5 ft. below the constant level of water in the tank. Water flows through the opening at the rate of 6 cu. ft. per min. Find the coefficient of discharge for the opening. (U.E.I.)

5. Explain the meaning of the expression "coefficient of discharge of an orifice". An orifice 1 in. in diameter discharges 19.6 gal. of water per minute under a head of 4 ft. Calculate the coefficient of discharge of the orifice. State how you would measure the discharge when making an experiment. (U.L.C.I.)

6. A jet of water $\frac{1}{2}$ in. in diameter issuing from a nozzle under a head of 20 ft. impinges normally on a flat stationary disc. Find (a) the weight of water issuing from the nozzle per second, (b) the momentum of the water issuing from the nozzle per second, and (c) the pressure exerted by the water on the disc. (U.L.C.I.)

7. Water is discharged through a nozzle of 1 in. diameter with a velocity of 50 ft. per sec. Calculate the reactive force of the jet.

(I.E.E.)

8. Calculate the force exerted by a jet of water, 1 in. in diameter, when it strikes a flat surface normally, with a velocity of 30 ft. per sec. Assume that the velocity of the jet, in the direction of motion, is completely destroyed, and that 1 cu. ft. of water weighs 62.3 lb.

(U.E.I.)

9. A bowl is suspended from a spring balance. Water is falling from a tap in a steady stream straight into the bowl. The sectional area of the stream is 1 sq. in. and the maximum velocity attained by it is 10 ft. per sec. Find the weight of water reaching the bowl per second, and its momentum, given that 1 cu. ft. of water weighs 62.3 lb. Does the spring balance record the correct weight of water in the bowl at any instant? If not, what is the amount of the error in pounds weight?

(U.E.I.)

10. Explain the meaning of the terms work, energy, power.

An engine pumps water from a well 22 ft. deep and discharges it with a velocity of 64 ft. per sec. Find the horse-power developed by the engine if the water is delivered at a rate of 800 gal. per min. (1 gallon of water weighs 10 lb., 1 h.p. = 550 ft.-lb./sec., $g = 32$ ft./sec.²).

(C.P.E.)

11. A jet of water issues from a nozzle of 1 in. diameter with a velocity of 80 ft. per sec. Determine: (i) the kinetic energy of each pound of water flowing through the nozzle; (ii) the weight of water passing through per second; (iii) the horse-power available from the jet. 1 cu. ft. of water weighs 62.4 lb.

(C.P.E.)

TABLE 6.—PROPERTIES OF SATURATED STEAM

Press. lb./in. ² abs	Temp. °F.		Heat in B.Th.U. per lb.			Vol. of 1 lb. dry sat. steam, cu. ft.	Heat in C.H.U. per lb.		Temp. °C.
	<i>t</i>	<i>h</i>	<i>L</i>	<i>H_g</i>	<i>V_g</i>		<i>h</i>	<i>H_g</i>	<i>t</i>
0.5	79.6	47.6	1048.5	1096.1	643.0		26.45	608.95	26.42
0.75	92.3	60.3	1041.5	1101.8	437.3		33.51	612.10	33.52
1.0	101.7	69.7	1036.1	1105.8	334.0		38.74	614.34	38.74
2.0	126.1	94.0	1022.2	1116.2	173.7		52.22	620.11	52.27
4.0	153.0	121.0	1006.7	1127.7	90.63		67.20	626.49	67.23
5.0	162.3	130.2	1001.6	1131.8	73.52		72.36	628.60	72.38
6.0	170.1	138.1	996.6	1134.7	61.98		76.71	630.33	76.72
8.0	182.9	151.0	988.5	1139.5	47.35		83.89	633.05	83.84
10.0	193.2	161.3	982.5	1143.8	38.42		89.61	635.43	89.58
14.69	212.0	180.1	970.6	1150.7	26.80		100.06	639.28	100.00
15	213.0	181.2	970.0	1151.2	26.28		100.65	639.5	100.57
18	222.4	190.6	964.0	1154.6	22.17		105.94	641.4	105.78
20	228.0	196.3	960.4	1156.7	20.09		109.05	642.6	108.87
30	250.3	219.0	945.6	1164.6	13.73		121.7	647.1	121.3
40	267.2	236.1	934.4	1170.5	10.50		131.2	650.4	130.7
50	281.0	250.2	924.6	1174.8	8.516		139.0	652.8	138.3
60	292.7	262.2	916.2	1178.4	7.175		145.6	654.8	144.9

80	312.0	282.1	901.9	1184.0	5.472	156.7	657.8	155.6
100	327.8	298.5	889.7	1188.2	4.434	165.8	660.1	164.4
120	341.3	312.5	878.9	1191.4	3.729	173.6	661.9	171.8
130	347.3	318.8	874.0	1192.8	3.456	177.1	662.7	175.2
140	353.0	324.9	869.1	1194.0	3.222	180.5	663.4	178.3
150	358.4	330.6	864.5	1195.1	3.015	183.7	664.0	181.3
160	363.6	336.6	860.1	1196.1	2.835	186.7	664.6	184.2
170	368.4	341.2	855.9	1197.1	2.677	189.5	665.2	186.9
180	373.1	346.1	851.9	1198.0	2.534	192.3	665.6	189.5
190	377.5	350.9	847.9	1198.8	2.407	194.9	666.0	192.0
200	381.8	355.5	844.0	1199.5	2.290	197.5	666.4	194.3
220	389.9	364.2	836.5	1200.7	2.085	202.3	667.2	198.8
250	401.0	376.1	826.0	1202.1	1.844	209.0	668.2	204.9
300	417.3	394.0	809.8	1203.8	1.543	219.0	668.9	214.1
350	431.7	409.9	795.0	1204.6	1.326	227.7	669.4	222.1
400	444.6	424.2	781.3	1205.5	1.161	235.8	669.6	229.2
450	456.3	437.4	768.2	1205.6	1.032	243.0	669.8	235.8
500	467.0	449.6	755.8	1205.4	0.928	249.8	669.7	241.7
600	486.2	471.8	732.4	1204.2	0.770	262.1	669.1	252.3
800	518.2	509.9	689.7	1199.6	0.569	283.3	666.4	270.1
1000	544.6	542.6	650.2	1192.8	0.446	301.4	662.7	284.8
2000	635.8	671.8	464.3	1136.1	0.186	373.2	631.2	335.4
3000	695.4	802.3	214.1	1016.4	0.086	446.2	564.7	368.6

• Extracted from the "Abridged Callendar Steam Tables", by permission of the publishers, Messrs. Edward Arnold & Co.

ANSWERS TO EXERCISES

CHAPTER II (p. 19)

3. (a) 14.19 sq. in. (b) $4\frac{1}{8}$ sq. in. 4. (a) 1016 kg. (b) 4828 m.
5. (a) 26.82 m./sec. (b) 7.031 kg/cm.² 6. (a) 20,160. (b) 352.
7. 5280 sq. cm. 8. 9.25 sq. in. 9. 7.42 sq. in. 10. 32.73 m.p.h.
14.62 m./sec.

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CHAPTER VI (p. 67)

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part. 7. 32.1 lb., 17° 39' W. of N. 8. 36° E. of N., 127 miles.
9. 11.66 lb. wt. at 59° from 6 lb. force. 10. 0.868 and 4.924 tons.
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string — 10.4 lb.

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1. 148.5 lb. 2. 540 lb. 3. R.H. $13\frac{1}{2}$ tons., L.H. $11\frac{1}{2}$ tons.
 4. 21 lb. in each. 5. 58.1 lb. 6. (a) $R_A = 93.88$ lb., $R_B = 102.12$ lb.
 (b) 196 lb. 8.86 ft. from A. 7. L.H. = 3.733 tons., R.H. 3.267 tons.
 8. 95 lb.-ft. 4.75 ft. from fixed end. 9. 14.76 lb.-in. anticlockwise,
 7.38 in. from A clockwise. 10. $\frac{1}{3}$ lb. anticlockwise. 11. (a) 15.4 lb.
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 2. Jib, 3.46 tons (comp.); Tie, 2 tons (tens.). 3. Yes; 7.72 lb. wt. tensile load in cord. 4. Jib, 23.2 tons (comp.); Tie, 16.9 tons (tens.).
 5. 0.654 tons in each. 6. $X = 16.9$ lb.; $R = 36\frac{1}{4}$ lb. 7. 0.74 and 0.89 ton (comp.); 0.62 ton (tens.); $R = \frac{3}{8}$ ton; $S = \frac{5}{8}$ ton.
 8. BD and DA = 125 lb. (comp.); DC = 433 lb. (tens.). 9. 30 lb. each. 10. 36 lb. 11. Jib, 173.2 lb. (comp.); Tie 100 lb. (tens.).
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 8. 77.76 ft.-lb. 9. 0.051 h.p. 10. (a) 192.7 lb. (b) 0.6306 h.p.
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 9. 120 lb.; 112.5 ft.-tons (approx.). 10. (i) 6,125,000 ft.-lb. (ii) 2.32 theoretical, 3.62 actual. (iii) 12.7 lb.-ft. (iv) 13.5 amp. 11. (i) 55.3 lb./in.² (ii) 11 in. diam. 12. 7.2 h.p.; 4.48 h.p. 13. 3.125 lb.; 9.44 m.p.h. 14. 638,400 ft.-lb.; 23.2 h.p.; 210,000 ft.-lb.
 16. 30.49 h.p. 17. (a) 50.6 amp. (b) 1 s. 10.77 d. 18. 11.7 lb.; 996.5 h.p.

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1. 75 per cent. 2. $66\frac{1}{3}$ lb.-ft. 3. 3.142 tons. 4. (a) 6.86. (b) 6.174. (c) 19.4 lb. 5. (i) 5. (ii) 50 ft.-lb. (iii) 75 per cent.
 6. 226; $1\frac{1}{3}$ lb. 7. M.A. = V.R. = 8; M.A. = 5.6, V.R. = 8; 70 per cent. 8. V.R. = 176, M.A. = $74\frac{2}{3}$, Eff. = $42\frac{1}{2}$ per cent.

10. 700 lb.; 35 per cent. 11. 38.9 per cent; 17,320 ft.-lb.
 12. 20.83 lb.; 4.17 lb. 13. 4.07 b.h.p. 14. 2.82 tons. 15. (i) and
 (ii) 2240 ft.-lb.; 30 b.h.p. 16. (a) 8.689 b.h.p. (b) 9.439 i.h.p.
 17. 11.05 i.h.p.; 113 lb.-ft. 18. (a) 89.4 per cent. (b) 108.6 lb.-ft.
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1. $E = 0.125W + 0.75$. 2. $8\frac{1}{2}$ lb. 4. $133\frac{1}{2}$ lb.
 6. (a) $E = 0.202W + 1.9$. (b) 8.77 lb.; 3.88; $24\frac{1}{4}$ per cent.
 7. $E = 0.064W + 0.3$; 84.9 per cent. 8. No. 10. V.R. = 3;
 25,200 ft.-lb./min. 11. 31.74 per cent. 12. V.R. = 3. (i) 55.6 lb.
 (ii) 95.2 lb.; $a = 0.397$; $c = 15.9$. 13. $E = 0.0088L + 10.5$;
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1. 20 r.p.m. Reversing motion. 2. V.R. = 22.5; 30 lb.; friction
 25 per cent. 3. $16\frac{2}{3}$ in.; motor $763\frac{7}{11}$ r.p.m.; machine $458\frac{2}{11}$ r.p.m.
 4. $13\frac{1}{2}$ r.p.m. 5. 616 r.p.m.; 78.6 r.p.m. 6. 34.4 in.; $229\frac{1}{2}$ r.p.m.
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 481.3 lb.; 1.6 sq. in. 10. 268.1 h.p. 11. 573 lb.

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1. Links 25,150 lb.; plunger 24,600 lb. 2. AD = 15.1 lb.;
 AB = 26.2 lb.; BC and DC = 30.2 lb., all tension; AC = 30.2 lb.
 thrust. 3. AB = 8.84 tons (comp.); BC = 6.25 tons (tens.).
 4. Lettering clockwise from L.H. side (Bow's notation), BF = 1.73,
 FA = 3.46, EF = 1.0, CE = 5.2 cwt., all compression; DE = 2.6
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 BD 1840 lb. (tens.). 7. $W = 32.6$ lb., $R = 48.2$ lb. at 86° (approx.)
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 10. 1.3 tons; 3.87 tons at 71° to horizontal. 11. $54\frac{1}{2}^\circ$; AC = 58.65
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 13. Ties—top and middle, 600 and 584 lb.; struts—848 and 1170 lb.
 14. 24.05 lb. 15. 25 lb. 16. 5.29 ft.; 13.22 lb. 17. $11\frac{1}{2}$ lb.;
 AB 25 lb., BC 15 lb., CA, $18\frac{3}{4}$ lb. 18. 16° to vertical; 104 lb.

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1. $R_O = 1\frac{1}{8}$ tons, $R_E = 4\frac{1}{8}$ tons. 2. (a) 7.28 and 4.47 tons
 (b) 78.86 tons-ft. (c) 3.03 tons. 3. 2160 lb.-ft. at 300-lb. load.
 5. B.M. 3780 lb.-ft.; S.F. 630 lb. 6. (a) 9 lb. each; 108 lb. in.;
 90 lb.-in. 7. $R_A = 16.5$ tons, $R_B = 17.5$ tons; max. B.M. occurs at
 21 t. load; max. B.M. 87.5 tons-ft.; S.F. changes from 3.5 to -17.5
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1. (d) $271\frac{1}{3}$ ft./sec. (e) 15 m.p.h. 2. 2.32 ft./sec.²; $22\frac{8}{11}$ sec.
 3. 120 ft. 4. 50 m.p.h.; 0.98 ft./sec.² 5. 5187 ft. 6. 38.5 m.p.h.;
 6 ft./sec.² 7. 0.004465 m.p.h.; 0.393 ft. 8. $1\frac{1}{2}$ rev./sec.²; $100\frac{1}{3}$ rev.
 9. 5 min.; $2\frac{2}{3}$ miles. 10. 149 ft. 11. 52.36 ft./sec.; 3142 ft.
 12. 1 mile. 13. (c) (i) 21.9 ft./sec. or 14.94 m.p.h. (ii) 389 ft.
 14. (a) 15,625 ft. (b) $31\frac{1}{2}$ sec. (c) 15,273.4 ft. 15. (a) 0.7866 ft./sec.²
 (b) 225 r.p.m. 16. 35 m.p.h.; 1 min. 43 sec.; 0.393 m. from 1st
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 19. (a) 4.034 ft./sec.² (b) $5\frac{5}{11}$ sec. (c) 4.03 radn./sec.² 20. 420 cm.
 21. 116 ft./sec.; $1\frac{1}{4}$ sec. 22. 1110 ft./sec. 23. (a) $2\frac{1}{2}$ sec. (b) 100 ft.
 24. $28^\circ 6'$ S. of E. 25. 14.73 sec.; 33.6 per cent.

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1. 1.166 ft./sec.² 2. (a) 5 metres/sec.² (b) 75 metres/sec. (c) 562.5
 metres. 3. (a) 18,266 lb. (b) 2922.6 h.p. 5. (a) 137,984 units.
 (b) 59.2 sec. 6. (a) $196\frac{1}{8}$ cm./sec.² (b) 24 gm. wt. (c) 392.4 cm.
 7. 6824 lb.; 3 ft./sec.; 9 ft. 8. 675 poundals, 20.95 lb. wt. 9. $16\frac{2}{3}$ ft./
 sec.²; 7.764 lb. 10. 8.049 ft. 11. $1\frac{1}{3}$ ft./sec.² 12. 1.12 sec.; 3.11 lb.
 13. 1320 h.p.; 107.44 + tractive resistance (18 lb.). 14. 0.261.
 15. 4 sec. 16. 138.4 lb.; 15.82 h.p. 17. 4709 lb. 18. 28,070 ft.;
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1. 3850 lb. 2. 103.8 r.p.m.; $18\frac{1}{4}$ lb.; $8\frac{1}{4}$ lb. 3. v ; $(3/\pi)(v^2/r)$ directed towards centre. 4. 195.9 lb.; 24 lb. 5. 83.2 lb.; 6800 and 6637 lb. 6. 2.92 in. 7. 18.15 tons. 8. 39.2 cwt. 9. $AZ = 11.17$ in.; angle $XAZ = 96^\circ$. 10. 32.7 lb. 11. 347.5 lb.; $C = 16.31$ lb. at 259° clockwise from AO . 12. 12.69 ft./sec.; 18 in.

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1. 12.8° C., 192.2° F. 2. (a) (i) $15\frac{5}{8}^\circ$ C. (ii) 17.8° C. (iii) 45° C. (iv) -20° C. (b) (i) 98.6° F. (ii) 2912° F. (iii) -38.02° F. (iv) 39.2° F. 3. 2.834 in. 4. Half = 66.6; three-quarters = 44.4; end = 33.3 lb./in.² 5. 10.21 cu. ft. 6. 454.4 cu. ft.; 426.3 cu. ft. 8. 24.3 atmospheres. 9. 980 cu. metres. 10. (a) 63.275° C. (b) 95.724° C. 11. (i) 0.0983 cm.²/sec. (ii) 0.492 cm.³/sec. 12. 124.5° C. 13. 90.42 cu. mm. 16. 0.0101 cu. ft.

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